Numerics of Dynamical Systems

Assignment 1

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1 Exercise 1

Abbildung 1: (i) Initial conditions: $x_0 = y_0 = -1$

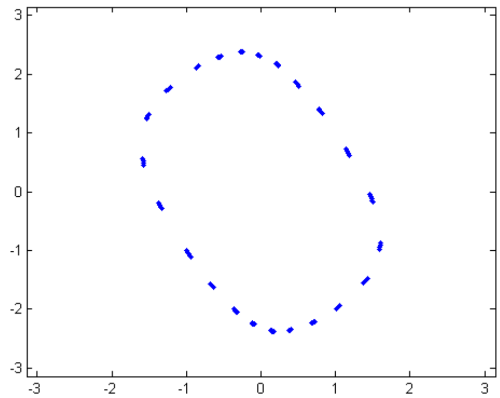
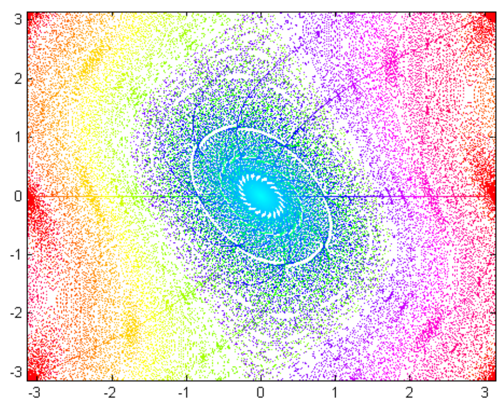


Abbildung 2: (ii) Initial conditions: $(x_0, 0)$ with x_0 between π and $-\pi$



Listing 1: Exercise 1(i).m

```
function [] = ass1()
%Initialization
x(1) = -1;
y(1) = -1;

a = -0.7; % Parameter
```

```
%(i)
for(i=2:100)
x(i) = mod(x(i-1) + a*sin(x(i-1)+y(i-1)),2*pi);
y(i) = mod(x(i-1) + y(i-1),2*pi);

if x(i)<-pi
        x(i)=x(i)+2*pi;
    elseif x(i)>pi
        x(i)=x(i)-2*pi;
end
if y(i)<-pi
        y(i)=y(i)+2*pi;
elseif y(i)>pi
        y(i)=y(i)-2*pi;
    end

end

plot(x,y,'.')
axis([-pi pi -pi pi])
end
```

Listing 2: Exercise 1(ii).m

```
function[] = ass1_1_2()
%Initialization
y(1) = 0;

a = -0.7; % Parameter

xAxis = -pi:0.01:pi;
Color = hsv(length(xAxis));

for j = 1:length(xAxis)
  x(1) = xAxis(j);

for i=2:100
x(i) = mod(x(i-1) + a*sin(x(i-1)+y(i-1)),2*pi);
y(i) = mod(x(i-1) + y(i-1),2*pi);
```

```
if x(i)<-pi
            x(i)=x(i)+2*pi;
        elseif x(i)>pi
            x(i)=x(i)-2*pi;
end
if y(i)<-pi
            y(i)=y(i)+2*pi;
elseif y(i)>pi
            y(i)=y(i)-2*pi;
        end
end
plot(x,y,'.','Color',Color(j,:),'MarkerSize',3);
hold on
end
axis([-pi pi -pi pi])
end
```

2 Exercise 2

A 2-periodic point is given with $(0, \pi)$:

$$f^2(\pi, 0) = f(f(\pi, 0)) = f(\pi, \pi) = (\pi, 0) \tag{1}$$

3 Exercise 3

Approximate values of 3-periodic points are for example $(-1.75, -2.25)$, $(-2.24, -0.12)$ and $(-2.28, 2.26)$:

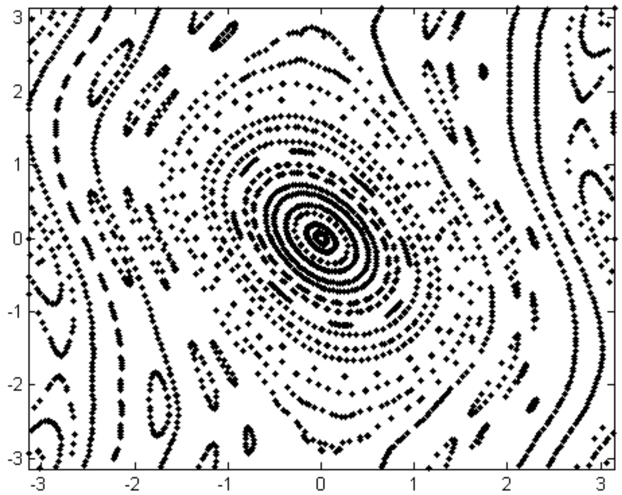
$$f^3(-1.75, -2.23) = (-1.7426, -2.3226)$$
$$f^3(-2.24, -0.12) = (-2.2205, -0.1462)$$
$$f^3(-2.28, 2.26) = (-2.2779, 2.2597)$$

4 Exercise 4

Approximate values of 6-periodic points are for example $(-1.75, -2.3)$:

$$f^6(-1.75, -2.3) = (-1.7333, -2.3260)$$

Abbildung 3: Graph with bigger step size of x_0 (to see the orbits).



Listing 3: Exercise 3 and 4.m

```
function[x,y] = per(x,y,per)
% calculates per steps of function f

%Initialization
x(1) = x;
y(1) = y;
a = -0.7; % Parameter

for i=2:(per+1)
x(i) = mod(x(i-1) + a*sin(x(i-1)+y(i-1)),2*pi);
y(i) = mod(x(i-1) + y(i-1),2*pi);

 if x(i)<-pi
                x(i)=x(i)+2*pi;
           elseif x(i)>pi
                x(i)=x(i)-2*pi;
end
 if y(i)<-pi
                y(i)=y(i)+2*pi;
 elseif y(i)>pi
                y(i)=y(i)-2*pi;
```

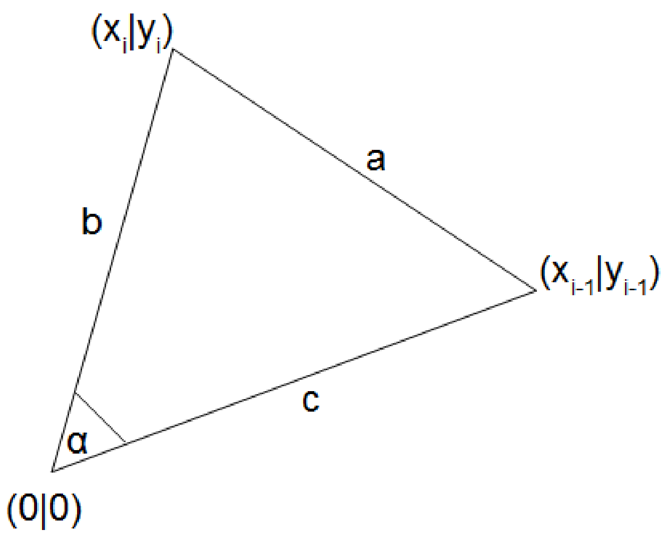
```
end
end
end
```

5 Exercise 5

For the initial conditions $-1.5 < x_0 < 1.5$ the solutions are one-periodic orbits around the equilibrium point $(0,0)$. At the left and right parts of the graph are parts with the 3 or 6 orbits of 3- and 6-periodic points, respectively.

6 Optional Exercise

The 3 points (0,0), (x_{i-1}, y_{i-1}) and (x_i, y_i) determine a triangle.



I calculated first the length of every side of this triangle:

$$a = \sqrt{(x_{i-1} - x_i)^2 + (y_{i-1} - y_i)^2}$$
$$b = \sqrt{(x_i)^2 + y_i)^2}$$
$$c = \sqrt{(x_{i-1})^2 + y_{i-1})^2}$$

Then I calculated the angle α_i with the law of cosines:

$$a^2 = b^2 + c^2 - 2bc \cdot cos(\alpha) \Rightarrow \alpha = arccos(\frac{b^2 + c^2 - a^2}{2bc}) \tag{2}$$

The roation number ρ will then be calculated by the formula for the (N-1) iterations:

$$\rho = \frac{\alpha_2 + ... + \alpha_N}{N - 1} \cdot \frac{1}{2\pi} \tag{3}$$

This is the plot: