UNIVERSITAT POLITÈCNICA DE CATALUNYA
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Full ___ de ___

QUALIFICACIÓ
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Assignatura: Teoria de grafos

Estudiant/a: [redacted]

Data: 09/01/2019

~~let G be an r-regular graph and let $r = \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ be the eigenvalues of its adjacency matrix~~
~~a) show that the stability number $\alpha(G)$ satisfies $\alpha(G) \leq \frac{-n\lambda_n}{r-\lambda_n}$~~
~~b) find an~~

4. Let G be an r-regular graph with n vertices and denote by $\tau(G)$ the number of spanning trees of G show that

$$\tau(G) \leq \frac{1}{n}\left(\frac{nr}{n-1}\right)^{n-1}$$

and characterize the case of equality

first let G be an r-regular graph with $\tau(G)$ number of spanning trees $\forall$.

we need to show 2 cases

1. $\tau(G) < \frac{1}{n}\left(\frac{nr}{n-1}\right)^{n-1}$, $\frac{nr^n}{(n-1)^n} \cdot \frac{(n-1)}{(nr)} \cdot \frac{1}{n} = \frac{r^{n}(n-1)}{nr(n-1)^n}$

$\tau(G) = \frac{1}{n}\left(\frac{nr}{n-1}\right)^{n-1}$

let us consider the case with

$2m \leq 3n-6$? that holds true if the graph is regular. ~~and~~ ~~not~~

r-regularity supports the fact that

nr in a graph is the equality of $n \cdot r = 2m$ with m standing for edges $n-1$ standing for the amount of edges of an ~~and~~ induced random tree ? (non-cyclic graph)

Since the amount embedded in the graph should depend on the amount of vertices in the original graph, we have the following equation $\frac{nr}{(n-1)}$

However, it does not work until the power would be of $n-1$ that stands for the amount of edges in a random tree embedded. The overall amount of trees should also be applied to each of the trees in particular, that is, divided by n vertices at the end

Ar. mean $\frac{\sum x_n}{n}$ $\frac{x \cdot n}{n}$ $\frac{1}{n}(x \cdot n)$

G. mean $\sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdot x_n}$ $(n \cdot x)^{\frac{1}{n}}$

$\frac{1}{n}(n \cdot x)^{\frac{1}{n}} \; n$ $\quad x \cdot n \quad \sqrt[n]{x n} \quad (n^{\frac{1}{n}} x)^n$

$\frac{1}{n} \quad \frac{1}{\quad}$ $\left(\frac{nr}{n-1}\right)^n \cdot \frac{1}{n}$

$\frac{1}{n}$ $\left(\frac{nr}{n-1}\right)$

$\left(\frac{nr}{n-1}\right)^{n-1} \quad \frac{1}{n}$

$n = 1$

$\tau(G) = 1$

$\tau(G)$

$n = 2$

$\tau(G) \le \frac{1}{2}\left(\frac{2 \cdot r}{2-1}\right)^{2-1} \le \frac{1}{2}\left(\frac{2r}{1}\right)^1 = r$

$n = 3$

$\tau(G) \le \frac{1}{3}\left(\frac{3 \cdot r}{3-1}\right)^{3-1} = \frac{1}{\cancel{3}}\left(\frac{\cancel{3}r}{2}\right)^2 = \frac{3r^2}{4}$

n vertices

→ n possible type of trees since it's regular, each of those vertices has r different ways of growth.