ASSIGNMENT 8 Computation of the Equilibrium Points of the RTBP. Stability for the Elliptic RTBP

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Given the program calc_eqpoints, that gives the values of $L1$ and $L2$, we modify it in order to find also $L3$ and the corresponding Jacobi integrals C_i.

Basically, from theory, we know that $L3$ is $(\mu + \psi, 0, 0, 0)$, and that ψ is found as an equilibrium point of

$$\sqrt[3]{\frac{(1-\mu)(1+\psi)^2}{1+2\mu+\psi(2+\mu+\psi)}}$$

A good initial point to find ψ is

$$\psi_0 = 1 - \frac{7\mu}{12}$$

We implement this in our program, and compute the Jacobi integrals $C = 2\Omega(x, 0)$

Once the program modified to run from $\mu \in (0, 0.5]$ the program reads:

```
      implicit real*8 (a-h,o-z)


      open(10,file='values.d',status='unknown')

C      write(*,*) 'xmu'
C      read(*,*) xmu

C      call peq(xmu,xl1,xl2,xl3,cl1,cl2,cl3)

C      write(*,*) xl1
C      write(*,*) xl2
C      write(*,*) xl3

C      write(*,*) cl1
C      write(*,*) cl2
C      write(*,*) cl3


      DO ii=1,50
```

```
        xmu=ii/100.d0
        call peq(xmu,xl1,xl2,xl3,cl1,cl2,cl3)

        write(10,*) xmu,xl1,xl2,xl3,cl1,cl2,cl3
        endDO

        end




c
c routine to compute xl1,xl2,xl3,Cl2,Cl2,Cl3
c
        subroutine peq(xmu,xl1,xl2,xl3,cl1,cl2,cl3)
        implicit real*8(a-h,o-z)
        a=1.d0/3.d0

        i=0
c to compute L2 (on the left hand side of the small primary)
        x=xmu/(3.d0*(1.d0-xmu))
        x=x**a
  1        den=3.d0-2.d0*xmu+x*(3.d0-xmu+x)
        f=xmu*(1.d0+x)**2/den
        f=f**a
        x1=xmu-1.d0-x
        if (dabs(x-f).le.1.d-15)then
c CALL .... and compute C(L2)
         xl2=X1
             call c(xmu,xl2,cl2)
             go to 3
        endif
        i=i+1
        x=f
        go to 1
   2    format(e25.16,',',e25.16,',',e25.16)
   3    continue

c
c L1 (between the primaries)
```

```
c
        i=0
        x=xmu/(3.d0*(1.d0-xmu))
        x=x**a
  10        den=3.d0-2.d0*xmu-x*(3.d0-xmu-x)
        f=xmu*(1.d0-x)**2/den
        f=f**a
        x1=xmu-1.d0+x
        if (dabs(x-f).le.1.d-15)then
c CALL .... and compute C(L1)
         XL1=X1
             call c(xmu,xl1,cl1)
             go to 4
        endif
        i=i+1
        x=f
        go to 10
   4    continue
c
c L3 (on the right hand side of the big primary)
c
        i=0
        x=1.d0- xmu*7.d0/12.d0
C        x=x**a

  15        den=1.d0+2.d0*xmu + x*(2.d0+xmu+x)

        f=(1.d0-xmu)*(1.d0+x)**2/den
        f=f**a

        x1=xmu+x
        if (dabs(x-f).le.1.d-15)then
c CALL .... and compute C(L1)
         XL3=X1
             call c(xmu,xl3,cl3)
             go to 5
        endif
        i=i+1
        x=f
        go to 15
```

```
   5    continue

        end




c 2. A routine to compute the Jacobi integral 2*Omega(x,y)-(x'^2+y'^2)=C
c  BUT for a collinear equilibrium point, it is simply
c      C=2*Omega(x,0)
c
c
        subroutine c(xmu,xl,cl)
        implicit real*8(a-h,o-z)

        r1=dsqrt((xl-xmu)*(xl-xmu))
        r2=dsqrt((xl-xmu+1)*(xl-xmu+1))

        cl=2.*( 0.5*xl*xl + (1-xmu)/r1 + xmu/r2 + 0.5*xmu*(1-xmu) )
        end
```

Once cheked the test values, we plot different graphics with gnuplot:

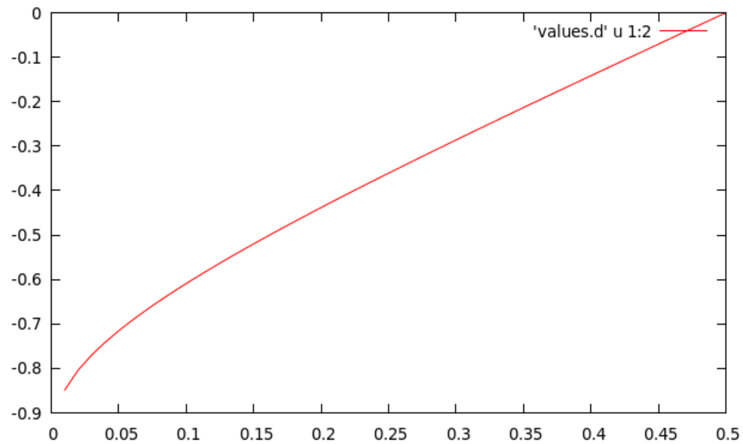

Figure 1: $(\mu, x(L1))$

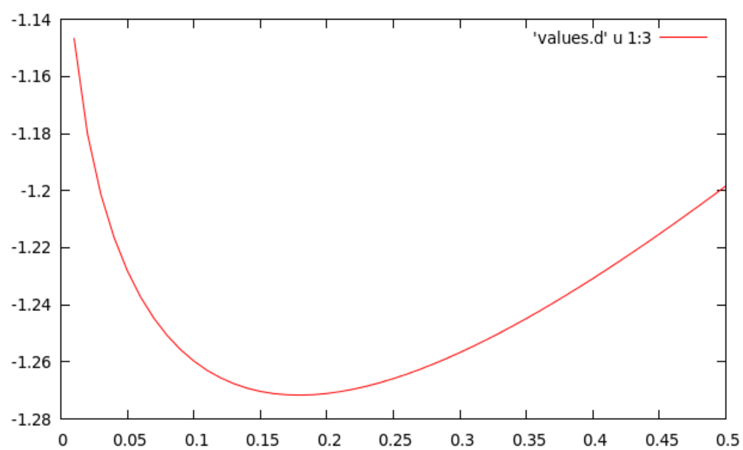

Figure 2: $(\mu, x(L2))$

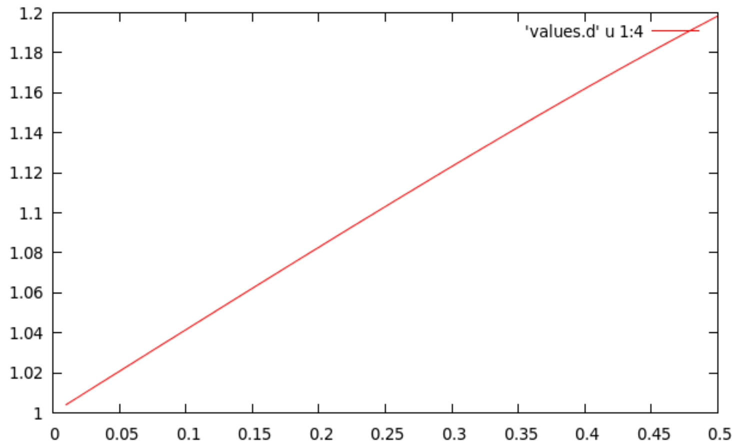

Figure 3: $(\mu, x(L3))$

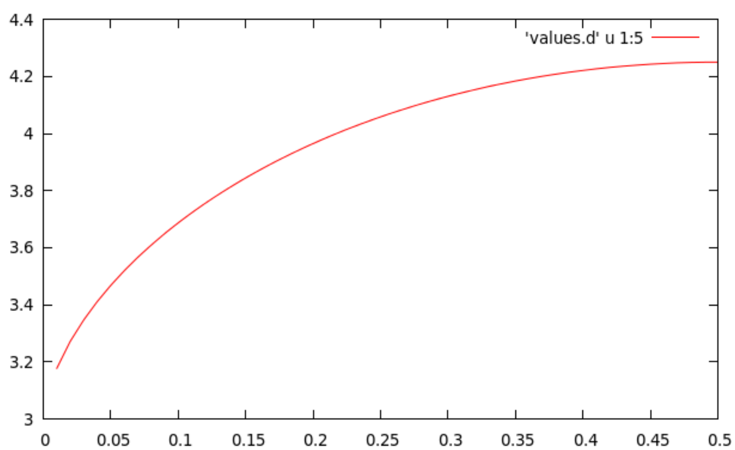

Figure 4: $(\mu, C1)$

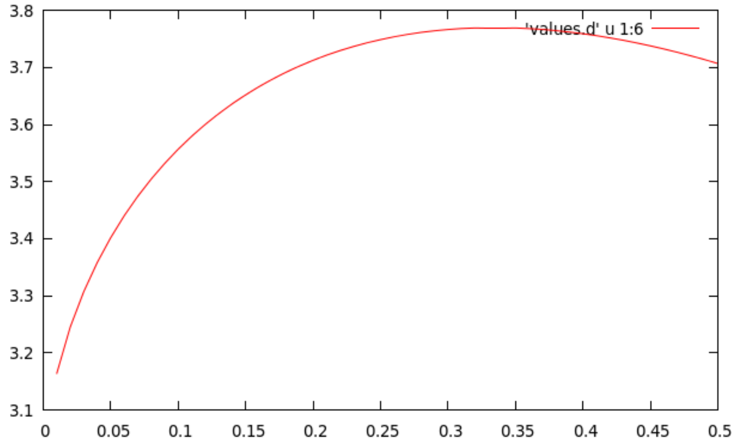

Figure 5: $(\mu, C2)$

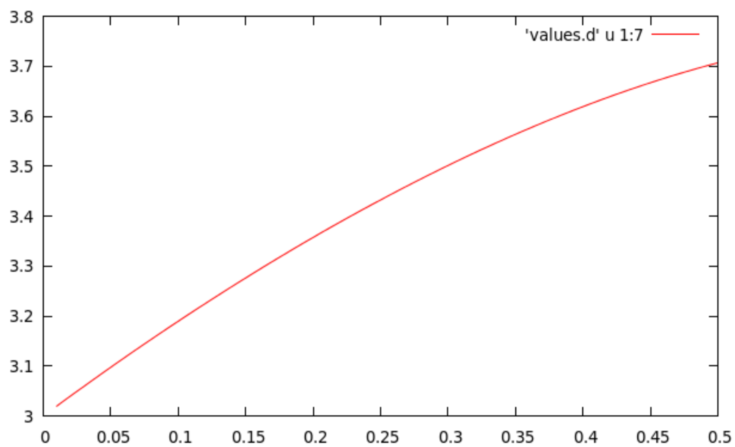

Figure 6: $(\mu, C3)$