Assignment 10 and optional part–Computation of homoclinic orbits of the equilibrium points

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1 Restricted Three-Body Problem

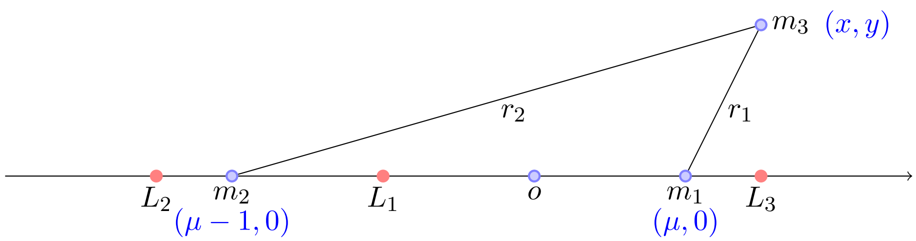


Figure 1: Restricted Three-Body Problem

The equations of motion are:

$$\begin{cases} x'' - 2y' & = \Omega_x \\ y'' + 2x' & = \Omega_y \end{cases} \tag{1}$$

And,

$$\Omega(x, y) = \frac{(x^2 + y^2)}{2} + \frac{1 - \mu}{r_1} + \frac{\mu}{r_2} + \frac{\mu(1 - \mu)}{2} \tag{2}$$

Here:

$$
\begin{cases}
\mu & = \frac{m_2}{m_1+m_2} \\
r_1 & = \sqrt{(x-\mu)^2+y^2} \\
r_2 & = \sqrt{(x-\mu+1)^2+y^2}
\end{cases} \tag{3}
$$

Let:

$$
\begin{cases}
x_1 & = x \\
x_2 & = y \\
x_3 & = x' \\
x_4 & = y'
\end{cases} \tag{4}
$$

The RTBP is expressed as:

$$
\begin{cases}
f_1 & = x_1' = x_3 \\
f_2 & = x_2' = x_4 \\
f_3 & = x_3' = 2x_4 + \Omega_{x_1} \\
f_4 & = x_4' = -2x_3 + \Omega_{x_2}
\end{cases} \tag{5}
$$

Where,

$$
\begin{cases}
\Omega_{x_1} & = x_1 - \frac{(1-\mu)(x_1-\mu)}{{r_1}^3} - \frac{\mu(x_1-\mu+1)}{{r_2}^3} \\
\Omega_{x_2} & = x_2(1 - \frac{1-\mu}{{r_1}^3} - \frac{\mu}{{r_2}^3})
\end{cases} \tag{6}
$$

2 Equilibrium points—L_1, L_2, L_3

The position of L_1 is:

$$
x_{L_1} = \mu - 1 + \xi \tag{7}
$$

where,

$$
f(\xi) = (\frac{\mu(1-\xi)^2}{3-2\mu-\xi(3-\mu-\xi)})^{\frac{1}{3}}
$$

$$
\xi_0 = (\frac{\mu}{3(1-\mu)})^{\frac{1}{3}}
$$

$$
\xi_{n+1} = f(\xi_n)
$$

The position of L_2 is:

$$
x_{L_2} = \mu - 1 - \xi \tag{8}
$$

where,

$$f(\xi) = (\frac{\mu(1+\xi)^2}{3-2\mu+\xi(3-\mu+\xi)})^{\frac{1}{3}}$$
$$\xi_0 = (\frac{\mu}{3(1-\mu)})^{\frac{1}{3}}$$
$$\xi_{n+1} = f(\xi_n)$$

The position of L_3 is:

$$x_{L_3} = \mu + \xi \tag{9}$$

where,

$$f(\xi) = (\frac{(1-\mu)(1+\xi)^2}{1+2\mu+\xi(2+\mu+\xi)})^{\frac{1}{3}}$$
$$\xi_0 = 1 - \frac{7}{12}\mu$$
$$\xi_{n+1} = f(\xi_n)$$

3 result

$\mu = 0.008$, $L_3 = 1.0033333053755920$
The first crossing with Poincare section y=0, the point is:

$$\begin{pmatrix} 0.59826229280258836 \\ 2.9575936557792854^{-14} \\ -0.26780932584763739 \\ 0.80590085785342314 \end{pmatrix}$$

The unstable manifold of L_3, with initial condition: $L_3 - 10^{-6} \cdot \overrightarrow{v}$.

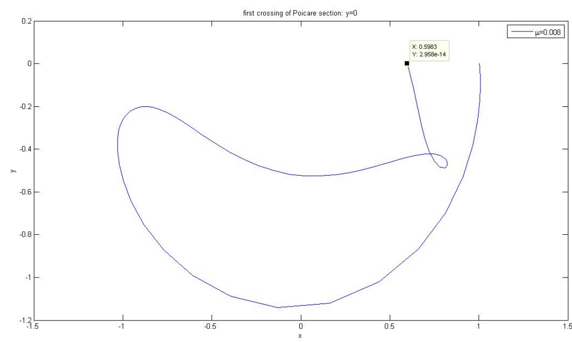


Figure 2: $\mu = 0.008, 1st$-crossing

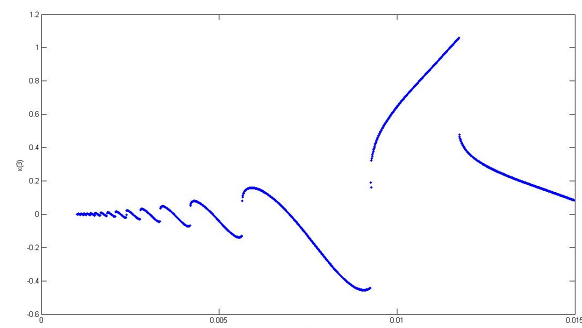


Figure 3: x' V.s μ in the interval (0.001,0.015)

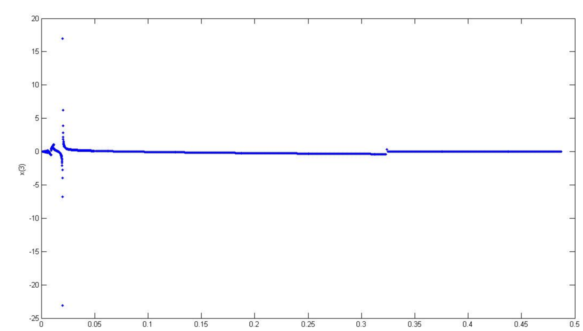


Figure 4: x' V.s μ in the interval (0.001,0.5)

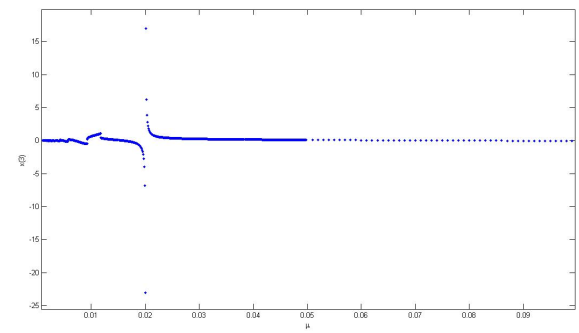


Figure 5: x' V.s μ in the interval (0.001,0.1)

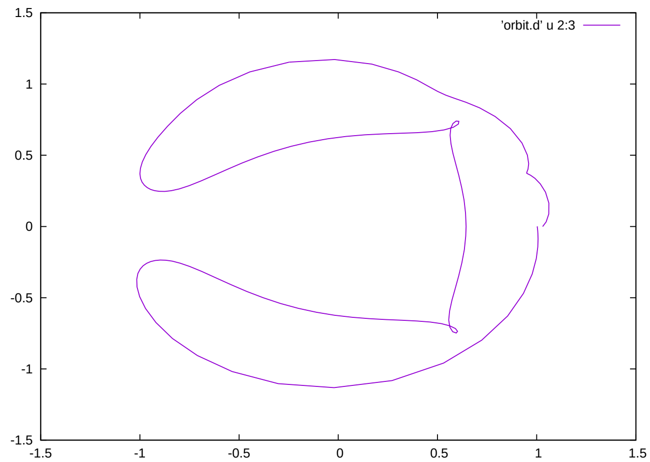


Figure 6: $\mu = 6.96499999^{-3}, 2nd$ crossing

4 Code

Main function:

```
      implicit real*8 (a-h,o-z)
      parameter (n=4)
      common/param/xmu
      dimension x(n),oa(n,n),rr(n),ri(n),vr(n,n),vi(n,n)
      open(10,file='orbit.d',status='unknown')
      open(11,file='relation.d',status='unknown')
      ti=0
      T=0.3138977039438897d01
      tmax=20.d0*T
      np=10
      p=-1
      write(*,*)'xincmu:'
      read(*,*) xincmu
      write(*,*)'xmuLow:'
      read(*,*) xmuLow
      write(*,*)'xmuUpp:'
      read(*,*) xmuUpp
      m=(xmuUpp-xmuLow)/xincmu-2
      write (*,*)'m:', m
      do j=1,m
         xmu=xmuLow+j*xincmu
         call peq(xmu,xl1,xl2,xl3,cl1,cl2,cl3)
         call jacobiA(xmu,xl3,n,oa)
         call vapvep(oa,n,rr,ri,vr,vi)
         x(1)=xl3
         x(2)=0.d0
         x(3)=0.d0
         x(4)=0.d0
         idir=dsign(1.d0,rr(3))
         x(1)=p*vr(1,3)*1.d-6+x(1)
         x(2)=p*vr(2,3)*1.d-6+x(2)
         x(3)=p*vr(3,3)*1.d-6+x(3)
         x(4)=p*vr(4,3)*1.d-6+x(4)
         call poinc1(n,x,tfinal,idir)
         write(11,*)xmu,x(1),x(3)
      enddo
      end


c
   ********************************************************************************
c
```

```
C
   ******************************************************************************

      SUBROUTINE POINC1(n,YI,tfinal,idirorig)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION YI(n),YF(n),DGG(n),F(n)
      icont=0
      idir=idirorig
      ti=0.d0
C   DETERMINATION OF THE FIRST PASSAGE OF THE ORBIT THROUGH
   y=0
      CALL SECCIO(YI,GG,DGG)
      IF(DABS(GG).LT.1.D-9)GG=0.d0
      GA=GG
      hab=.1e-16
      hre=.1e-16
      pabs=dlog10(hab)
      prel=dlog10(hre)
      istep=1
      pas=5d0
      ht=0.d0
      t=ti
1      tmax=t+idir*pas
      CALL taylor_f77_eq_rtbp_(t,yi,idir,istep,pabs,prel,
     &  tmax,ht,iordre,ifl)
      CALL SECCIO(YI,GG,DGG)
      IF(GG*GA.LT.0.D0)go to 22
      write(10,*)t,(yi(ii),ii=1,4)
      GA=GG
      GO TO 1
C
C   REFINEMENT OF THE INTERSECTION POINT YF(*) USING NEWTON
   METHOD
C   TO GET A ZERO OF THE FUNCTION GG (SEE SUBROUTINE SECCIO)
C
22    continue
      icont=icont+1
      if (icont.gt.20)then
          write(*,*)'problems finding the section'
          return
      endif
      CALL FIELD(T,YI,N,F)
      P=0.D0
      DO 3 I=1,N
3     P=P+F(I)*DGG(I)
      H=-GG/P
      if (h.ge.0.d0)idir=1
```

```
      if (h.lt.0.d0)idir=-1
      tmax=t+h

      CALL taylor_f77_eq_rtbp_(t,yi,idir,istep,pabs,prel,
     & tmax,ht,iordre,ifl)
      CALL SECCIO(YI,GG,DGG)
      IF(DABS(GG).GT.1.D-13)GO TO 22
      DO 4 I=1,N
4     YF(I)=YI(I)
      tfinal=t
      write(10,*)t,(yf(ii),ii=1,4)
      return
      end


C
    ************************************************************************

C

    *
C     THE SURFACE g OF SECTION,IN THIS CASE
C          INPUT PARAMETERS:
C     Y(*)        POINT
C          OUTPUT PARAMETERS:
C     GG           FUNCTION THAT EQUATED TO 0 GIVES THE
    SURFACE OF
C                  SECTION
C     DGG(*)       GRADIENT OF FUNCTION GG
C

    *
C
    ************************************************************************

      SUBROUTINE SECCIO(Y,GG,DGG)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Y(4),DGG(4)
      GG=Y(2)
      DO 1 I=1,4
 1    DGG(I)=0.D0
      DGG(2)=1.d0
      RETURN
      END


C
C
    ************************************************************************
```

```
C
C     EQS OF MOTION IN synodical VARIABLES
C     X           TIME
C     Y(*)        POINT (Y(1),Y(2),....Y(n))
C     NEQ         NUMBER OF EQUATIONS
C          OUTPUT PARAMETERS:
C     F(*)        VECTOR FIELD
C
C
    ********************************************************************************

      subroutine field(t,x,neq,f)
      implicit real*8 (a-h,o-z)
      common/param/xmu
      dimension x(neq),f(neq)
      umu=1.-xmu
      d1=x1-xmu
      d2=x1+umu
      r12=d1*d1+x2*x2
      r22=d2*d2+x2*x2
      r0=sqrt(r12)
      r1=sqrt(r22)
      r032=r12*r0
      r132=r22*r1
      omex=x1-(umu*(-xmu+x1)/r032)-(xmu*(x1+umu)/r132)
      omey=x2*(1.-(umu/r032)-(xmu/r132))
      f(1)=x(3)
      f(2)=x(4)
      f(3)=2*x(4)+omex
      f(4)=-2*x(3)+omey
      return
      end

C
    ********************************************************************************

C
C
    ********************************************************************************

       subroutine jacobiA(xmu,x,n,oa)
        implicit real*8(a-h,o-z)
        dimension oa(n,n)
        umu=1.d0-xmu
        x1=x
        xd1=dabs(x1-xmu)
        xd2=dabs(x1+umu)
        r032=xd1**3
```

```
      r132=xd2**3
      r052=xd1**5
      r152=xd2**5
      xa=3*umu*(-xmu+x1)/r052
      xb=3*xmu*(x1+umu)/r152
      omexx=1.d0-umu/r032+xa*(-xmu+x1)-xmu/r132+xb*(x1+umu
     )
      omeyy=1.d0-umu/r032-xmu/r132
      oa(1,1)=0.d0
      oa(1,2)=0.d0
      oa(1,3)=1.d0
      oa(1,4)=0.d0
      oa(2,1)=0.d0
      oa(2,2)=0.d0
      oa(2,3)=0.d0
      oa(2,4)=1.d0
      oa(3,1)=omexx
      oa(3,2)=0.d0
      oa(3,3)=0.d0
      oa(3,4)=2.d0
      oa(4,1)=0.d0
      oa(4,2)=omeyy
      oa(4,3)=-2.d0
      oa(4,4)=0.d0
      return
      end
```

5 optional part–some particular cases

5.1 The curve (μ, x)

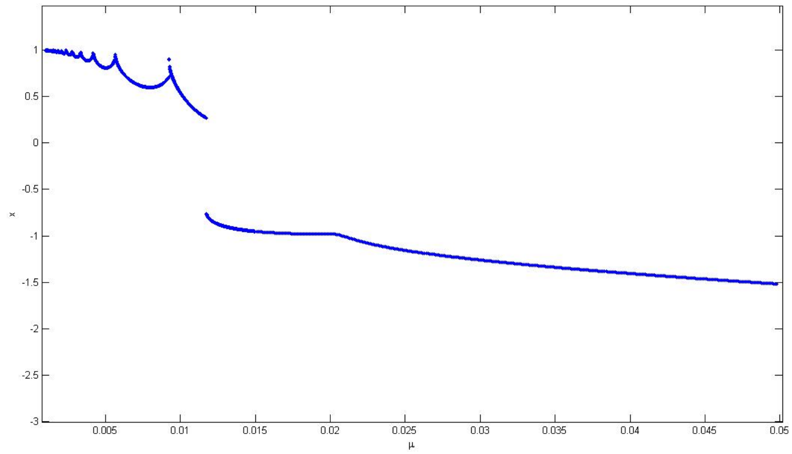


Figure 7: x V.s μ in the interval (0.001,0.015)

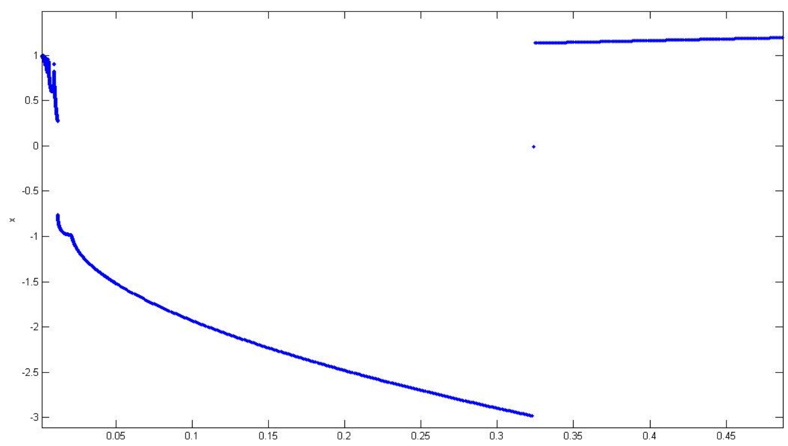


Figure 8: x V.s μ in the interval (0.001,0.5)

5.2 Some fixed μ and comments

From the relationship between μ and x, i.e. figures 7 and 8, we can see in interval $\mu \in (0.001, 0.5)$, it is separated into three parts by $\mu \approx 0.01174$ and $\mu \approx 0.3245$. In the first part, the value of x is around 1, it means the first crossing is closed to the equilibrium point L_3, which is showed in figures

9,**??** and 11. The overall trend of first crossing point goes farther away from the L_3 as μ increasing, despiting the oscillations at the beginning. When μ is bigger than 0.01174, it reaches the second part, where the first crossing is around -1. We can see the examples from figures 12, 13 and 14. In the third part, the first crossing is around 1 again.
From the relationship between μ and x', i.e. figures 3 and 4, we can see when μ is smaller than 0.01, there are several jumps in the x'. i.e. in one point(the first crossing), there are two different x', which brings a small loop. And this loop is symmetric with respect to x-axis, as showed in figure 10b($\mu = 0.00567$, $2nd$ crossing). Besides, we can see, there is a singular point $\mu \approx 0.02$. It represents that the first-crossing collides with m_2.

For the μ is equal to 0.0056, 0.00567, 0.0057 respectively.
For the μ is equal to 0.01179, 0.0159375, 0.083 respectively.

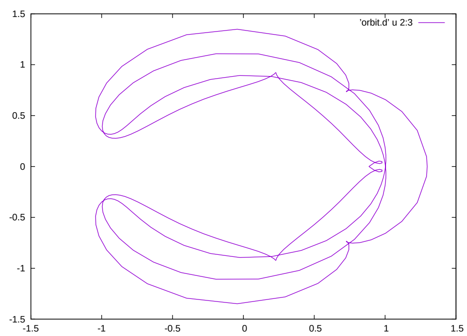
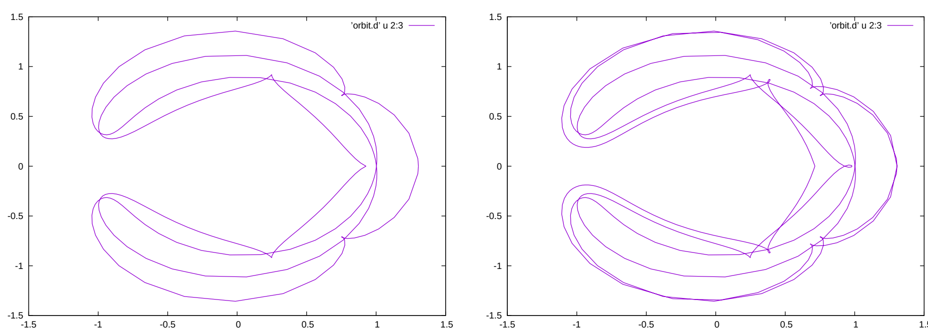


Figure 9: $\mu = 0.0056$



(a) 1st crossing

(b) 2nd crossing

Figure 10: $\mu = 0.00567$

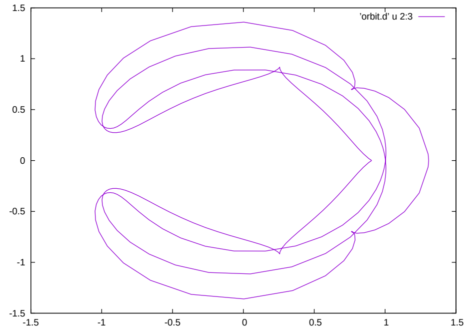


Figure 11: $\mu = 0.0057$

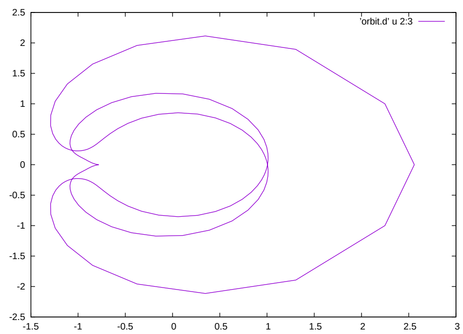


Figure 12: $\mu = 0.01179$

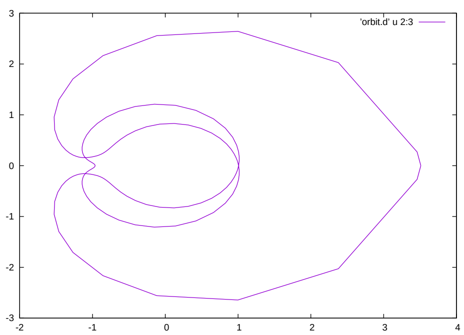


Figure 13: $\mu = 0.0159375$

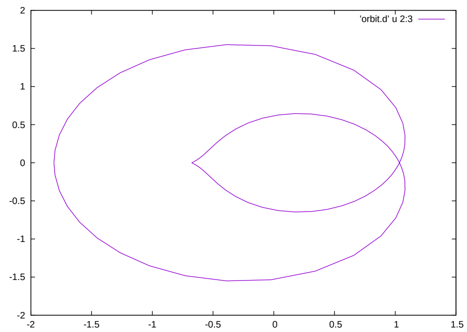


Figure 14: $\mu = 0.083$

6 Code

Main function:

```
      implicit real*8 (a-h,o-z)
      parameter (n=4)
      common/param/xmu
      dimension x(n),oa(n,n),rr(n),ri(n),vr(n,n),vi(n,n)
      open(10,file='orbit.d',status='unknown')
      open(11,file='relation.d',status='unknown')
      ti=0
      T=0.3138977039438897d01
      tmax=20.d0*T
      np=10
      p=-1
      write(*,*)'mu:'
      read(*,*)xmu

     call peq(xmu,xl1,xl2,xl3,cl1,cl2,cl3)
     call jacobiA(xmu,xl3,n,oa)
     call vapvep(oa,n,rr,ri,vr,vi)
      do k=1,2
         p=-1*p
         write(*,*)'p:', p
         do j=3,4
            x(1)=xl3
            x(2)=0.d0
            x(3)=0.d0
            x(4)=0.d0
            idir=dsign(1.d0,rr(j))
            write(*,*)'idir:', idir
            x(1)=p*vr(1,j)*1.d-6+x(1)
```

```
            x(2)=p*vr(2,j)*1.d-6+x(2)
            x(3)=p*vr(3,j)*1.d-6+x(3)
            x(4)=p*vr(4,j)*1.d-6+x(4)
            write(*,*)'Initial t:'
            write(*,*)ti
            write(*,*)'Initial cond:'
            write(*,*)(x(ii),ii=1,n)
            write(*,*) 'm times crossing'
            read(*,*) m
            do i=1,m
               call poinc1(n,x,tfinal,idir)
               t=tfinal+t
               write(*,*)'t:'
               write(*,*)t
            enddo
            write(10,90)
90          format()
         enddo
      enddo
      end
```