0/12

# Part 2 of the exam

[redacted]

*Monday, June 08, 2015*

The device of the scientist provides that data:

$$Y_i = \begin{cases} 0 & \text{if} \quad -1 \leq X_i \leq 1, \\ X_i & \text{if} \quad |X_i| > 1 \end{cases}$$

In first place, we load the data.

```
y <- read.table("results.txt", col.names=FALSE)[,1]
y
```

```
##   [1] -1.914131  1.054858  2.668882 -4.191395  1.358249  1.512112  0.000000
##   [8]  0.000000  0.000000 -1.280076  0.000000 -1.496773 -1.052508  0.000000
##  [15]  2.418988  0.000000  0.000000 -1.322391 -1.174343  5.331670  0.000000
##  [22]  0.000000  0.000000  1.419179  0.000000 -2.396410  1.649511 -1.547311
##  [29]  0.000000 -1.371897  2.704595  0.000000  0.000000  0.000000 -2.758187
##  [36] -1.835239 -3.860079 -2.181986  0.000000  0.000000  3.398993 -1.637285
##  [43] -1.210729  0.000000 -1.488680 -1.437029 -1.714636 -2.003972  0.000000
##  [50]  0.000000 -3.112063  0.000000 -1.717779 -1.529924  0.000000  1.626112
##  [57]  3.795635 -1.046707  3.711819 -1.815617  1.813177  5.597982  0.000000
##  [64]  0.000000  0.000000  4.054169 -1.777215  3.235654  3.159130  1.172946
##  [71]  0.000000  0.000000  0.000000  1.796573  4.640542  0.000000 -2.281402
##  [78]  0.000000  1.016524  0.000000  0.000000  0.000000 -2.244604  0.000000
##  [85]  2.200465  1.895217  1.599995  0.000000  0.000000 -1.889056  0.000000
##  [92]  1.010392  3.911928  2.503027  0.000000  1.211101 -1.769216  2.256407
##  [99]  2.445834  4.742234
```

She wants to estimate the bidimensional parameter $\theta = (\mu, \sigma)$.

0

**1. What is the contribution to the likelihood function of the $i$-th experiment result when the observation $y_i$ is equal to 0? And what is the contribution when $|y_i| > 1$?** We can measure the contribution with the following function:

$$\phi(z)\Phi(z)\frac{1}{\sigma}\phi(\frac{x-\mu}{\sigma})\Phi(\frac{x-\mu}{\sigma})$$

$$x_i|(Y_i = 0)x_i|(|Y_i| > 1)$$

$\sigma$

Aquí no hay ninguna

Z1

Where

$$P(Y_i = 0) = \tau_1 P(|Y_i| > 1) = 1 - \tau_1$$

0

**2. Write down the log-likelihood function for the complete data, and the log likelihood for the observed data.** Complete data:

$$\ell(p, \theta_0, \theta_1; \tilde{X}, \tilde{Y}) = \sum_{i=1}^{n} X_i \log \frac{p}{1-p} + n \log(1-p) + \sum_{i=1}^{n} X_i \log f(Y_i; \theta_1) + \sum_{i=1}^{n} (1 - X_i) \log f(Y_i; \theta_0)$$

Observed data:

$$\ell_{\text{obs}(\theta;Y)} = \ell(\theta; Y) - \log f_2(Y_{\text{mis}} | Y_{\text{obs}}; \theta)$$

**3. R script for the E and M steps.** Remember: N(0,1)

```
# General Steps
  T1 <- tau_1 * dnorm( x, mu_1 )
  T2 <- tau_2 * dnorm( x, mu_2 )

  P1 <- T1 / (T1 + T2)
  P2 <- T2 / (T1 + T2) ## note: P2 = 1 - P1

  tau_1 <- mean(P1)
  tau_2 <- mean(P2)
# E step
em <- function(x, a, b) {
  if
  fx <- (f*x)/((F*b)-(F*a))
  else {
    fx <- 0
  }
}
mu + ((phi*((a-mu)/sigma) - phi*((b-mu)/sigma))/(Phi*((b-mu)/sigma) - Phi*((a - mu)/sigma)))*sigma
Var <- (sigma^2)
# M Step
if(yi > 1) {
  tyi <- yi } else {

  }
um1 <- 1/n * sum(tyi)
sigma2m1 <- 1/n * sum(tyi - um1)^2
}
```

```
# Reading the table
y <- read.table("results.txt", col.names=FALSE)[,1]
```

**Appendix: R Code**