Numerics of Dynamical Systems

Assignment 9

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1 Programme

Since vebrep from package_alg does not calculate the correct eigenvalues and eigenvectors I corrected them manually.

Listing 1: main_rtbp_flow9.f

```
c***********************************************************
c
c   MAIN_RTBP_FLOW9.f
c
c       We integrate the harmonic oscillator field with Taylor
c       from t=ti up to t=tmax
c       idir= +1 (integration forward in time); =-1 (backward)
c       np= number of intermediate points (apart from the initial one)
c           that we want to write on the file orbit.d. If np=1
c           only the initial and final points are written
c
c   input: xi,ti,tmax,idir,np
c***********************************************************
       implicit real*8 (a-h,o-z)
       parameter (n=4,m=4)
       dimension xi(n),x(n),O(m,m),A(n,n),RR(n),RI(n),VR(n,n),VI(n,n)
       dimension v(n),p(2),yf(n)
       common/param/xmu
       open(10,file='orbit.d',status='unknown')
       write(*,*)'sign'
       read(*,*) iregion
       write(*,*) 'idir?'
       read(*,*) idir
       write(*,*) 'ncrossing?'
       read(*,*) ncrossing
       ti=0.d0
       tmax = 6.28
       np=30
       xmu=0.1

        call peq(xmu,xl1,xl2,xl3,cl1,cl2,cl3)
        write(*,*) 'xl3', xl3
        write(*,*) 'cl3', cl3

       C=cl3
```

```
         x(1)=xl3
         x(2)=0
         x(3)=0
         x(4)=0
         call JacobiMatrix(n,x,xmu,A)
         write(*,*) 'A'
            do i=1,n
              write(*,*) (A(i,j),j=1,n)
            enddo
         call vapvep(A,n,RR,RI,VR,VI)
c******* Correction of eigenvalues
         RR(1) = 0
         RR(2) = 0
         RR(3) = -0.5016
         RR(4) = 0.5016
c         RR = SNGL(RR)
c         RI = SNGL(RI)
c         VR = SNGL(VR)
c         VI = SNGL(VI)
         if(idir.gt.0) then
           do i=1,n
             if(RR(i).gt.0) then
             k=i
             endif
           enddo
         else
           do i=1,n
             if(RR(i).lt.0) then
             k=i
             endif
           enddo
         endif
c****** Correction of eigenvectors
         VR(1,3) = -0.2894
         VR(2,3) = -0.8457
         VR(3,3) = 0.1452
         VR(4,3) = 0.4242
         VR(1,4) = -0.2894
         VR(2,4) = 0.8457
         VR(3,4) = -0.1452
         VR(4,4) = 0.4242
```

```
         do i=1,n
         v(i) = VR(i,k)
         enddo
         p(1) = RR(k)
c          p(2) = RI(2)
         write(*,*) 'Eigenvalue', p(1)
c, '+i*', p(2)
         write(*,*) 'v', (v(i),i=1,n)
         s = 1.d-6
          if(iregion.lt.0) then
         s = -s
         endif
         x = x + s * v
         write(*,*) 'initial point', (x(i),i=1,n)

          call jacobi(x,C,xmu,n)

         ti=0
         do j = 1,ncrossing
         t=0.d0
           write(10,*)t,(x(i),i=1,n)
           call jacobi(x,C,xmu,n)
           call poinc1(j,xmu,n,m,x,yf,tfinal,idir,ti)
           ti = ti + tfinal
         end do
         end

c*********Computes Jacobi-Constant
        subroutine jacobi(x,C,xmu,n)
        implicit real*8 (a-h,o-z)
        dimension x(n)
        ro = dsqrt((x(1) - xmu)*(x(1) - xmu))
        rt = dsqrt((x(1) - xmu + 1.d0)*(x(1) - xmu + 1.d0))
        ome =0.5d0*(x(1)*x(1))+(1.d0-xmu)/ro+xmu/rt+0.5d0*xmu*(1.d0-xmu)
        Cnew = 2*ome
        Cdiff = dabs(C - Cnew)
        if (Cdiff.gt.1.d-3) then
        write(*,*) 'Jacobi constant not conserved'
        endif
        end
```

```
C*************************************************************************
c Input:
c n dimension of the vectors yi and yf
c yi initial point
c idirorig: +1 integration forwards in time; -1 backwards
c yf final point
c tfinal final time
c
C*************************************************************************
      SUBROUTINE POINC1(j,xmu,n,m,YI,YF,tfinal,idirorig,ti)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION YI(n),YF(n),DGG(n),F(n)
         icont=0
       idir=idirorig
c
c  we assume initial time t=0.
c
c          ti=0.D0
C   DETERMINATION OF THE FIRST PASSAGE OF THE ORBIT THROUGH y=0
C
      CALL SECCIO(YI,GG,DGG)
      IF(DABS(GG).LT.1.D-9)GG=0.d0
      GA=GG
      hab=.1e-16
      hre=.1e-16
      pabs=dlog10(hab)
      prel=dlog10(hre)
      istep=1
c reasonable step:
      pas=0.4d0
      ht=0.d0
      t=ti
c |tmax| must be big enough
1      tmax=t+idir*pas
      CALL taylor_f77_eq_rtbp_var_(t,yi,idir,istep,pabs,prel,
     & tmax,ht,iordre,ifl)
c computation of first integral to be done
C
      CALL SECCIO(YI,GG,DGG)
      IF(GG*GA.LT.0.D0)go to 22
      write(10,*)t,(yi(ii),ii=1,n)
```

```
          GA=GG
          GO TO 1
C
C    REFINEMENT OF THE INTERSECTION POINT YF(*) USING NEWTON'S METHOD
C    TO GET A ZERO OF THE FUNCTION GG (SEE SUBROUTINE SECCIO)
C
  22    continue
        icont=icont+1
        if (icont.gt.20) then
            write(*,*)'problems finding the section'
            stop
        endif
        CALL FIELD(xmu,T,YI,N,F)
        P=0.D0
        DO 3 I=1,N
3       P=P+F(I)*DGG(I)
        H=-GG/P
c check p is not (or very close to) 0:  to be done
        if (h.ge.0.d0)idir=1
        if (h.lt.0.d0)idir=-1
        tmax=t+h
c           write(*,*)icont,'   refining: h and time ',h,tmax
c           write(*,*)'refining   t point ',t,yi(1),yi(2)
        CALL taylor_f77_eq_rtbp_var_(t,yi,idir,istep,pabs,prel,
     &  tmax,ht,iordre,ifl)
        CALL SECCIO(YI,GG,DGG)
        IF(DABS(GG).GT.1.D-13)GO TO 22
        DO 4 I=1,N
4       YF(I)=YI(I)
        tfinal=t
c check first integral: to be done
        write(*,*)'tfinal point time ',tfinal
c        call checkperiod(j,tfinal)
        write(*,*)(yf(ii),ii=1,n)
        write(10,*)t,(yf(ii),ii=1,n)
c        call matrix(yf,m,n)
        return
        t = tfinal
        end
```

```
C************************************************************************
C                                                                       *
C     THE SURFACE g OF SECTION, IN THIS CASE
C          INPUT PARAMETERS:
C     Y(*)       POINT
C          OUTPUT PARAMETERS:
C     GG          FUNCTION THAT EQUATED TO 0 GIVES THE SURFACE OF
C                 SECTION
C     DGG(*)      GRADIENT OF FUNCTION GG
C                                                                       *
C************************************************************************
        SUBROUTINE SECCIO(Y,GG,DGG)
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION Y(2),DGG(2)
        GG=Y(2)
        DO 1 I=1,2
 1      DGG(I)=0.D0
        DGG(2)=1.d0
        RETURN
        END


C
C FIELD.F
C
C************************************************************************
C
C     EQS OF MOTION IN synodical VARIABLES
C     X          TIME
C     Y(*)       POINT (Y(1),Y(2),....Y(n))
C     NEQ        NUMBER OF EQUATIONS
C          OUTPUT PARAMETERS:
C     F(*)       VECTOR FIELD
C
C************************************************************************
      subroutine field(xmu,t,x,neq,f)
      implicit real*8 (a-h,o-z)
      dimension x(neq),f(neq)
c
      umu=1.d0-xmu
      d1=x(1)-xmu
```

```
␣␣␣␣␣␣d2=x(1)+umu
␣␣␣␣␣␣r12=d1*d1+x(2)*x(2)
␣␣␣␣␣␣r22=d2*d2+x(2)*x(2)
␣␣␣␣␣␣r0=dsqrt(r12)
␣␣␣␣␣␣r1=dsqrt(r22)
␣␣␣␣␣␣r032=r12*r0
␣␣␣␣␣␣r132=r22*r1
␣␣␣␣␣␣r052=r12*r032
␣␣␣␣␣␣r152=r22*r132
␣␣␣␣␣␣c1=-umu/r032-xmu/r132
␣␣␣␣␣␣c2=3.d0*umu/r052
␣␣␣␣␣␣c3=3.d0*xmu/r152
␣␣␣␣␣␣␣␣omex=x(1)-(umu*(-xmu+x(1))/r032)-(xmu*(x(1)+umu)/r132)
␣␣␣␣␣␣␣␣omey=x(2)*(1.d0-(umu/r032)-(xmu/r132))
␣␣␣␣␣␣␣␣omexx=c1+c2*d1*d1+c3*d2*d2+1.d0
␣␣␣␣␣␣␣␣omexy=c2*d1*x(2)+c3*d2*x(2)
␣␣␣␣␣␣␣␣omeyy=c1+(c2+c3)*x(2)*x(2)+1.d0
␣␣␣␣␣␣f(1)=x(3)
␣␣␣␣␣␣f(2)=x(4)
␣␣␣␣␣␣f(3)=2.*x(4)+omex
␣␣␣␣␣␣f(4)=-2.*x(3)+omey
␣␣␣␣␣␣return
␣␣␣␣␣␣end
c**********Jacobi-Matrix␣of␣x
␣␣␣␣␣␣subroutine␣JacobiMatrix(n,x,xmu,A)
␣␣␣␣␣␣implicit␣real*8␣(a-h,o-z)
␣␣␣␣␣␣dimension␣x(n),A(n,n)
c
␣␣␣␣␣␣umu=1.d0-xmu
␣␣␣␣␣␣d1=x(1)-xmu
␣␣␣␣␣␣d2=x(1)+umu
␣␣␣␣␣␣r12=d1*d1+x(2)*x(2)
␣␣␣␣␣␣r22=d2*d2+x(2)*x(2)
␣␣␣␣␣␣r0=dsqrt(r12)
␣␣␣␣␣␣r1=dsqrt(r22)
␣␣␣␣␣␣r032=r12*r0
␣␣␣␣␣␣r132=r22*r1
␣␣␣␣␣␣r052=r12*r032
␣␣␣␣␣␣r152=r22*r132
␣␣␣␣␣␣c1=-umu/r032-xmu/r132
␣␣␣␣␣␣c2=3.d0*umu/r052
```

```
      c3=3.d0*xmu/r152
        omex=x(1)-(umu*(-xmu+x(1))/r032)-(xmu*(x(1)+umu)/r132)
        omey=x(2)*(1.d0-(umu/r032)-(xmu/r132))
        omexx=c1+c2*d1*d1+c3*d2*d2+1.d0
        omexy=c2*d1*x(2)+c3*d2*x(2)
        omeyy=c1+(c2+c3)*x(2)*x(2)+1.d0
      A(1,1)=0
      A(1,2)=0
      A(1,3)=1
      A(1,4)=0
      A(2,1)=0
      A(2,2)=0
      A(2,3)=0
      A(2,4)=0
      A(3,1)=omexx
      A(3,2)=omexy
      A(3,3)=0
      A(3,4)=2
      A(4,1)=omexy
      A(4,2)=omeyy
      A(4,3)=-2
      A(4,4)=0
      return
      end
```

2 a)

$$xl3 = 1.0416089091893053$$

$$Cl3 = 3.1895781531155980$$

$$A = \begin{pmatrix} 0.0000000000000000 & 0.0000000000000000 & 1.0000000000000000 & 0.0000000000000000 \\ 0.0000000000000000 & 0.0000000000000000 & 0.0000000000000000 & 0.0000000000000000 \\ 3.1833839641328692 & 0.0000000000000000 & 0.0000000000000000 & 2.0000000000000000 \\ 0.0000000000000000 & -9.1691982066434363E-002 & -2.0000000000000000 & 0.0000000000000000 \end{pmatrix}$$

3 b)

For idir = 1:

$$\lambda_1 = -0.5016$$

$$v_1 = (-0.2894, -0.8457, 0.1452, 0.4242)$$

For idir = -1

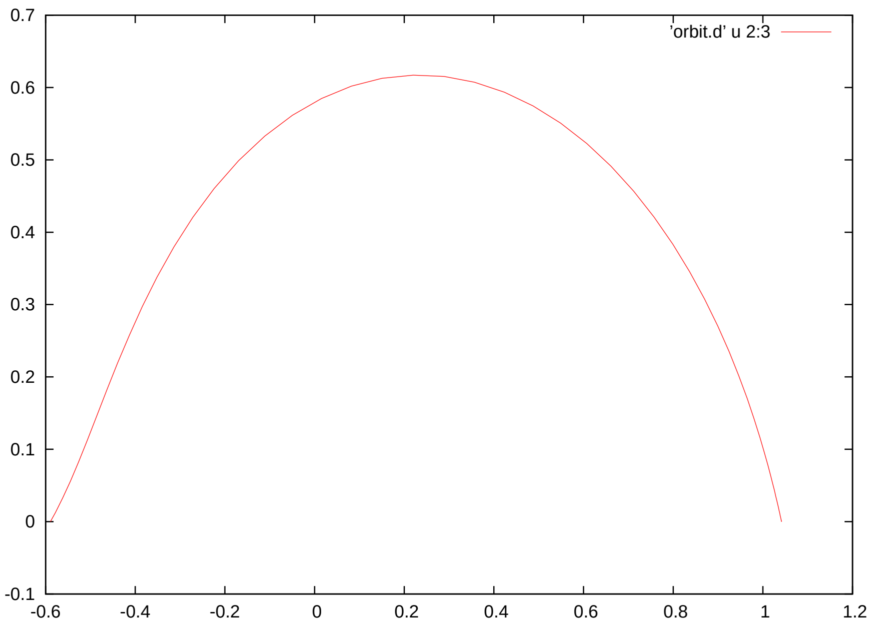
$$\lambda_2 = 0.5016$$

$$v_2 = (-0.2894, 0.8457, -0.1452, 0.4242)$$

3.1 ncrossing = 1, iregion = 1, idir = 1

$x_{init} = (1.0416086197892940, 8.4570002555847169E{-}007, -1.4519999921321868E{-}007, 4.2419999837875363E$

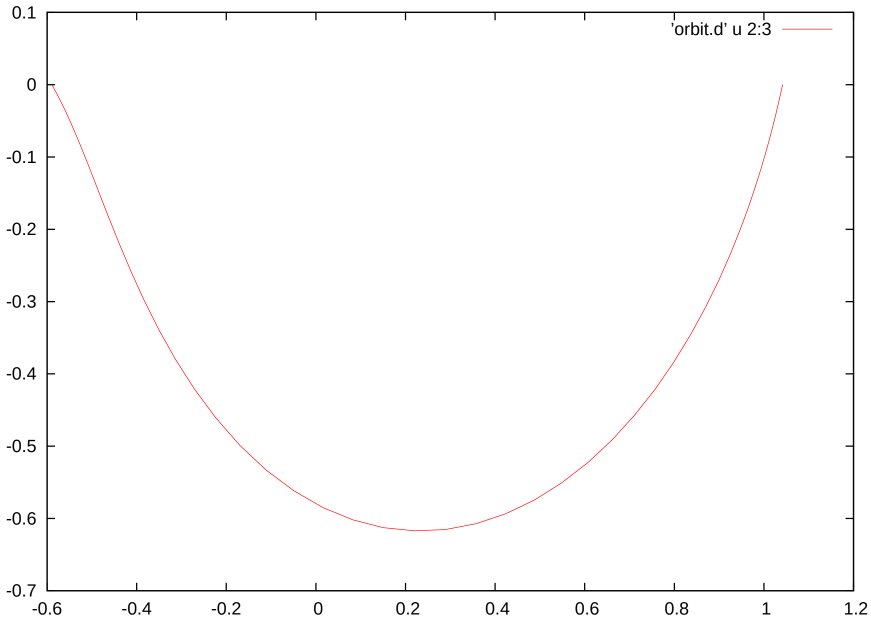
$x_{final} = (-0.58927579507814054, -3.3113893342737215E{-}016, -0.48044497057252672, -0.5214734895794196$



3.2 ncrossing = 1, iregion = 1, idir = -1

$x_{init} = (1.0416086197892940, -8.4570002555847169E{-}007, 1.4519999921321868E{-}007, 4.2419999837875363E$

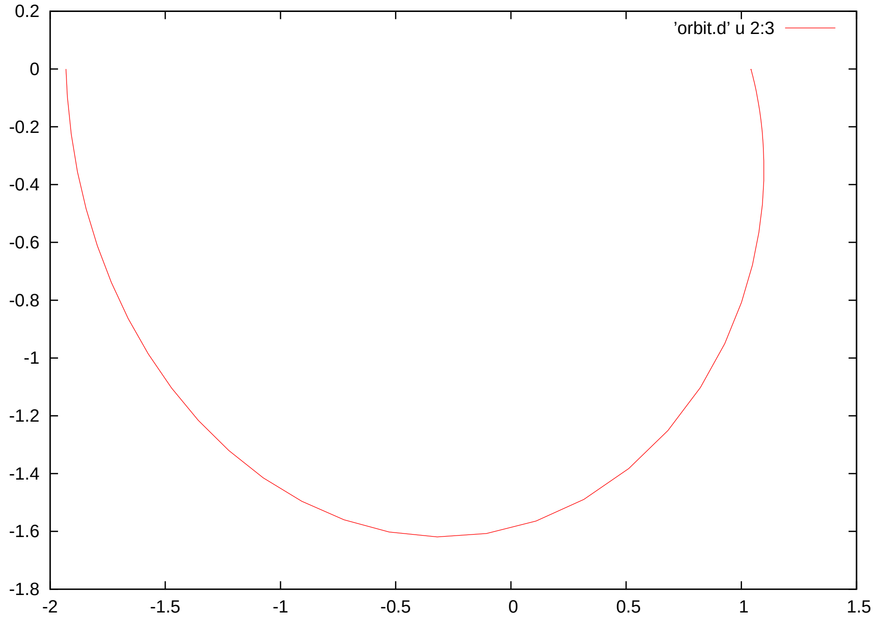
$x_{final} = (-0.58927579507814276, 7.2191906222848855E{-}016, 0.48044497057253011, -0.52147348957941553)$



3.3 ncrossing = 1, iregion = -1, idir = 1

$x_{init} = (1.0416086197892940, -8.4570002555847169E{-}007, 1.4519999921321868E{-}007, -4.2419999837875363$

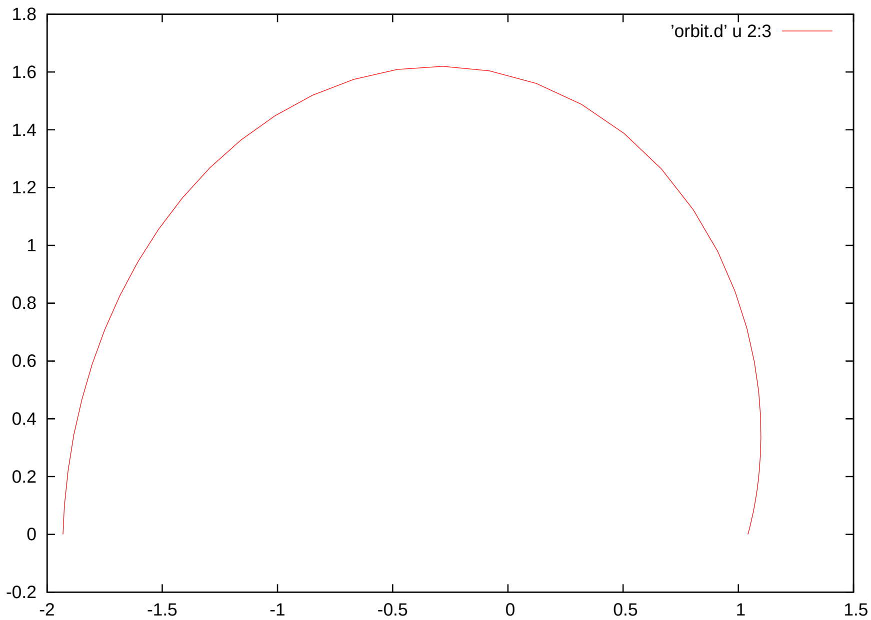
$x_{final} = (-1.9309863889976497, 3.4228581677494186E{-}015, -4.4509982786329026E{-}002, 1.306677844975557$



3.4 ncrossing = 1, iregion = -1, idir = -1

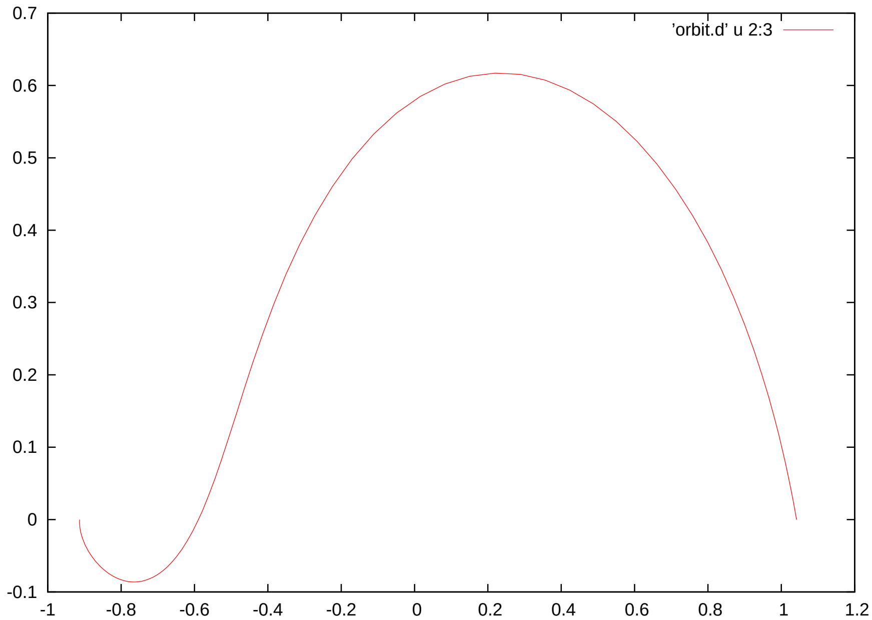
$x_{init} = (1.0416086197892940, 8.4570002555847169E{-}007, -1.4519999921321868E{-}007, -4.2419999837875363$

$x_{final} = (-1.9309863889976489, -3.1732117345874358E{-}015, 4.4509982786328686E{-}002, 1.306677844975569$



3.5 ncrossing = 2, iregion = -1, idir = 1

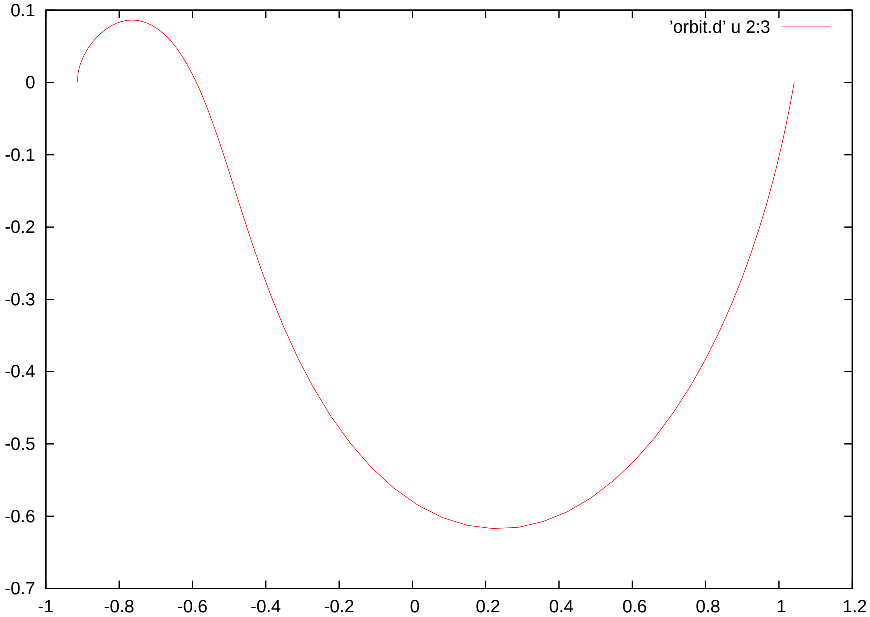
Abbildung 1: iregion = 1, idir = 1, ncrossing = 2



3.6 ncrossing = 2, iregion = 1, idir = -1

$x_{init} = (1.0416086197892940, -8.4570002555847169E{-}007, 1.4519999921321868E{-}007, 4.2419999837875363E$

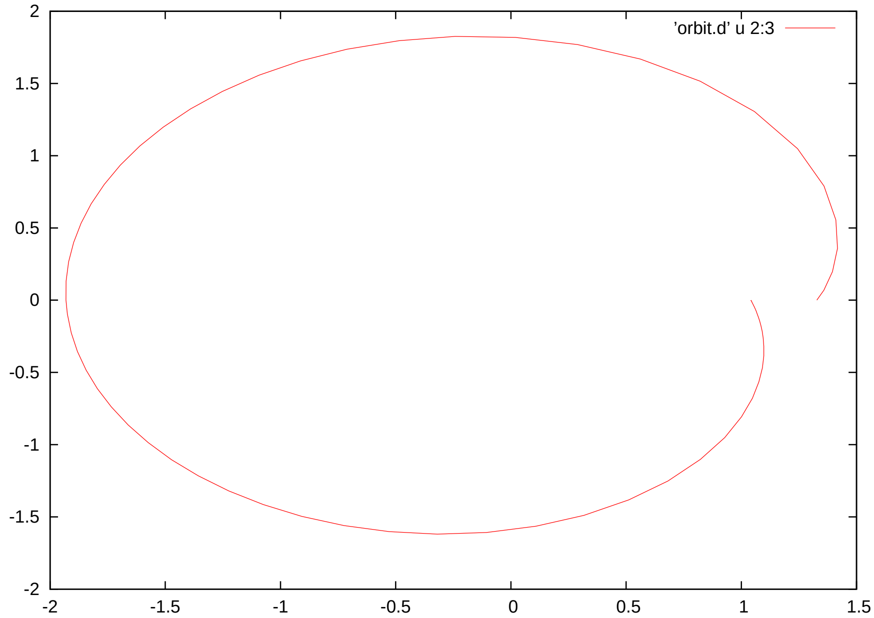
$x_{final} = (-0.913022373485010923.2618286945488629E{-}014{-}0.527569447763149963.8197833807735542)$



3.7 ncrossing = 2, iregion = -1, idir = 1

$x_{init} = (1.0416086197892940, -8.4570002555847169E{-}007, 1.4519999921321868E{-}007, -4.2419999837875363$

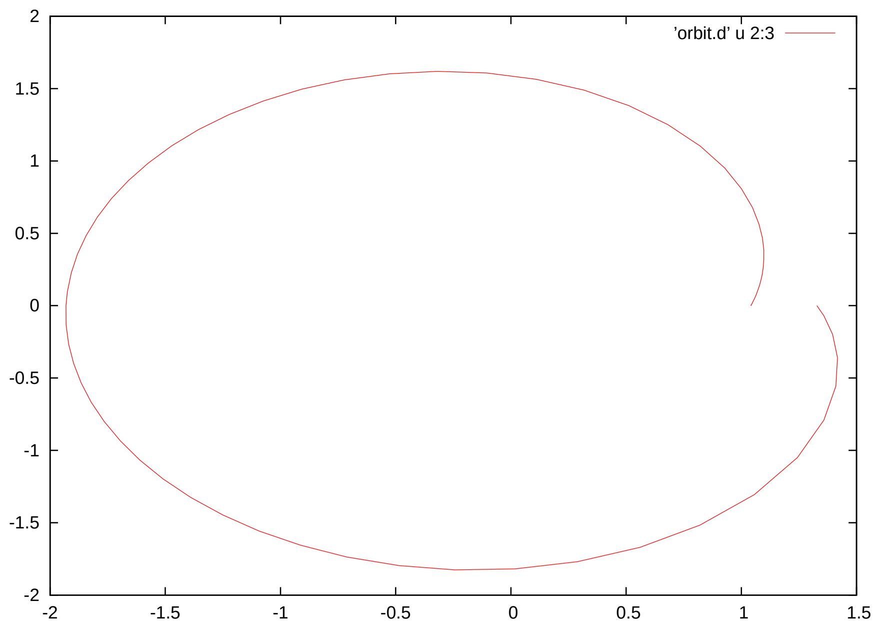
$x_{final} = (1.3278684162625420, 1.1778250122459057E{-}015, -0.20828628078659464, -0.41952382597430521)$



3.8 ncrossing = 2, iregion = -1, idir = -1

$x_{init} = (1.0416086197892940, 8.4570002555847169E{-}007, -1.4519999921321868E{-}007, -4.2419999837875363$

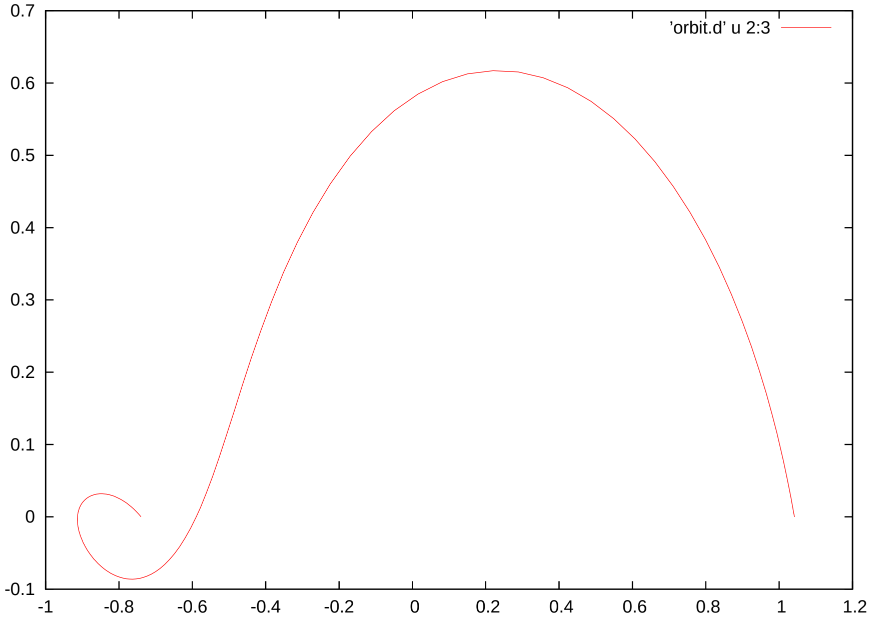
$x_{final} = (1.3278684162625227, 1.2336957766279402E{-}015, 0.20828628078659864, -0.41952382597427490)$



3.9 ncrossing = 3, iregion = 1, idir = 1

$x_{init} = (1.0416086197892940, 8.4570002555847169E{-}007, -1.4519999921321868E{-}007, 4.2419999837875363E$

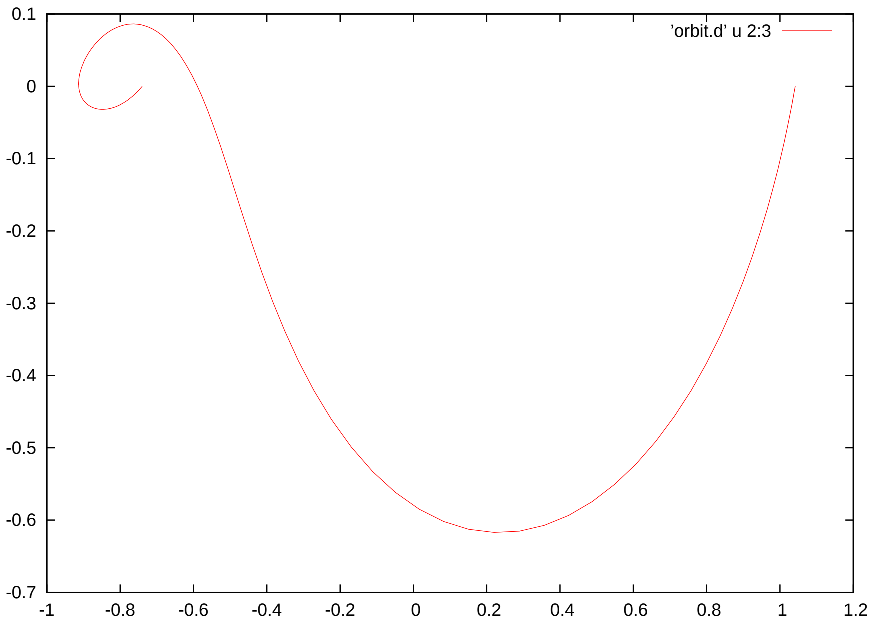
$x_{final} = (-0.73983563139680131, -8.5064738771797817E{-}015, 0.78432829518588376, -0.47392138431732977)$



3.10 ncrossing = 3, iregion = 1, idir = -1

$x_{init} = (1.0416086197892940, -8.4570002555847169E{-}007, 1.4519999921321868E{-}007, 4.2419999837875363E$

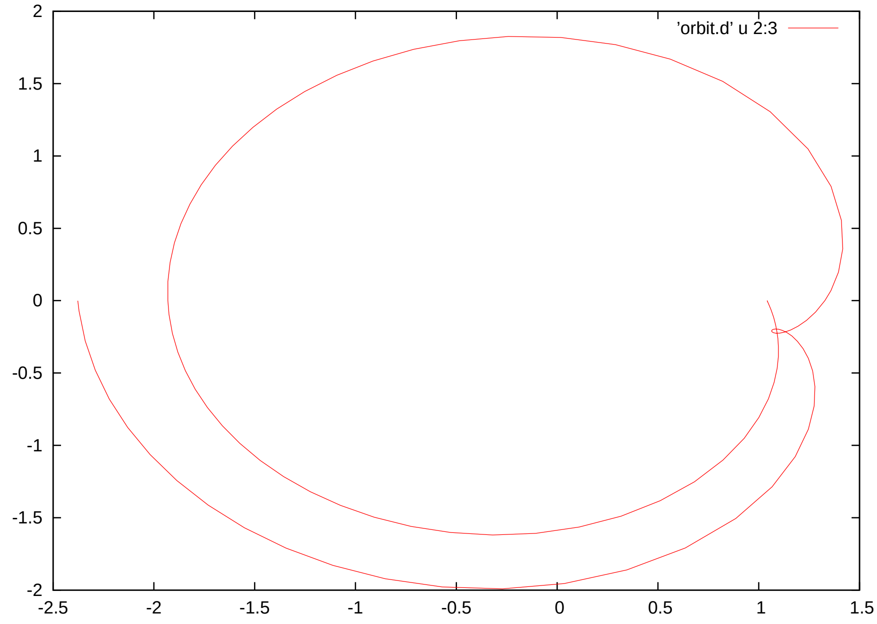
$x_{final} = (-0.73983563139677788, 2.0274205879574222E{-}015, -0.78432829518597946, -0.47392138431727626)$



3.11 ncrossing = 3, iregion = -1, idir = 1

$x_{init} = (1.0416086197892940, -8.4570002555847169E{-}007, 1.4519999921321868E{-}007, -4.2419999837875363$

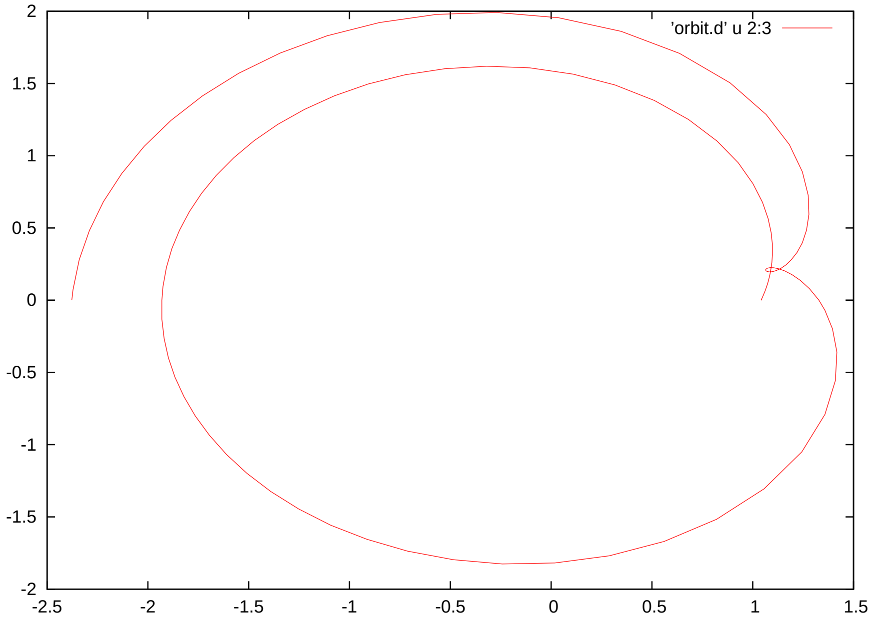
$x_{final} = (-2.3776891788032488, -1.6790423540189206E{-}015, -0.12845655294865468, 1.8436801322508460)$



3.12 ncrossing = 3, iregion = -1, idir = -1

$x_{init} = (1.0416086197892940, 8.4570002555847169E{-}007, -1.4519999921321868E{-}007, -4.2419999837875363$

$x_{final} = (-1.9309863889976489, -3.1732117345874358E{-}015, 4.4509982786328686E{-}002, 1.306677844975569$


'orbit.d' u 2:3
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5