ASSIGNMENT 11 Computation of Periodic Orbits for the RTBP

Ignacio Coca

In this assay we will compute symmetric periodic orbits of the RTBP around L3.

For a given C (starting in $C = c(l3)$), we find the first value of x (starting in $x = x(l3)$) such that the first crossing with the Poincare section $y = 0$ is such that $x' = 0$. Doing this for many values of C we find the characteristic curve (C, x).

We will start with $C = c(L3)$ and use an decreasing factor of $1.d - 3$ until we reach $C = 2.9$. For values below 2.9 we find too many singularities.

The program reads:

```
implicit real*8 (a-h,o-z)


parameter (n=4)
dimension x11(n), yf(n), xi(n), x22(n), x11poinc(n), x22poinc(n)
dimension xmiddle(n), xbis1(n), xbis2(n)
common/param/xmu
 open(10,file='orbit.d',status='unknown')

xmu=0.1
call peq(xmu,xl1,xl2,xl3,cl1,cl2,cl3)



xmiddle(1)=xl3
xmiddle(2)=0.d0
xmiddle(3)=0.d0
xmiddle(4)=0.d0



 C=cl3

  xincC=1.d-3
  xincx=1.d-5
```

```
   x11=xmiddle(1)

   x11(2)=0.d0
   x11(3)=0.d0
   call yprime(x11(1),-1, C, x4)
   x11(4)=x4

   x11poinc=x11

   call poinc1(n,x11poinc,yf,tfinal,1)
   x11(3)=yf(3)


DO io=1,500
   C=C-xincC
   if(C.lt.2.9d0)then
   goto 55
   endif

   x11(1)=xmiddle(1)+xincx

   x11(2)=0.d0
   x11(3)=0.d0
   call yprime(x11(1),-1, C, x4)
   x11(4)=x4

   x11poinc=x11

   call poinc1(n,x11poinc,yf,tfinal,1)
   x11(3)=yf(3)



DO i=1,1000



   x22(1)=x11(1)+xincx
   x22(2)=0.d0
```

```
            x22(3)=0.d0
            call yprime(x22(1),-1, C, x4)
            x22(4)=x4

            x22poinc=x22

            call poinc1(n,x22poinc,yf,tfinal,1)
            x22(3)=yf(3)

C            if(x22(3).lt.1.d-12)then
C            go to 50
C            end if

            if((x11(3)*x22(3)).lt.0.d0)then

C            write(*,*) (x11(ik), ik=1,4)
C            write(*,*) (x22(ik), ik=1,4)
            write(*,*) 'found point'

         xbis1=x11
         xbis2=x22

         call bisection(xbis1, xbis2, xmiddle, C)


         write(*,*) x11
         write(*,*) xmiddle
         write(*,*) x22
         write(*,*) C
         write(10, *) C, xmiddle(1)


            goto 50


            endif

            if((x11(3)*x22(3)).gt.0.d0)then

            x11=x22
```

```
          endif

        end DO

50     continue
         xincx=1.d-3
       end DO

55      continue


      end


        subroutine peq(xmu,xl1,xl2,xl3,cl1,cl2,cl3)
        implicit real*8(a-h,o-z)
        a=1.d0/3.d0

        i=0
c to compute L2 (on the left hand side of the small primary)
        x=xmu/(3.d0*(1.d0-xmu))
        x=x**a
  1        den=3.d0-2.d0*xmu+x*(3.d0-xmu+x)
        f=xmu*(1.d0+x)**2/den
        f=f**a
        x1=xmu-1.d0-x
        if (dabs(x-f).le.1.d-15)then
c CALL .... and compute C(L2)
         xl2=X1
             call c(xmu,xl2,cl2)
             go to 3
        endif
        i=i+1
        x=f
        go to 1
   2    format(e25.16,',',e25.16,',',e25.16)
   3    continue

c
```

```
c L1 (between the primaries)
c
       i=0
       x=xmu/(3.d0*(1.d0-xmu))
       x=x**a
  10       den=3.d0-2.d0*xmu-x*(3.d0-xmu-x)
       f=xmu*(1.d0-x)**2/den
       f=f**a
       x1=xmu-1.d0+x
       if (dabs(x-f).le.1.d-15)then
c CALL .... and compute C(L1)
        XL1=X1
            call c(xmu,xl1,cl1)
            go to 4
       endif
       i=i+1
       x=f
       go to 10
   4   continue
c
c L3 (on the right hand side of the big primary)
c
       i=0
       x=1.d0- xmu*7.d0/12.d0
C       x=x**a

  15       den=1.d0+2.d0*xmu + x*(2.d0+xmu+x)

       f=(1.d0-xmu)*(1.d0+x)**2/den
       f=f**a

       x1=xmu+x
       if (dabs(x-f).le.1.d-15)then
c CALL .... and compute C(L1)
        XL3=X1
            call c(xmu,xl3,cl3)
            go to 5
       endif
       i=i+1
       x=f
```

```
      go to 15
5     continue

      end


      subroutine c(xmu,xl,cl)
      implicit real*8(a-h,o-z)

      r1=dsqrt((xl-xmu)*(xl-xmu))
      r2=dsqrt((xl-xmu+1)*(xl-xmu+1))

      cl=2.*( 0.5*xl*xl + (1-xmu)/r1 + xmu/r2 + 0.5*xmu*(1-xmu) )
      end


    subroutine bisection(xa, xb, xmig, C)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION xa(4), xb(4), xmid(4), xmig(4)

    call f(xa,fa)
    call f(xb,fb)


    DO ip=1,100

      xmid(1)=0.5d0*(xa(1)+xb(1))
      xmid(2)=0.d0
      xmid(3)=0.d0
      call yprime(xmid(1),-1, C, x4)
      xmid(4)=x4

      call f(xmid,fmid)
      xmid(3)=fmid

      if (dabs(xmid(3)).le.1.0d-11)go to 2

      if ((fa*fmid).le.0.d0)then
            xb=xmid
      endif
```

```
          if (fb*fmid.le.0.d0)then
                xa=xmid
          endif
C          write(*,*) fmid
        end do



  2     xmig=xmid
        return
        end


        subroutine f(x,valor)
        implicit real*8(a-h,o-z)
        DIMENSION temp(4), yf(4), x(4)

        temp=x
        call poinc1(4,temp,yf,tfinal,1)
        valor=yf(3)
        return
        end


        subroutine yprime(x1,isignx4, C, x4)
        implicit real*8(a-h,o-z)
        common/param/xmu

        x2=0.d0

        r1=dsqrt((x1-xmu)*(x1-xmu)+x2*x2)
         r2=dsqrt((x1-xmu+1.d0)*(x1-xmu+1.d0)+x2*x2)
         omeg=0.5d0*(x1*x1+x2*x2)+(1.d0-xmu)/r1

       .  +xmu/r2+0.5d0*(1.d0-xmu)*xmu

        x4=isignx4*dsqrt((2.d0*omeg)-C)

        return
        end
```

```
      SUBROUTINE POINC1(n,YI,YF,tfinal,idirorig)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION YI(n),YF(n),DGG(n),F(n)

         icont=0
      idir=idirorig
c
c  we assume initial time t=0.
c
      ti=0.D0
C   DETERMINATION OF THE FIRST PASSAGE OF THE ORBIT THROUGH y=0
C
      CALL SECCIO(YI,GG,DGG)
      IF(DABS(GG).LT.1.D-9)GG=0.d0
      GA=GG
      hab=.1e-16
      hre=.1e-16
      pabs=dlog10(hab)
      prel=dlog10(hre)
      istep=1
c reasonable step:
      pas=0.4d0
      ht=0.d0
      t=ti
c |tmax| must be big enough
1      tmax=t+idir*pas
      CALL taylor_f77_eq_rtbp_(t,yi,idir,istep,pabs,prel,
     & tmax,ht,iordre,ifl)
c computation of first integral to be done
C
      CALL SECCIO(YI,GG,DGG)
      IF(GG*GA.LT.0.D0)go to 22
C      write(10,*)tfinal,(yi(ii),ii=1,2)
      GA=GG
      GO TO 1
C
C   REFINEMENT OF THE INTERSECTION POINT YF(*) USING NEWTON'S METHOD
C   TO GET A ZERO OF THE FUNCTION GG (SEE SUBROUTINE SECCIO)
```

```
C
  22    continue
        icont=icont+1
        if (icont.gt.20)then
            write(*,*)'problems finding the section'
            stop
        endif
        CALL FIELD(T,YI,N,F)
        P=0.D0
        DO 3 I=1,N
3       P=P+F(I)*DGG(I)
        H=-GG/P
c check p is not (or very close to) 0:  to be done
        if (h.ge.0.d0)idir=1
        if (h.lt.0.d0)idir=-1
        tmax=t+h
c          write(*,*)icont,'  refining: h and time ',h,tmax
c          write(*,*)'refining  t point ',t,yi(1),yi(2)
        CALL taylor_f77_eq_rtbp_(t,yi,idir,istep,pabs,prel,
     &  tmax,ht,iordre,ifl)
        CALL SECCIO(YI,GG,DGG)
        IF(DABS(GG).GT.1.D-13)GO TO 22
        DO 4 I=1,N
4       YF(I)=YI(I)
        tfinal=t+tfinal
c check first integral: to be done
C        write(*,*)'tfinal point time ',tfinal
C        write(*,*)(yf(ii),ii=1,n)
C        write(10,*)tfinal,(yf(ii),ii=1,2)
        return
        end


C*******************************************************************************
C                                                                              *
C     THE SURFACE g OF SECTION,IN THIS CASE
C          INPUT PARAMETERS:
C     Y(*)       POINT
C          OUTPUT PARAMETERS:
C     GG          FUNCTION THAT EQUATED TO 0 GIVES THE SURFACE OF
```

```
C                 SECTION
C     DGG(*)      GRADIENT OF FUNCTION GG
C                                                                           *
C****************************************************************************
        SUBROUTINE SECCIO(Y,GG,DGG)
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION Y(2),DGG(2)
        GG=Y(2)
        DO 1 I=1,2
 1      DGG(I)=0.D0
        DGG(2)=1.d0
        RETURN
        END



C
C FIELD.F
C
C****************************************************************************
C
C     EQS OF MOTION IN synodical VARIABLES
C     X          TIME
C     Y(*)       POINT (Y(1),Y(2),....Y(n))
C     NEQ        NUMBER OF EQUATIONS
C          OUTPUT PARAMETERS:
C     F(*)       VECTOR FIELD
C
C****************************************************************************
        subroutine field(t,u,neq,f)
        implicit real*8 (a-h,o-z)
        common/param/xmu
        dimension u(4),f(4)
c


        umu=1.-xmu
        d1=u(1)-xmu
        d2=u(1)+umu
```

```
        r12=d1*d1+u(2)*u(2)
        r22=d2*d2+u(2)*u(2)
        r0=dsqrt(r12)
        r1=dsqrt(r22)

        r032=r12*r0
        r132=r22*r1
        r052=r12*r032
        r152=r22*r132

        omex=u(1)-(umu*(-xmu+u(1))/r032)-(xmu*(u(1)+umu)/r132)
        omey=u(2)*(1.-(umu/r032)-(xmu/r132))

omexx=1.-(umu*((r0*r0)-3.*d1)/(r0*r0*r0*r0*r0))
     .    -(xmu*( (r1*r1)-(3.*(umu+u(1))*(umu+u(1))) )/(r1*r1*r1*r1*r1))
omexy=u(2)*(((3.*umu*d1)/(r0*r0*r0*r0*r0))
     .    +(3.*xmu*(u(1)+umu))/ (r1*r1*r1*r1*r1))
omeyy=(1.-(umu/(r0*r0*r0))-(xmu/(r1*r1*r1))) + ( u(2)* ((3.
     .    *umu*u(2))/ (r0*r0*r0*r0*r0) )+ (xmu*3.*u(2))
     .      / (r1*r1*r1*r1*r1) )



        f(1)=u(3)
        f(2)= u(4)
        f(3)=2.*u(4)+omex
        f(4)=-2.*u(3)+omey



        return
        end
```

We note that in order to find the value of x for which we have $x' = 0$ we fist look for two values with different sign of their x-derivative, and then use the bisection method for them.

Ploting the characteristic curve, we have:

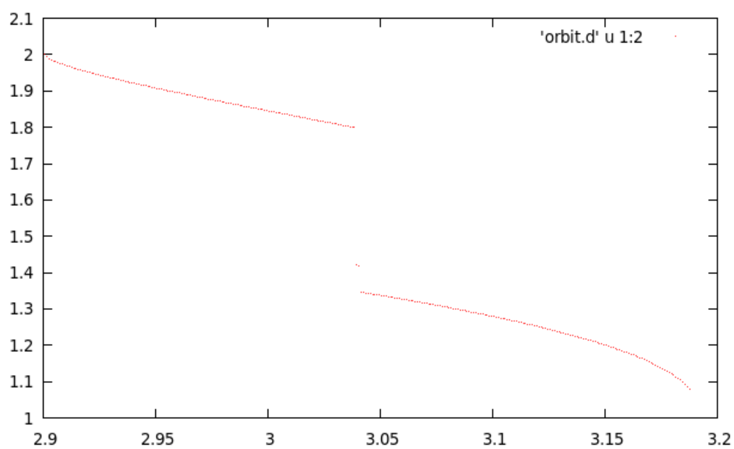

Figure 1: (C, x)