# Examen part 2

[redacted]

*Monday, June 08, 2015*

7'5/12

Answer the following questions.

1'5

1. What is the contribution to the likelihood function of the i-th experiment result when the observation $y_i$ is equal to 0? And what is this contribution when $|y_1| > 1$?

   The contribution when $y_i = 0$ will be the probability of y_i equal to 0. However, this probability will be, in fact, the probability of x_i between 1 and -1. In other words, the contribution will be:

   $$P(Y_i = 0) = P(-1 \le X_i \le 1) = \Phi\left(\frac{1-\mu}{\sigma}\right) - \Phi\left(\frac{-1-\mu}{\sigma}\right)$$

   The contribution when $|y_i| > 1$ will be:

   $$P(Y_i = y_i) = P(X_i = y_i) = \frac{1}{\sigma}\phi(\frac{y_i-\mu}{\sigma})$$

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2. Write down the log-likelihood function for the complete data, and the log-likelihood for the observed data.

   Using what has been previously said, we can write the log-likelihood for both the observed and the complete data.

   Complete data: $(X_1, ..., X_n) = (x_1, ..., x_n)$

   $$L^c(\mu, \sigma; x) = \prod_{i=1}^{n} \frac{1}{\sigma}\phi(\frac{x_i-\mu}{\sigma})$$

   Hence,

   $$l^c(\mu, \sigma; x) = \log L^c(\mu, \sigma; x) = -n \log \sigma + \sum_{i=1}^{n} \log\left(\phi\left(\frac{x_i-\mu}{\sigma}\right)\right)$$

   Observed data: $(Y_1, ..., Y_n) = (y_1, ..., y_n)$. Let $\delta_i$ be 1 if $y_i = 0$ and 0 otherwise; and $n_0 = \sum \delta_i$.

   $$L^o(\mu, \sigma; x) = \prod_{i=1}^{n} \left(\Phi\left(\frac{1-\mu}{\sigma}\right) - \Phi\left(\frac{-1-\mu}{\sigma}\right)\right)^{\delta_i} \left(\frac{1}{\sigma}\phi(\frac{y_i-\mu}{\sigma})\right)^{1-\delta_i}$$

   Therefore,

$$l^o(\mu,\sigma;x) = \log L^o(\mu,\sigma;x) = n_0 \log\left(\Phi\left(\frac{1-\mu}{\sigma}\right) - \Phi\left(\frac{-1-\mu}{\sigma}\right)\right) - (n-n_0)\log\sigma + \sum_{i=1}^{n}(1-\delta_i)\log\left(\phi\left(\frac{y_i-\mu}{\sigma}\right)\right)$$

3. E step in the EM algorithm.
4. M step in the EM algorithm.

No están hechos

5. Write an R code implementing this EM algorithm.

```
Estep <- function(sample, mu, sigma) {
    ## Calculates the Expectation using current mu and sigma.  Formula from 3. I
    ## use 4. results

    expr <- mu + sigma * (dnorm((-1 - mu)/sigma) - dnorm((1 - mu)/sigma))/(pnorm((1 -
        mu)/sigma) - pnorm((-1 - mu)/sigma))
    sample2 <- ifelse(abs(sample) > 1, sample, expr)
    return(sample2)
}

Mstep <- function(sample2, mu, sigma) {
    ## update mu and sigma (those maximizing Estep.  maximize Estep, or
    ## equivalently use 4. results.

    mu <- mean(sample2)
    sigma <- sqrt(1/length(sample2) * sum((sample2 - mu)^2))
    return(c(mu, sigma))
}

EMalgorithm <- function(mu0, sigma0, sample, its) {
    mu <- NULL
    mu[1] <- mu0
    sigma <- NULL
    sigma[1] <- sigma0
    i <- 2

    while (i <= its) {
        sample2 <- Estep(sample, mu[i - 1], sigma[i - 1])
        aux <- Mstep(sample2, mu[i - 1], sigma[i - 1])
        mu[i] <- aux[1]
        sigma[i] <- aux[2]
        i <- i + 1
    }
    return(list(mu = mu, sigma = sigma))
}
```

6. Use your EM algorithm for estimating $(\mu,\sigma)$ by maximum likelihood.

```
y <- read.table("results.txt",col.names=FALSE)[,1]
EMalgorithm(mu=mean(y), sigma=sd(y), y, its=100)
```

```
## $mu
```

```
##   [1] 0.2585526 0.2666553 0.2669991 0.2670104 0.2670107 0.2670108 0.2670108
##   [8] 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108
##  [15] 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108
##  [22] 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108
##  [29] 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108
##  [36] 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108
##  [43] 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108
##  [50] 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108
##  [57] 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108
##  [64] 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108
##  [71] 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108
##  [78] 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108
##  [85] 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108
##  [92] 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108 0.2670108
##  [99] 0.2670108 0.2670108
##
## $sigma
##   [1] 1.975969 1.965025 1.964983 1.964981 1.964981 1.964981 1.964981
##   [8] 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981
##  [15] 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981
##  [22] 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981
##  [29] 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981
##  [36] 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981
##  [43] 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981
##  [50] 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981
##  [57] 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981
##  [64] 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981
##  [71] 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981
##  [78] 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981
##  [85] 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981
##  [92] 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981 1.964981
##  [99] 1.964981 1.964981
```

Fiexem-nos que ambdós paràmetres convergeixen, obtenint $\mu = 0.267$ i $\sigma = 1.965$ aproximadament.

OK