ASSIGNMENT 1 Standard Map

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```
      implicit real *8 (a-h, o-z)
      dpi=8.d0*datan(1.d0)
      pi=dpi/2.d0
      open(10,file='xy.dat',status='unknown')
      a=-0.7d0

      niter=500
      XA=-pi-0.05d0

2     n=0
      XA=XA+0.05d0
      if(XA.gt.pi) stop
      x=XA
      y=0.d0
      write(*,*)'ic ',x,y
1     x1=x+a*dsin(x+y)
      y1=x+y
      x1=dmod(x1,dpi)
      y1=dmod(y1,dpi)
      if(x1.lt.-pi) x1=x1+dpi
      if(y1.lt.-pi) y1=y1+dpi
      if(x1.gt.pi) x1=x1-dpi
      if(y1.gt.pi) y1=y1-dpi
        write(*,*) x1,y1
      write(10,*) x1,y1
      n=n+1
      x=x1
      y=y1
      if(n.ge.niter) go to 2
      go to 1
      end
```

Figure 1: Code

We first simulate the standard map using the code in Figure 1.

In the code we can note that we have done 500 iterations for each initial condition, running from $-\pi$ to π, with step $h = 0.05$.

This means that we start with $x_{0_0} = -\pi$, we compute 500 iterates, and we follow with $x_{0_1} = -\pi + 0.05$ and compute 500 more iterates. This method runs until we reach $x_{0_N} = \pi$, always taking $y_{0_i} = 0$

The dynamics of the system can be seen in the Figure 2, done with gnuplot. We can easily find initial conditions of a n-periodic orbit just looking at the graphic.

Some n-periodic points occur, approximately taking as initial condition:

- 2-periodic point: $(\pi, 0)$

- 3-periodic point: (2.27,0)
- 6-periodic point: (2.58,0)

Some comments on the dynamics we see on the plot can be done:

- At the center of the plot we can see some stable behavior, since we find periodic orbits more or less concentric.
- Surrounding these periodic orbits we can see a chaotic behavior (isolated dots).
- Fixed points are $(x, y) = (0, 0)$, and $(x, y) = (0, \pi)$ (or $(0, -\pi)$).
- Some periodic behavior occurs if we take as initial condition x_0 once the chaotic behavior ends (approximately from $-\pi$ to -1.5 and from 1.5 to π). We can see, as pointed before, different periodic points and orbits.
- The graphic is antisymmetric ($f(x, y) = -f(-x, -y)$).

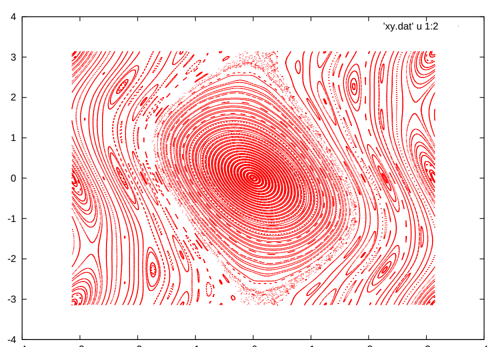

Figure 2: Dynamic System Plot

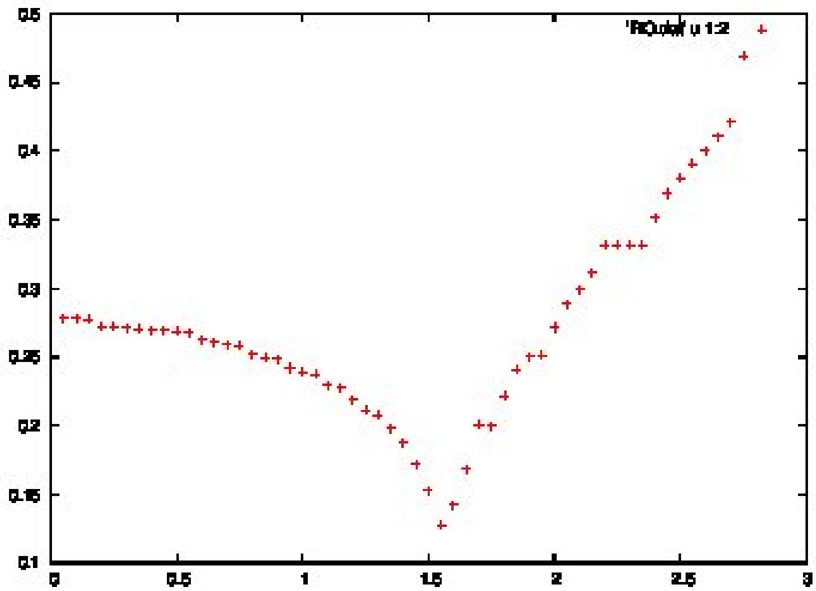

Figure 3: Rotation Number

Optional Assignment

In Figure 3 we see the plot (x, ρ), where ρ denotes the rotation number.

In the plot we can see that between 0.05 and 1.50 the rotation number decreases, where the orbits showed in Figure 2 become bigger. between 1.5 and 2.75 the rotation number increases, and we can see some regions where it remains shortly constant, precisely where we find periodic points in Figure 2.

The same graphic in .pdf is Figure 5, and the code to do this is Figure 4

```
      implicit real *8 (a-h, o-z)
      dpi=8.d0*datan(1.d0)
      pi=dpi/2.d0
      open(11,file='RO.dat',status='unknown')
      a=-0.7d0

      niter=100
      XA=0d0

2     ro=0
      n=0
      XA=XA+0.05d0

      if(XA.gt.2.75) stop
      x=XA
      y=0.d0

1      x1=x+a*dsin(x+y)
      y1=x+y
      x1=dmod(x1,dpi)
      y1=dmod(y1,dpi)
      if(x1.lt.-pi) x1=x1+dpi
      if(y1.lt.-pi) y1=y1+dpi
      if(x1.gt.pi) x1=x1-dpi
      if(y1.gt.pi) y1=y1-dpi

      deltaro=datan((y1)/(x1))-datan(y/x)
      IF(deltaro.le.0) deltaro=dpi+deltaro

      ro=ro+deltaro

      n=n+1
      x=x1
      y=y1

      IF(n.ge.niter) THEN
        ro=ro/(200.d0*pi)
        write(11,*) XA,ro
      go to 2

      ELSE
      go to 1

      END IF

      end
```

Figure 4: Code Optional

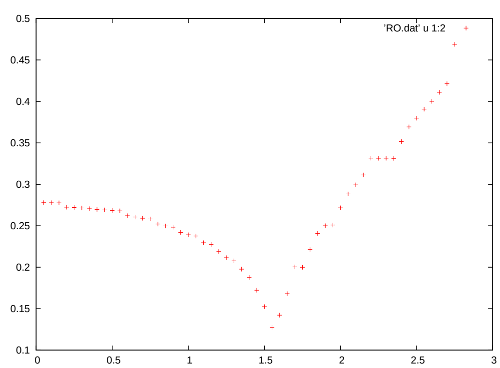

Figure 5: Rotation Number