ASSIGNMENT 4 Integrating a Linear System of ODE using Taylor

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We want to plot 3 phase portraits of different equilibrium points classes: a center, a focus and a saddle.

In order to do that, we implement a code (a modification of the main_ls_flow.f program) that will allow us to compute the orbits of a ODE in the following form:

```
diff(x1,t)=aa*x1+bb*x2;
diff(x2,t)=cc*x1+dd*x2;
```

The terms aa, bb, cc and dd form a matrix, and will be the input of our program, with the following code:

```
      implicit real*8(a-h,o-z)
      parameter(n=2)
      DIMENSION xi(n),x(n)
      DIMENSION A(n,n)
      common/param/aa,bb,cc,dd

      write(*,*) 'a,b,c,d'
      read(*,*) aa,bb,cc,dd

      write(*,*) 'ti,tmax,np (number of points)'
      read(*,*)TI0,TMAX0,NP0

      write(*,*) 'number of orbits'
      read(*,*)norbs

      open(10,file='orbit.d',status='unknown')


      DO ii=1,norbs
      ti=TI0
      tmax=TMAX0
      np=NP0
      write(*,*) 'Initial condition'
      read(*,*) (xi(i),i=1,n)
```

```
c particular example integration up to t=pi
c        pi=4.d0*datan(1.d0)
c        tmax=pi/2.d0
        if (tmax.ge.ti)then
c         'idir (=1 forward in time, =-1 backward)'
            idir=1
        else
            idir=-1
        endif
        do i=1,n
         x(i)=xi(i)
        enddo
         write(*,*)ti,'   initial t, initial cond:'
         write(*,*)(x(i),i=1,n)
c REMARK: xinctime positive
       xinctime=dabs(tmax-ti)/np
         write (10,*)ti,(x(ii),ii=1,n)
       do 20 i=1,np
         call flow(ti,n,x,idir,xinctime)
         write (10,*)ti,(x(ii),ii=1,n)
 20    continue


       write(*,*)ti,'  final t,  final point:'
       write(*,*)(x(i),i=1,n)
       write(10,50)
50     format()
        end DO

        end

       subroutine flow(t,n,x,idir,xinctemps)
       IMPLICIT REAL*8 (A-H,O-Z)
       dimension x(n)
       tmax=t+idir*xinctemps
c
c parameters for the integration
c
        hab=0.1e-16
```

```
        hre=0.1e-16
        pabs=dlog10(hab)
        prel=dlog10(hre)
c Option of control of step
        istep=1
        ht=0.d0
 1        CALL taylor_f77_eq_ls_(t,x,idir,istep,pabs,prel,
     &  tmax,ht,iordre,ifl)
c        write(10,100) t,(x(i),i=1,n)
        if (idir.eq.1.and.t.lt.tmax)go to 1
        if (idir.eq.-1.and.t.gt.tmax)go to 1
c check t=tmax
        if (dabs(t-tmax).le.1.d-13)return
        write(*,*)'problems in taylor'
        stop
c 100     format(f15.8,2f22.15)
        return




        end
```

To write an undefined number of orbits, we asked the user to give the number of orbits (norbs) to print, and we do a do-loop to compute them. After each orbit computation, we print a blank line (format()) in order to recognize the end of the orbit and the beginning of the next one. We also initialize the initial time after each iteration.

With this program we find the orbits of a center ($aa = 2$, $bb = -5$, $cc = 1$ and $d = -2$) and a focus ($aa = 3$, $bb = -2$, $cc = 4$ and $d = -1$). The corresponding plots are shown in figures 1 and 2 respectively.

To compute the saddle we did a different program in order to plot also the stable and unstable manifolds, as shown in the following code:

```
        implicit real*8 (a-h,o-z)
        parameter (n=2)
        common/param/aa,bb,cc,dd
```

```
dimension xi(n),x(n),a(n,n),vapr(n),vapi(n),vepr(n,n),vepi(n,n)


open(10,file='orbit.d',status='unknown')

write(*,*) 'a,b,c,d'
read(*,*) aa,bb,cc,dd

write(*,*) 'ti,tmax ,np (number of points)'
read(*,*)ti,tmax ,np

write(*,*) 'number of orbits'
read(*,*)norbs

t0=ti
m=-1
xinctime=dabs(tmax -ti)/np
a(1,1)=aa
a(1,2)=bb
a(2,1)=cc
a(2,2)=dd

call vapvep(a,n,vapr,vapi,vepr,vepi)

do k=1,2
m=-1*m
  do j=1,2
  ti=t0
  x(1)=0.d0
  x(2)=0.d0
  idir=dsign(1.d0,vapr(j))
  x(1)=m*vepr(1,j)*1.d-6+x(1)
  x(2)=m*vepr(2,j)*1.d-6+x(2)

  write (10,*)ti ,(x(ii),ii=1,n)
      do i=1,np
        call flow(ti,n,x,idir ,xinctime)
        write (10,*)ti ,(x(ii),ii=1,n)

      enddo
```

```
          write (10,50)
          write(*,*)ti,' final t'
          write(*,*)(x(i),i=1,n),' final point'

          enddo
        enddo

        do j=1,norbs
          ti=t0
          write(*,*) 'Initial condition x(1),x(2)'
          read(*,*) (xi(i),i=1,n)
          idir=-1
          do i=1,n
            x(i)=xi(i)*1.d-6
          enddo

          write(*,*)ti,' initial t, initial cond:'
          write(*,*)(x(i),i=1,n)
          write (10,*)ti ,(x(ii),ii=1,n)
          do i=1,np
            call flow(ti,n,x,idir ,xinctime)
            write (10,*)ti ,(x(ii),ii=1,n)
          enddo

          write (10,50)
          write(*,*)ti,' final t'
          write(*,*)(x(i),i=1,n),' final point'
        enddo

50      format()
        end

       subroutine flow(t,n,x,idir,xinctemps)
       IMPLICIT REAL*8 (A-H,O-Z)
       dimension x(n)
       tmax=t+idir*xinctemps
c
c parameters for the integration
c
```

```
        hab=0.1e-16
        hre=0.1e-16
        pabs=dlog10(hab)
        prel=dlog10(hre)
c Option of control of step
        istep=1
        ht=0.d0
 1        CALL taylor_f77_eq_ls_(t,x,idir,istep,pabs,prel,
     &  tmax,ht,iordre,ifl)
c        write(10,100) t,(x(i),i=1,n)
        if (idir.eq.1.and.t.lt.tmax)go to 1
        if (idir.eq.-1.and.t.gt.tmax)go to 1
c check t=tmax
        if (dabs(t-tmax).le.1.d-13)return
        write(*,*)'problems in taylor'
        stop
c 100     format(f15.8,2f22.15)
        return

        end
```

and we plot it in figure 3

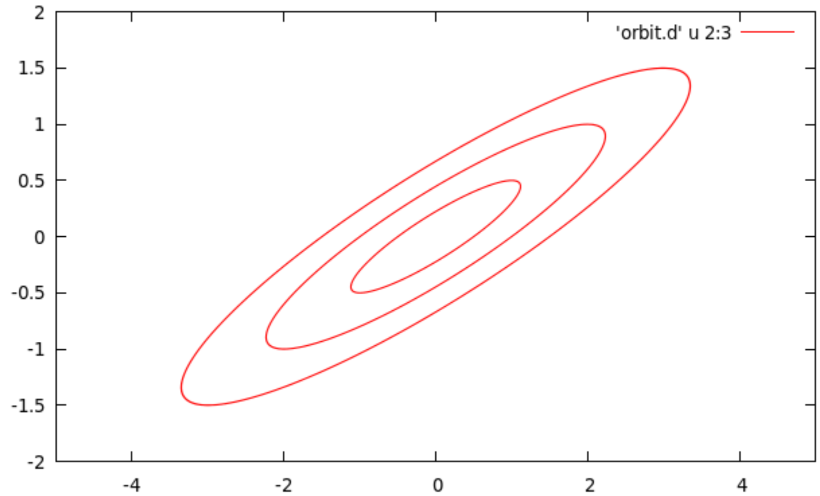

Figure 1: Center

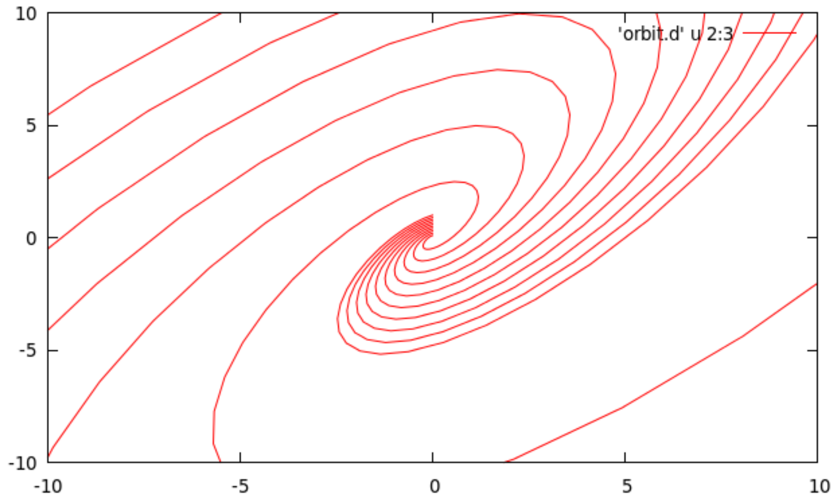

Figure 2: Focus

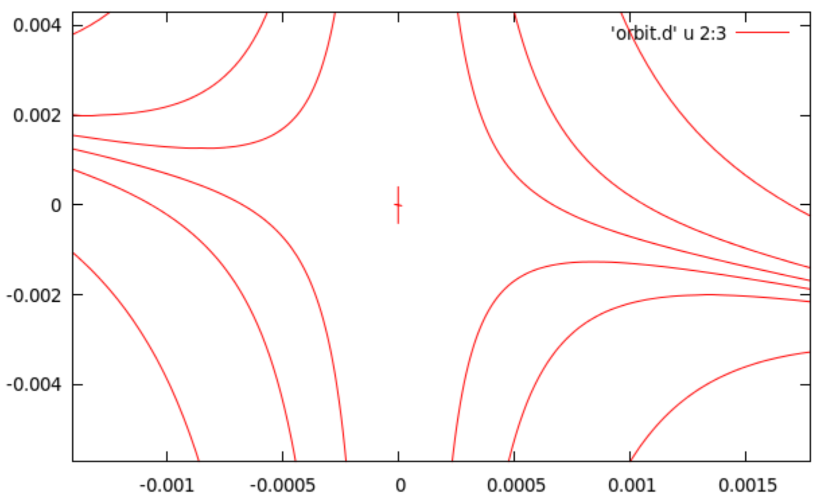

Figure 3: Saddle