# Assignment 1–Standard Map

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Get standard map of function $f(x, y) = (x + asin(x + y), x + y)$, where $a$ is a known parameter. And analysize it.

## 1 Standard Map

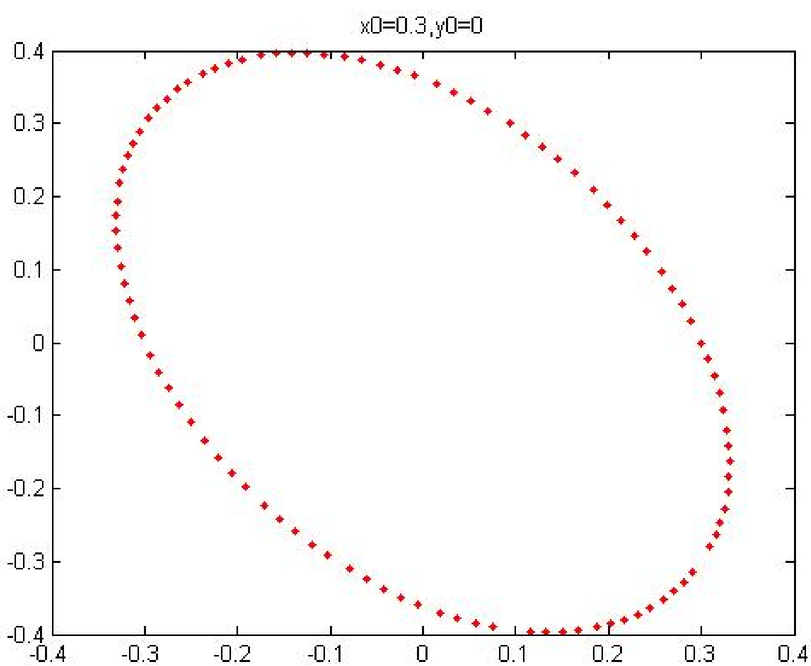



Figure 1: initial condition: $x_0 = 0.3, y_0 = 0$

Here, 100 pairs of initial conditions within $(-\pi, \pi)$ are took in figure 2.

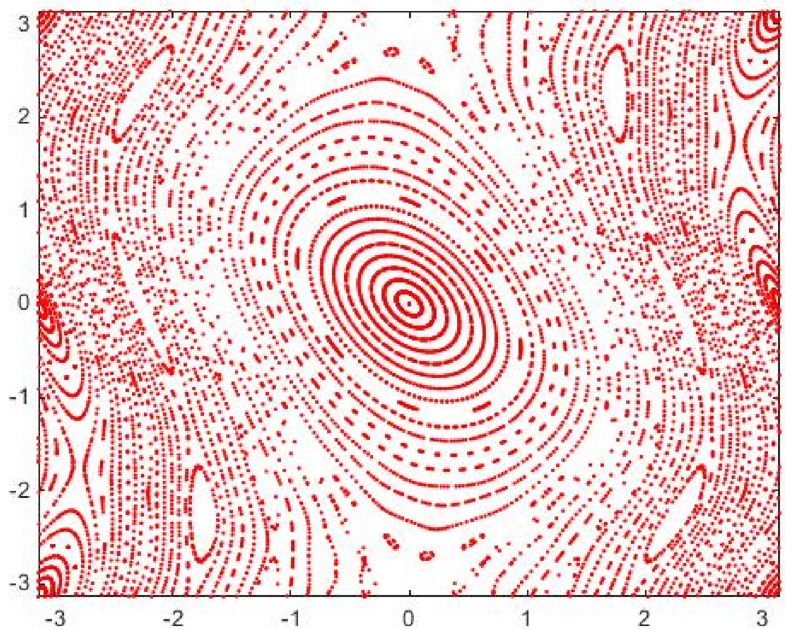
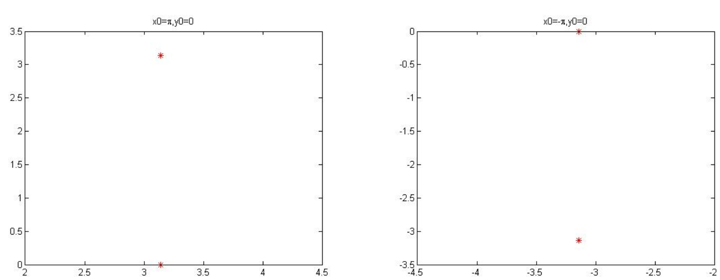



Figure 2: standard map with $a = -0.7$

## 2 2-periodic point




(a) $x_0 = \pi, y_0 = 0$ (b) $x_0 = -\pi, y_0 = 0$

Figure 3: 2-periodic points

2-periodic point—-initial condition pairs $(x_0, y_0)$ : $(-\pi, -\pi)$, $(-\pi, 0)$,

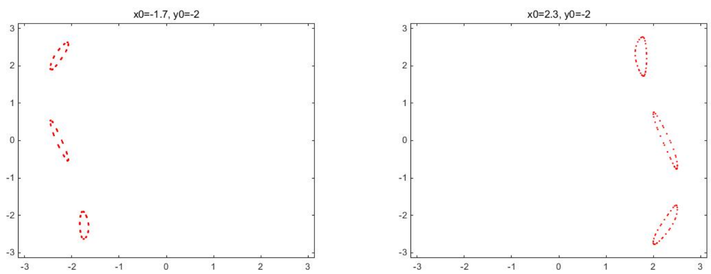
$(\pi, \pi)$, $(\pi, 0)$.

$$f(\pi, 0) = (\pi, \pi) \to f^2(\pi, 0) = f(\pi, \pi) = (\pi, 0)$$

$$f^2(\pi, \pi) = f(\pi, 0) = (\pi, \pi)$$

The same happens to $(-\pi, -\pi)$, $(-\pi, 0)$.

# 3 3-periodic point




(a) $x_0 = -1.7, y_0 = -2$ (b) $x_0 = 2.3, y_0 = -2$

Figure 4: 3-periodic points

# 4 6-periodic point

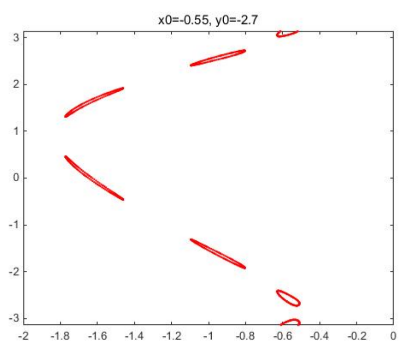



Figure 5: initial condition: $x_0 = -0.55, y_0 = -2.7$

## 5 Comments

In this Standard Map, $(0,0)$ is the fix point. It is an elliptic point. There are invariant curves surrounding it. It is stable. $(0,\pi)$ and $(0,-\pi)$ are the hyperbolic equilibrium points. At $x=-\pi$, we can see the 2-periodic points. As we increase the value, we can see the $n$-periodic points $(n=3,4..)$. $n$ increases as the $x$ increasing. Until $x$ reaches around -0.8. Besides, as $x$ increases, the change of value influences more about the dynamics, i.e. a small perturbation of the value of initial point will change the orbit. From $(0.8,/pi)$, it goes in the opposite way.

## 6 Rotation Number

Definition of the rotation number

$$\rho = \lim_{N\to\infty}(\frac{\phi_1+\phi_2+...+\phi_N}{N})(\frac{1}{2\pi})$$

Here, we can see the part: $\lim_{N\to\infty}\frac{\phi_1+\phi_2+...+\phi_N}{N}$ is the mean angle.

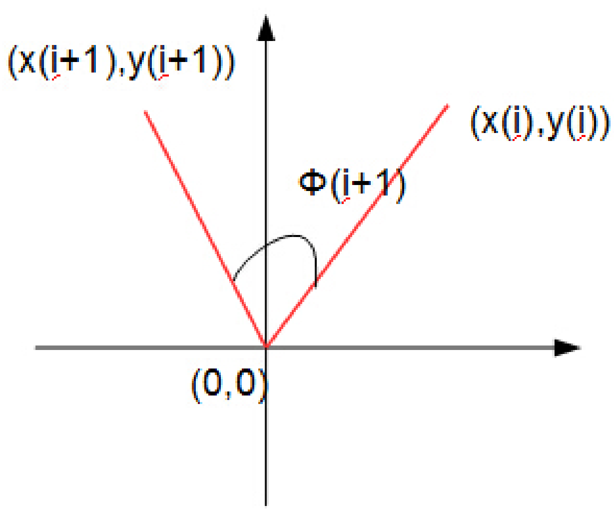



Figure 6: angle $\phi_i$

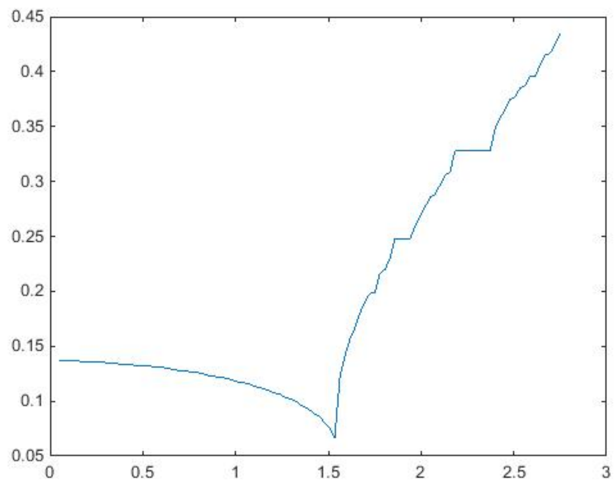



Figure 7: Rotation Number

# 7 Matlab Code

```
function getMap()
    n=100;
    totalPhi=[];
    rotationNum=[];
    x0=[0.05:2.7/n:2.75];
    y0(n+1)=[0];
    ini={};
    for i=1:(n+1)
        ini{i}=[x0(i),y0(i),y0(i)/x0(i),0];
    end
    points=getPoints(ini);
    figure
    for i=1:(n+1)
        m=size(points{i},1);
        totalPhi(i)=sum(points{i}(:,4));
        rotationNum(i)=totalPhi(i)/(m*2*pi);
        plot(points{i}(:,1),points{i}(:,2),'r.',
                'MarkerSize',4.5);
```

```
        hold on
    end
    figure
    plot(x0,rotationNum)
end
```

```
function points=getPoints(ini)
    a=-0.7;
    points=ini;
    n1=size(points,2);
    n2=100;

    for j=1:n1
        line=points{j};
        x=line(1);
        y=line(1,2);
        phi=line(1,3);
        phiN=0;
        b=0;
% get x & y
        for i=1:n2
            xn=x+a*sin(x+y);
            yn=x+y;
            x=xn;
            y=yn;
            x=sign(x)*mod(abs(x),2*pi);
            y=sign(y)*mod(abs(y),2*pi);
            if(abs(x)>pi)
                x=x-sign(x)*2*pi;
            end
            if(abs(y)>pi)
                y=y-sign(y)*2*pi;
            end
% get angle \phi
            if x<0
                phi=atan(y/x)+pi;
            elseif y<0
                phi=atan(y/x)+2*pi;
            else
```

```
                    phi=atan(y/x);
                end
                line(i+1,1:3)=[x,y,phi];
% get /deltaPhi
                deltaPhi=line(i+1,3)-line(i,3);
                if deltaPhi<0
                    deltaPhi=deltaPhi+2*pi;
                end
                line(i+1,4)=deltaPhi;
            end
            points{j}=line;
        end
end
```