UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Facultat de Matemàtiques i Estadística

E2

Full 1 de

QUALIFICACIÓ
9

Assignatura: Àlgebra

Estudiant/a: [redacted]

Data: 8/1/21

1] $V = \dim 2$ $\quad W = \dim 2$ $\quad v = (0,0,0,1,1) \in V \cap W$

A $\mathbb{R}^5/V$ $\quad (V+W)/V$ $\quad u_1 = V$ $\quad u_2 = (1,0,0,0,1)$ $\quad u_3 = (2,1,0,0,1)$

$u_4 = (1,0,0,1,2)$

a) $\dim V+W = \dim V + \dim W - \dim V \cap W$

$$\begin{pmatrix} 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 2 \end{pmatrix} \underset{f_5 \to f_5 - f_4}{\sim} \begin{pmatrix} 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \underset{f_1 \to f_1 - f_5}{\sim} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \underset{f_2 \to f_2 - f_1}{\sim}$$

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

$z = 0$
$x = -t = y$

Base $[(1,0,1,-1)]$ de $\mathbb{R}^5/V$

~~$\dim (V+W)/V = 1$~~

$\boxed{\dim V \cap W = 1}$?

~~$\Rightarrow \dim V+W = \dim V + \dim W - \dim V \cap W = 2+2-1 = 3$~~

$\boxed{\dim (V+W)/V} = \dim(\text{Base } \mathbb{R}^5/V) = \boxed{1} = \dim V + \dim W - \dim V \cap W - \dim V$?

$= \dim W - \dim V \cap W = 2 - 1 = 1$

sigui $V = (a_1, \dots, v_1, \dots, a_n)$, $W = (b_1, \dots, v_1, \dots, b_n)$

$\boxed{\text{Base } V+W = [(a_1, \dots, v_1, \dots, a_n, \dots, b_1, \dots, v_1, \dots, b_n)]}$?

b) $u_3 - u_2 - 7v \in V$ ~~doneu-ne explícita~~ B^* de $(V+W)/V$

Sigui $B = [(1, 0, 1, -1)]$ i $u_3 - u_2 - 7v = (1, 1, 0, -7, -7) \in V$

$1 = a\alpha + b\beta - 7c\gamma - 7d\delta$

$0 = a\alpha$

$1 = x\alpha + z\beta - \gamma\delta$

$1 = x\alpha + z\gamma$

$0 =$

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Ex 1)

b) Sigui $B = [(1,0,1,-1)] = [e_1]$ $\quad B^* = [e_1^*]$

i $w_1 = v$, $w_2 = (1,0,0,0,1)$, $w_3 = (2,1,0,0,1)$, $w_4 = (1,0,0,1,2)$

~~$(a,b,c,d) = (1,0,1,-1) = \alpha(0,0,0,1,1) + \beta(1,0,0,0,1) + \gamma(2,1,0,0,1) +$~~
~~$+ \sigma(1,0,0,1,2) = (\beta + 2\gamma + \sigma, \gamma, 0, \alpha + \sigma, \alpha + \beta + \gamma + 2\sigma)$~~

~~$\beta + 2\gamma + \sigma = 1$~~
~~$\gamma = 0$~~
~~$0 = 1$~~

??

$(1,0,1,-1) = (a, 0, c, -d) = \alpha(0,0,0,1,1) + \beta(1,0,0,0,1) + \gamma(2,1,0,0,1) + \sigma(1,0,0,1,2)$

$$\left.\begin{array}{l} a = \beta + 2\gamma + \sigma \\ 0 = \gamma \\ c = 0 \\ -d = \alpha + \beta + \gamma + 2\sigma \end{array}\right| \quad \begin{array}{l} a = \beta + \sigma \\ -d = \alpha, \beta + 2\sigma \Rightarrow d = -\alpha - \beta - 2\sigma \end{array}$$

$$\begin{pmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -2 \end{pmatrix}$$

β^*

$e_1 = e_1 \Rightarrow e_1 = 1$

$f(e_1^*)_{e_1} = (\beta + \sigma)\, e_1 = \beta + \sigma$

$f(e_1^*)_{e_2} = (-\alpha - \beta - 2\sigma) e_2 = 0$

$e_2 = 0$ perquè $e_1 \neq e_2$

$\boxed{B^* \{(0,1,0,1)\}}$

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ex 2)

a)

$$f^n(u) = a_0 u + a_1 f(u) + \dots + a_{n-1} f^{n-1}(u)$$

$$m_f(x) = x^n - a_{n-1} x^{n-1} \dots - a x - a_0$$

$$f^n(u) = a_0 u + a_1 f(u) + \dots + a_{n-1} f^{n-1}(u) =$$

$$= a_0 (x^0, \dots) + a_1 (x^0, x^1, \dots) + \dots + a_{n-1} (x^0, \dots, x^{n-1})$$

què vol dir això?

$$A = \begin{pmatrix} a_0 & a_1 & \cdots & a_{n-1} \\ 0 & a_1 & & \vdots \\ \vdots & & \ddots & a_{n-1} \\ 0 & \cdots & 0 & \end{pmatrix}$$

NO $\longrightarrow M_B(f) = \begin{bmatrix} 0 & 0 & \cdots & a_0 \\ 1 & 0 & \cdots & a_1 \\ 0 & 1 & & \vdots \\ \vdots & \vdots & & \\ 0 & 0 & \cdots & a_{n-1} \end{bmatrix}$

$$P_f(x) = \begin{vmatrix} a_0 - x & a_1 & \cdots & a_{n-1} \\ 0 & a_1 - x & & \vdots \\ \vdots & & \ddots & a_{n-1} \\ 0 & \cdots & 0 & a_{n-1} - x \end{vmatrix} = \prod_{i=0}^{n-1} (a_i - x)$$

$\boxed{0.2}$

$m_f(x) \mid P_f(x) \iff P_f(x) = m_f(x) \cdot q(x) + r(x)$ on $r(x) = 0$

b) $\det f = 0 \iff f^n \in [f(u), \dots, f^{n-1}(u)]$

$$\det \begin{vmatrix} a_1 & \cdots & a_{n-1} \\ a_1 & & \vdots \\ 0 & \ddots & \vdots \\ \vdots & & \\ 0 & 0 & a_{n-1} \end{vmatrix} \underset{f_2 \to f_2 - f_1}{=} \begin{vmatrix} a_1 & \cdots & a_{n-1} \\ 0 & \cdots & 0 \\ 0 & a_2 & a_{n-1} \\ & 0 & \\ 0 & 0 & a_{n-1} \end{vmatrix} = 0$$

$\exists$ una fila tota de zeros

Aquesta NO és la matriu que has escrit a l'apartat a)

Aquesta matriu és quadrada?

$$P_f(\lambda) = \begin{vmatrix} a_2 & \cdots & a_{n-1} \\ a_2 & & \vdots \\ a_2 - \cancel{X} & & \vdots \\ 0 & \ddots & \\ 0 & 0 & a_{n-1} - X \end{vmatrix} = \prod_{i=2}^{n-1} a_i - X \quad \text{on } \lambda_i = a_i \quad a_{\lambda_i} = 1$$

Tenim i VAPs de multiplicitat alg. 1, anem

?

~~Per λ_i~~ a veure la multiplicitat geomètrica per saber si f diagonalitza segons el teorema de diagonalització que diu que

$$f \text{ diagonalitza} \iff \begin{cases} - f \text{ descompon totalment a } K \\ - g_{\lambda_i} = a_{\lambda_i} \end{cases}$$

Per λ_i tenim

$$(A - \lambda_i I) = \begin{pmatrix} a_2 & \cdots & a_{n-1} \\ a_2 & & \vdots \\ (a_2 - a_i) & & \\ 0 & \ddots & 0 \\ 0 & & \\ 0 & 0 & (a_{n-1} - a_i) \end{pmatrix} \begin{matrix} \Rightarrow a_{2_i} - a_i = 0 \\ \\ \Rightarrow a_{n-3} = 0 \\ \Rightarrow a_{n-2} = 0 \\ \Rightarrow a_{n-1} = a_i \end{matrix} \Bigg\} \; (0, \dots, 0)$$

$\dim \underbrace{\mathrm{Nuc}(A - \lambda_i I)}_{\{\vec{0}\}} = g_{\lambda_i} \neq a_{\lambda_i} \Rightarrow f$ no diagonalitza

Si λ_i és VAP, $\dim \mathrm{Nuc}(A - \lambda_i I d) \geq 1$!!!

$\boxed{0.2}$

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ex 21

c) f és invertible $\Leftrightarrow$ f és un isomorfisme $\Leftrightarrow$ f és bijectiva

f és bijectiva $\Leftrightarrow$ f és exhaustiva $\wedge$ f injectiva.

f injectiva $\Leftrightarrow \mathrm{Nuc}\, f = \{\vec{0}\}$

f exhaustiva $\Leftrightarrow \dim \mathrm{Im} f = \dim \mathbb{R}^n = n$

$$\begin{pmatrix} a_0 & \cdots & a_{n-1} \\ & \ddots & \vdots \\ & & a_{n-1} \end{pmatrix}^{-1} = \frac{1}{\prod_{i=0}^{n-1} a_i} \cdot \begin{pmatrix} a_0 & 0 & \cdots & 0 \\ \vdots & \ddots & & 0 \\ a_{n-1} & \cdots & & a_{n-1} \end{pmatrix}^{*} =$$

$$= \frac{1}{d} \cdot \begin{pmatrix} (-1)^{i+j} \cdot d/a_0 & 0 & \cdots & 0 \\ \vdots & \ddots & & 0 \\ (-1)^{i+j} d/a_{n-1} & \cdots & & (-1)^{i+j} d/a_{n-1} \end{pmatrix} =$$

sigui $d = \prod_{i=0}^{n-1} a_i$

$$= \boxed{\begin{pmatrix} (-1)^{i+j} \frac{1}{a_0} & 0 & \cdots & 0 \\ \vdots & \ddots & & \vdots \\ (-1)^{i+j} \frac{1}{a_{n-1}} & \cdots & (-1)^{i+j} & \frac{1}{a_{n-1}} \end{pmatrix}}$$ on $a_i \neq 0$.

Res de tot això el pertany al que demana l'enunciat.
Has de fer servir el teorema. Cayley-Hamilton.

$\boxed{0.1}$

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$E = \mathbb{R}_2(x) \Rightarrow n = 2$

ex3] Sigui $p(x) = p_n x^n + p_{n-1} x^{n-1} + \dots + p_1 x + p_0$ i $q(x)$ sigui

$q(x) = q_n x^n + q_{n-1} x^{n-1} + \dots + q_1 x + q_0$

0,25

Aquí el mig falta la matriu M del prod. escalar.

a) $\langle p(x), q(x) \rangle = (p_n x^n \; p_{n-1} x^{n-1} \cdots p_1 x \; p_0) \begin{pmatrix} q_n x^n \\ q_{n-1} x^{n-1} \\ \vdots \\ q_1 x \\ q_0 \end{pmatrix} = \begin{pmatrix} p_n q_n x^{2n} \\ p_{n-1} q_{n-1} x^{2n-2} \\ \vdots \\ p_1 q_1 x^2 \\ p_0 q_0 \end{pmatrix} = p_n q_n x^{2n} + \dots + p_1 q_1 x^2 + p_0 q_0$

~~$p(x) q(x) = p_n (q_n x^{2n} + q_{n-1} x^{2n-1} \dots q_0)$~~

~~$\int_{-1}^{1} p(x) q(x)\, dx = \int_{-1}^{1} p_n (q_n x^{2n} + q_{n-1} x^{2n-1} + \dots + q_1 x^{n+1} + q_0 x^n) + \dots + p_1 (q_n x^{n+1} + \dots + q_1 x^2 + q_0 x)$~~

~~$+ p_0 (q_n x^n + \dots + q_1 x + q_0)\, dx = p_n \int_{-1}^{1} q_n x^{2n} + q_{n-1} x^{2n-1} + \dots + q_1 x^n \, dx + \dots + p_0 \int_{-1}^{1} q$~~

1- def. positiva? $\Leftrightarrow$ ~~$\langle p(x), q(x) \rangle \geq 0 \wedge \langle p(x), q(x) \rangle = 0 \Leftrightarrow$~~

$\langle p(x), p(x) \rangle \geq 0$

Sigui $\langle p(x), p(x) \rangle = p_n^2 x^{2n} + \dots + p_1^2 x^2 + p_0^2$, tots els termes del producte escalar

són positius, per tant $\langle p(x), p(x) \rangle \geq 0$.

i és $\langle p(x), p(x) \rangle = 0 \Leftrightarrow p(x) = 0$

2- simètrica? $\Leftrightarrow \langle p(x), q(x) \rangle \stackrel{?}{=} \langle q(x), p(x) \rangle$

$(p_n x^n \cdots p_1 x \; p_0) \begin{pmatrix} q_n x^n \\ \vdots \\ q_1 x \\ q_0 \end{pmatrix} = (q_n x^n \cdots q_1 x \; q_0) \begin{pmatrix} p_n x^n \\ p_{n-1} x^{n-1} \\ \vdots \\ p_1 x \\ p_0 \end{pmatrix} = p_n q_n x^{2n} + \dots + p_0 q_0 = q_n p_n x^{2n} + \dots + q_0 p_0$

on $p_i q_i = q_i p_i$ per $i = 0 \dots n \Rightarrow$ és simètrica
commutem

3- bilineal? $\bullet \langle a_1 p_1 + a_2 p_2, q\rangle = a_1 \langle p_1, q\rangle + a_2 \langle p_2, q\rangle$

$$\langle a_1 p_1(x) + a_2 p_2(x), q(x)\rangle = a_1 \langle p_1(x), q(x)\rangle + a_2 \langle p_2(x), q(x)\rangle =$$

$$= a_1 \cdot \begin{pmatrix} p_{n_1} x^n & \cdots & p_{1_1} x & p_{0_1} \end{pmatrix} \begin{pmatrix} q_n x^n \\ \vdots \\ q_1 x \\ q_0 x \end{pmatrix} + a_2 \begin{pmatrix} p_{n_2} x^n & \cdots & p_{1_2} x & p_0 x \end{pmatrix} \begin{pmatrix} q_n x^n \\ q_{n-1} x^{n-1} \\ \vdots \\ q_1 x \\ q_0 \end{pmatrix} =$$

$\langle a_1 p_1 + a_2 p_2, q\rangle$

$$= \begin{pmatrix} (a_1 p_{n_1} + a_2 p_{n_2}) x^n & \cdots & (a_0 p_{0_1} + a_2 p_{0_2}) \end{pmatrix} \begin{pmatrix} q_n x^n \\ \vdots \\ q_0 \end{pmatrix} = (a_1 p_{n_1} + a_2 p_{n_2}) q_n x^{2n} + \cdots + (a_1 p_{1_1} + a_2 p_{1_2}) q_1 x^2 +$$

$$a_1 p_1 + a_2 p_2 = (a_1 p_{n_1} x^n + a_1 p_{n-1_1} x^{n-1} + \cdots + a_1 p_0) + (a_2 p_{n_2} x^n + \cdots + a_2 p_{1_2} x + a_0 p_{0_2}) =$$

$$= (a_1 p_{n_1} + a_2 p_{n_2}) x^n + \cdots + (a_1 p_{1_1} + a_2 p_{1_2}) x + (a_1 p_{0_1} + a_2 p_{0_2})$$

$$(a_1 p_{0_1} + a_2 p_{0_2}) q_0 = a_1 p_{n_1} q_n x^{2n} + a_2 p_{n_2} q_n x^{2n} + \cdots + a_1 p_{0_1} q_0 + a_2 p_{0_2} q_0 =$$

$$= a_1 \langle p_1, q\rangle + a_2 \langle p_2, q\rangle$$

$\bullet \langle p(x), a_1 q_1(x) + a_2 q_2(x)\rangle = a_1 \langle p(x), q_1(x)\rangle + a_2 \langle p(x), q(x)\rangle$

$$\begin{pmatrix} p_n x^n & \cdots & p_1 x & p_0 \end{pmatrix} \begin{pmatrix} a_1 q_{n_1} x^n + a_2 q_{n_2} x^n \\ \vdots \\ a_1 q_{1_1} x + a_2 q_{1_2} x \\ a_1 q_{0_1} + a_2 q_{0_2} \end{pmatrix} = \underbrace{p_n a_1 q_{n_1} x^{2n} + p_n a_2 q_{n_2} x^{2n} + \cdots + p_0 a_1 q_{0_1} + p_0 a_2 q_{0_2}}_{\substack{\parallel \\ a_1 \langle p(x), q_1(x)\rangle + a_2 \langle p(x), q_2(x)\rangle}} =$$

$$= (a_1 q_{n_1} + a_2 q_{n_2}) p_n x^{2n} + \cdots + (a_1 q_{1_1} + a_2 q_{1_2}) p_1 x^2 + (a_0 q_{0_1} + a_0 q_{0_2}) p_0$$

$\Downarrow$

és bilineal

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ex3

a) Ara donarem la matriu en la base canònica $\{1, x, x^2\}$

$$\langle p(x), q(x)\rangle = \langle p_2x^2 + p_1x + p_0,\ q_2x^2 + q_2x + q_0\rangle = p_2q_2x^4 + p_1q_1x^2 + p_0q_0$$

$$M = \begin{pmatrix} \langle 1,1\rangle & \langle 1,x\rangle & \langle 1,x^2\rangle \\ \langle x,1\rangle & \langle x,x\rangle & \langle x,x^2\rangle \\ \langle x^2,1\rangle & \langle x^2,x\rangle & \langle x^2,x^2\rangle \end{pmatrix} = \begin{pmatrix} 2 & 0 & 2/3 \\ 0 & 2/3 & 0 \\ 2/3 & 0 & 2/5 \end{pmatrix}$$

b) $p_1(x) = 1 \quad p_2(x) = x$ sigui u un element de E

$\langle p_1(x), u\rangle = 0 \Rightarrow p_1(x) \perp u$

$\langle p_2(x), u\rangle = 0 = p_2(x) \perp u$

$\langle 1, u\rangle = 0 = \langle x, u\rangle$

$u = ux \iff \boxed{u=0} \vee \boxed{x = 1 \}\ \text{\~{}~~tu~~}}$ ~~Xxxx~~

c) b.o.n on $\langle p, q\rangle = 0 \wedge$ norma $= 1 = \sqrt{p+q}$

d) $F^{\perp} = E/F = (\underbrace{a_2x^2 + a_1x + a_0x_0}_{E=\mathbb{R}^2[x]}) - (x^2) = \boxed{a_2 + a_1x + a_0x}$

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ex 4)

a) Sigui $P_f(x) = \begin{vmatrix} a-x & b & c & c \\ b & a-x & c & c \\ c & c & a-x & b \\ c & c & b & a-x \end{vmatrix} \underset{f_1 \to f_1+f_2+f_3+f_4}{=}$

$= \begin{vmatrix} a-x+b+2c & a-x+b+2c & a-x+b+2c & a-x+b+2c \\ b & a-x & c & c \\ c & c & a-x & b \\ c & c & b & a-x \end{vmatrix} = (a-x+b+2c) \begin{vmatrix} 1 & 1 & 1 & 1 \\ b & a-x & c & c \\ c & c & a-x & b \\ c & c & b & a-x \end{vmatrix}$

$= (a-x+b+2c) \left(\begin{vmatrix} a-x & c & c \\ c & a-x & b \\ c & b & a-x \end{vmatrix} + b \begin{vmatrix} 1 & 1 & 1 \\ c & a-x & b \\ c & b & a-x \end{vmatrix} + c \begin{vmatrix} 1 & 1 & 1 \\ a-x & c & c \\ c & b & a-x \end{vmatrix} + \right.$

$+ c \begin{vmatrix} 1 & 1 & 1 \\ a-x & c & c \\ c & a-x & b \end{vmatrix} = (a-x+b+2c)\left((a-x)^3 + 2c^2 b - 2c^2(a-x) - b^2(a-x) + \right.$

$+ b \cdot \left((a-x)^2 + 2bc - 2(a-x)c - b^2 \right) + c\left(c(a-x) + \cancel{c^2} + b(a-x) - \cancel{c^2} - bc - (a-x)^2 \right) +$

$\left. + c \cdot \left(cb + \cancel{c^2} + (a-x)^2 - \cancel{c^2} - c(a-x) - b(a-x) \right) \right) =$

$= (a-x+b+2c)\left((a-x)^3 + 2c^2 b - 2c^2(a-x) - b^2(a-x) + b(a-x)^2 + 2b^2 c - 2b(a-x)c - b^3 + \right.$

$\left. + c^2(a-x) + \cancel{2}bc(a-x) - \cancel{bc^2} - \cancel{c(a-x)^2} + \cancel{c^2 b} + \cancel{c(a-x)^2} - \cancel{bc(a-x)} \right) =$

$= (a-x+b+2c)\left((a-x)^3 + 2c^2 b - c^2(a-x) - b^2(a-x) + b(a-x)^2 - 2b^2 c - 2b(a-x)c + \ldots \right.$

$= (a-x+b+2c)\left((a-x)^3 + b(a-x)^2 + (a-x)(-c^2 - b^2 + b - 2bc) \right.$

ex 4

a)

Sigui $P_f(x) = \begin{vmatrix} a-x & b & c & c \\ b & a-x & c & c \\ c & c & a-x & b \\ c & c & b & a-x \end{vmatrix}$

Per λ_1,

$Nuc(A-\lambda I) = (1,1,1,1)$

$(a-\cancel{\lambda}) + b + 2c = 0 \Rightarrow \boxed{\lambda_1 = a+2c+b}$

~~$b + a - \lambda + 2c = 0$~~

~~$2c + a - \lambda + b = 0$~~

~~$2c + b + a - \lambda = 0$~~

Per λ_2,

$Nuc(A-\lambda_2 I) = (1,1,-1,-1)$

$a - \lambda_2 + b - 2c = 0 \Rightarrow \boxed{\lambda_2 = -2c + b + a}$

VAP$_2$

Per λ_3,

$Nuc(A-\lambda_3 I) = (1,-1,1,-1)$

$a - \lambda - b + c - c = 0 \Rightarrow \boxed{\lambda_3 = a - b}$

~~Per λ_4,~~

~~$Nuc(A-\lambda_4 I) =$~~

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$b_{o.n} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{pmatrix}$

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ex 4

b)

$\sigma = 1/\lambda$ on σ és SV i λ és VAP de A

$$\boxed{\sigma_1 = 1/_{a+2b+c} \qquad \sigma_2 = 1/_{a+b-2c} \qquad \sigma_3 = 1/_{a-b}}$$

$$\begin{pmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{a+2b+c} & 0 \cdots & \cdots 0 \\ 0 & \frac{1}{a+b-2c} & 0 \\ \vdots & & \\ 0 \cdots & \cdots 0 & \frac{1}{a-b} \end{pmatrix}$$

c) $A = \begin{pmatrix} 4 & 2 & 1 & 1 \\ 2 & 4 & 1 & 1 \\ 1 & 1 & 4 & 2 \\ 1 & 1 & 2 & 4 \end{pmatrix}$

$$\begin{pmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{9} & & \\ & \frac{1}{4} & \\ & & \frac{1}{2} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{9} & & \\ & \frac{1}{4} & \\ & & \frac{1}{2} \end{pmatrix}^{-1} \Bigg(\ \cancel{4}$$