ASSIGNMENTS COURSE 2014-15, NUMERICAL METHODS FOR DYNAMICAL SYSTEMS, MAMME

ASSIGNMENT 1

```
1. (i) Simulate the standard map dynamics taking 100 iterates
   for each initial condition. Try with different initial conditions.
   (ii) Modify the code such that the program takes NP initial
        conditions on the x axis i.e. (x,0) with x between xmin and xmax.
   Plot in x,y variables.
2. Find exact initial condition of a 2-periodic point.
3. Find approximate initial condition of a 3-periodic point.
4. Find approximate initial condition of a 6-periodic point.
5. Make some comments on the dynamics you see on the plot.
6. Send the plot file (.ps or .pdf), the comments and the code
   (Fortran, C, matlab,....) to merce.olle@upc.edu

 RECALL: with gnuplot

>gnuplot
 plot .....
 set term post enhanced color solid eps
 set output 'name_file.ps'
 replot
 exit

 and the file name_file.ps is created

RECALL how to generate a pdf file with math text and figures.
 Edit a file  name_of_file.tex (follow instructions given in
 the first pc session)


OPTIONAL ASSIGNMENT

Compute for each initial condition $(x,0)$ for $x\in (0.05,2.75)$
the rotation number. Make a plot in the variables $(x,\rho )$.
 Make some comments comparing this plot and the plot obtained in
 1. (ii).
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ASSIGNMENT 2: Using Taylor integrator

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*********************************************************
 PART 1: we integrate the harmonic oscillator
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1) Edit a file called eq_os.eq
 and type the equations on this file
    Remarks:
     -each line with a ; at the end
     -write the code using real numbers (for example 3. NOT 3.D0
      since we will use a C compiler to generate the taylor integrator
      (see 2))
     -save the file
2) On a terminal, type

 taylor (or ./taylor) -name eq_os -o taylor_eq_os.c -step
-jet -f77 -header -sqrt eq_os.eq

Now taylor_eq_os.c (the time stepper is created)

3) If there is NOT a common, go to 4)

4) (i) Type
      gcc -c taylor_eq_os.c

   Now taylor_eq_os.o is created.
   And go on with the compilation of other routines:

   (ii) For example type
          gfortran -c main_os_flow.f

        (main_os_flow.o is created).
     REMARK: check that the integrator call is done by the instruction
        call taylor_f77_eq_os_(.........)
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   (iii) gfortran -o main_os_flow main_os_flow.o taylor_eq_os.o
        (the executable file main_os_flow is created)
   (iv) ./main_os_flow (or if you create a folder /work,
       then type
        cd work
        ../main_os_flow)

Some tests:

1.   Enter initial conditions: 1.,0.
     ti=0., idir=1, tmax: 6.28 (better: tmax=2.d0*pi,
     with pi=4.d0*datan(1.d0), np=30
     Plot the data

2. Similar but idir=-1
     Plot the data

3. Similar but idir=1 and tmax=pi/2
     Plot the data

4.   Enter initial conditions: 1.,0.
     ti=0., idir=1, tmax: 12., np=2
     Plot the data
     (4.1 Edit the main_os_flow.file
      4.2 Remove 'C' in routine flow
c          write(10,100) t,(x(i),i=1,n) , so it becomes
        write(10,100) t,(x(i),i=1,n)
      4.3 Run the program again and look at the 'intermediate' points)

5. Add in the program the code to check the first integral

***************************************************************
 PART 2: we integrate the harmonic oscillator + variational eqs
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PRELIMINARY

2.1 Write a main program to compute the determinant of a given matrix A
    of dimension n.
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*************************************************************
 PART 3:  The code
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3.1 Write a file eq_os_var.eq with 6 ODE
3.2 Proceed as in part 1.: Write a new file main_os_flow_var.f.

 In the file, some remarks:
    - type the initial condition for x3=1.,x4=0.,x5=0.,x6=1. on the program
    - Recall that in the flow routine you must write
        call taylor_f77_eq_os_var_(.........)
    - Once you have the final point for tmax, call a routine that has as input
        x and as output a matrix A
    - Compute the determinant of the matrix


TEST:
3.3 As initial condition enter 1.,0. (for x1,x2) tmax=dpi (ie 2.*pi)
 (the matrix A should be the identity)
3.4 Send the code and a file .eps with a plot of the periodic orbit,
 (x,y) coordinates, to merce.olle@upc.edu
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ASSIGNMENT 3: Computation of a Poincare section

```
 Implement the computation of the Poincare section y=0 in the harmonic
 oscillator problem.
 1. Done for n_crossing=1 (ie first crossing with the P section).
 2. Implement the computation of n_crossing > 1.
 3. Check test: with the different initial conditions: a point on the x axis,
   a point on the y axis, any point (x,y) and different
   values of n_crossing.
 4. Check also that the orbit is well computed: for example plot
    the orbit in (x,y) coordinates.
 5. Send
  a) the output (the final time of integration and the
    final point) given by the program excecution.
    Take initial conditions (x,y)=(1.,0.), number of crossings 2
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   (try both cases: idir=+1, idir=-1)
   Take initial conditions (x,y)=(0.,1.), number of crossings 2
   (try both cases: idir=+1, idir=-1)

 b) the four corresponding plots in (x,y)
 c) and the code (ALL inside one file .pdf) to merce.olle@upc.edu
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ASSIGNMENT 4: we integrate a linear system of ODE

```
0. Compute the eigenvalues and eigenvectors of a square matrix A
    of dimension n
1. Implement the integration of a linear system of dimension 2 (ctant
   coefficients aa,bb,cc,dd)
   You want to simulate different systems, so you consider
   variables aa,bb,cc,dd as parameters.

2. TESTS to be done:
   (i) A center: aa=2, bb=-5, cc=1, dd=-2
   (ii) A focus: aa=3, bb=-2, cc=4, dd=-1 (in  this case you might play
       with a program to ompute the Poincare section)
   (iii) A saddle:  aa=-1, bb=0, cc=3, dd=2.
       3.1 Compute the eigenvalues and eigenvectors of the Jacobian
             matrix at (0,0).
       3.2 Compute and plot also the unstable and stable manifolds
             of the origin (computing previously the eigenvalues
             and eigenvectors of the Jacobian matrix)
       3.3 Plot the phase portrait.

Remarks:
  * Once aa,bb,cc,dd are fixed, consider a set of initial conditions.
    For each initial condition, save the associated orbit.
  * Choose a reasonable window for each plot.

3. Send me the code and also 3 plots
   (with the corresponding phase space).
```

ASSIGNMENT 5: a. COMPUTATION OF THE EQUILIBRIUM POINTS OF THE RTBP b. Stability for the ELLIPTIC RTBP

```
* Circular RTBP
 1. Write a program such that given xmu, computes L1,L2,L3
    and C(L1),C(L2),C(L3).
     Test:

 xmu=0.1:
 xL1,xL2,xL3:
 -0.60903511002320210       -1.2596998329023312        1.0416089085710609
 C(L1),C(L2),C(L3)
   3.6869532298798946        3.5566844258406487        3.1895781504493819

 xmu=0.2:
 xL1,xL2,xL3:
 -0.43807595853836628       -1.2710486907398808        1.0828394642022439
 C(L1),C(L2),C(L3)
   3.9646532763063704        3.7123933328511765        3.3573204210059799

 2. Generate a plot in (mu,x) where mu varies in (0,0.5] and
    x=x(Li), i=1,2,3 (so there appear 3 curves).
 3. Generate a plot in (mu,C) where mu varies in (0,0.5] and
    C=C(Li), i=1,2,3  (so there appear 3 curves).
 4. Send the 2 plots and code to merce.olle@upc.edu.



* Elliptic RTBP (OPTIONAL)
 5. Stability of the triangular points L4 and L5 varying
 mu and eccentricity. Find a plot in (mu,exc)
 6. Send me the code and the plot.
```

ASSIGNMENT 6: integration of the RTBP

```
 Input: mu (a double precision variable), initial time,
 final time, initial conditions and number
of points along the orbit, idir =1 or -1 (forward or backward in time)
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2. Implementation:
Check that at each point the JAcobi integral is conserved

TEST: Using the RTBP with initial conditions and mass parameter given in ci_rtbp.dat (download from web page),

1. Integrate up tp the period. Check it is a periodic orbit: the final condition is equal to the initial one, at least the first 13 digits (otherwise it is wrong)
2. Check that the Jacobi integral is conserved

OUTPUT to be sent:
a. the plot of the periodic orbit (x,y) projection
b. Send me the output of the run :
   idir =1
   idir=-1
   test on the (final condition-initial condition).
c. Code used.

ASSIGNMENT 7: Integration of the RTBP + variational eqs

1. Implement the integratio of the flow for the RTBP taking into account the 20 ODE (RTBP+variational eqs). Implement the two checks (conservation of the Jacobi first integral and determinant of monodromy matrix ("matrix solution" of the variational eqs after a period equal to 1 -for the initial conditions of a periodic orbit (PO)-)

2. Check the two tests so that the integration of the RTBP works fine both when idir=1 (integration forward in time) and idir=-1 (backwards in time). Take as initial conditions the file from the web page. REMARK that only the half period is given.

3. Check also that the final point is the initial one (they MUST coincide at least on the first 12 digits).

Send the output (and the code) to me.

ASSIGNMENT 8: Computation of a Poincare section for the RTBP

```
1. Implement the computation of the Poincare section y=0 taking
   into account the 20 ODE (RTBP+variational eqs). From assignments
   5 and 6, the two checks (first integral and determinant of variational
   eqs equal to 1) are already implemented and tested OK.
2. Implement the computation of n_crossing > or = 1.

3. Check that the integration of the RTBP works fine both
   when idir=1 (integration forward in time) and idir=-1 (backwards
   in time).

As a test:
TAke number of crossings =2.
A.1 consider the initial conditions of the PO (the period as the time
    to integarate),
    try idir=1 (it becomes a retrograde orbit), and try idir=-1
    (a direct one).
A.2 Check that the final point is the initial one (they MUST
    coincide at least on the first 13 digits)
A.3 Check that the time of integration
    must be equal to the period, ie the difference in absolute value
    must be less than 10^(-12).
Send the output (and the code) to me.
```

ASSIGNMENT 9: COMPUTATION OF THE INVARIANT MANIFOLDS ASSOCIATED WITH THE COLLINEAR EQUILIBRIUM POINTS OF THE RTBP

```
Compute the unstable and stable manifolds of the equilibrium points
L1,L2 and L3 of the RTBP for a given mu.

1. Given xmu and i (i can be 1,2,3) compute Li.
2. For the same xmu, and given a point Li, i=1 or 2 or
    3, compute the Jacobian matrix of the RTBP at Li. We call A such
    matrix.
3. Compute the eigenvalues and eigenvectors of A.
 (I) For the UNSTABLE manifold: Wu
    3.1 we want the eigenvector that
          corresponds to the eigenvalue with lambda >0.
    3.2 Enter as input a sign (isig=+1 or isig=-1) and as ouput obtain
          the ASSOCIATED eigenvector. We call it v.
    3.3 Consider as initial condition L_i+sv with s=1.d-6, integrate
          forward in time this orbit (which will be the 'real' unstable
          manifold of L_i) up to a given section (y=0) and
          a given crossing (call it n_crossing) and try
                    n_crossing=1, 2 and 3.
    3.4 Repeat the computation with s =1.d-5 and s=1.d-7 and check
          that the output points are 'the same'.

    Remark: 1. Check for each point of the integration that the Jacobi
          constant remains constant.


OUTPUT to be sent: Take xmu=0.1, write on a file:
      point L3 and C(L3)
      the jacobian matrix A
    for L3, and isig=+1,
      eigenvalue and eigenvector
      initial condition taking s=1.d-6
      final one after nomtall=1, 2 and 3
      variation of the Jacobi constant
    -plot of the manifold: (x,y) coordinates
    for L3, and isig=-1, the same


(II)For the STABLE manifold: Ws
    Proceed in a similar way but now consider the
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     eigenvector associated with the eigenvalue with lambda<0.
     Now you must integrate backwards in time.

 For the simulation take xmu=0.1
 OUTPUT:    the same as for the unstable manifold.

 (III) CHECK the symmetry of the manifolds in the plot.

Send the output and code to merce.olle@upc.edu. Just one pdf file please.


TEST: check that for mu=0.001

xl3=1.0004166666122849      C(L3)=3.0019989789680306
 A
 row           1
   0., 0., 1.,0.
 row           2
   0., 0., 0.,1.
 row           3
   3.0017508024238229        0.        0.       2.
 row           4
   0.      -8.75401211911499351E-004  -2.       0.
 eigenvalue    5.12167128556700879E-002
 eigenvector 3.40898211106462692E-002 -0.99810900974950223
    1.74596857912521445E-003 -5.11198625509974511E-002
```

ASSIGNMENT 10: COMPUTATION OF HOMOCLINIC ORBITS OF EQUILIBRIUM POINTS

```
We will compute symmetric homoclinic orbits to L3 of the RTBP

 1. Given xmu, compute L3
 2. Consider isig=-1, Generate the unstable manifold up to first crossing
    with the Poincare section (PS) y=0. Save in a file variables xmu,x,x'
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   of point x,y=0,x',y' in PS.
 3. Plot, for example, the orbit (up to the first crossing)
    for mu=0.008.
 4. Consider xmu <-xmu+xincmu, repeat steps 1. and 2.
 5. Plot  (xmu,x') for xmu in (0.001,.5).

CONSIDER 3 intervals in  mu
 a. xincmu 0.00001, mu in (0.001,0.015)
 b. xincmu 0.0001, mu in (0.015,0.05)
 a. xincmu 0.001, mu in (0.05,0.49)

 6. Send to merce.olle@upc.edu
 6.a  plot 3.
 6.b  plot 5.: for x in (0.001,0.5)
 6.c  plot 5.: for x in (0.001,0.1)   (ie a zoom of previous plot)
 6.d  the code
```

(OPTIONAL) ASSIGNMENT 10 bis: COMPUTATION OF HOMOCLINIC ORBITS OF EQUILIBRIUM POINTS. SOME PARTICULAR CASES.

```
Once we have the curve (mu,x') at the first crossing (taking Wu,-(L3)),
we want to understand it.

Generate the following plots:
1. the one you already had: the curve (mu,x') for mu in [.001,.1]
2. the curve (mu,x1) where x1 is the value of the x component of the manifold
    at the first crossing
3. For mu=0.0056, 0.00567,0.0057 plot the 3 manifolds in (x,y) coordinates and
    explain what is happening.
4. For mu=0.01179, 0.0159375,0.083 plot the 3 manifolds in (x,y) coordinates and
    explain the evolution of the shape of the manifolds when increasing mu.

Send the plots and comments to me
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ASSIGNMENT 11: COMPUTATION OF PERIODIC ORBITS FOR THE RTBP

```
We will compute symmetric periodic orbits for the RTBP, in particular
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a family of periodic orbits around L3.

 A) Implement a bisection method for f(x)=0.

 ASSUME A GIVEN mu. For example mu=0.1

 B) Assume C fixed. Implement a routine such that, given an initial x,
    sign of y' and C, computes (as output) y', such that the initial condition
    of the orbit will be (x,0,0,y').
 C) Assume C fixed. Implement a routine such that given an initial x
    and the 'associated' initial condition (x,0,0,y'), returns as
    output F(x)=x' (at a given crossing).
 D) Assume C fixed. Vary x such that you find x1 and x2 such that F(x)
    changes sign.
 E) Assume C fixed. Now refine x, using A), such that F(x)=0.
    (ie |F(x)|<1.d-12)

 F) Once A) to E) work, compute the family of the Lyapunov periodic
    orbits around L3, varying the Jacobi constant in the interval
    (2.1,3.189).

 G) Send the plot of the characteristic curve (plane (C,x)) of the family,
    a plot with some orbits of the family (projection (x,y)) (say for
    C=2.1,2.5,3.15) and the code to merce.olle@upc.edu


 HINT: it works with
 mu=0.1, cmin=2.1 cmax=3.189, xincC=1.d-3, xincini=1.d-5, xincx=1.d-3
```

ASSIGNMENT 12: COMPUTATION OF INVARIANT MANIFOLDS OF PERIODIC ORBITS

```
We will compute the invariant manifolds of a give Lyapunov periodic orbit
of the RTBP (as a particular example)

 ASSUME A GIVEN mu, an initial condition of a periodic orbit and its
period (download file ci_lpo_l3.dat from the web page)
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```
xmu=1.d-2
x=1.033366313746765d0 y=x'=0, y'=-.05849376854515592d0
HALF period= 3.114802556760205d0


Follow the explanations from the theoretical sessions.


 Send the plot of some orbits of the unstable manifold $W^{u,-}$
(projection (x,y)) up to the first crossing with the Poincar\'e section
x=0 (remark: NOT y=0)
and the code to merce.olle@upc.edu

Proceed similarly with $W^{u,+}$, $W^{s,-}$ and $W^{s,+}$.
Check the symmetry.
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