QUALIFICACIÓ

Assignatura: Computational Mechanics

Estudiant/a: [redacted]

Data: 4/6/19

EXERCISE 1

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a) 5 We have
$$\begin{cases} \rho \dfrac{d^2 u}{dt^2} = \rho b + \nabla \cdot \sigma \\ \sigma = \eta \, \varepsilon^{dev} \end{cases}$$

where $\varepsilon^{dev} = \varepsilon - \frac{1}{3}(tr\,\varepsilon)\,Id.$ ✓

We know that $\sigma = \eta \varepsilon \overset{\text{dev}}{=} \eta \sum_j B_j(x) u_j$, where

$$B_j = \begin{pmatrix} \dfrac{\partial N_j(x)}{\partial x} & 0 \\ 0 & \dfrac{\partial N_j(x)}{\partial y} \\ \dfrac{\partial N_j(x)}{\partial y} & \dfrac{\partial N_j(x)}{\partial x} \end{pmatrix}$$ (N_j base functions)

Computing the weak form and discretizing we get that
$$K_{ij}^e = \int_{V_e} B_i(x)^T B_j(x) \, dV_e$$

0

b) Since we are in 3D and we have 8 nodes per element, the K matrix will be $\cancel{8 \times 3}$.
So, its maximum rank will be $\cancel{3}$.

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EXERCISE 2

a) We know that $\omega^2 = \text{eigenvalues}(M^{-1}K)$

$$M^{-1} = \frac{1}{h\rho}\begin{pmatrix} 4 & -2 \\ -2 & 4 \end{pmatrix}, \quad K = \frac{E}{h}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$M^{-1}K = \frac{6E}{h^2\rho}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

So, $\omega^2 = \text{eigenvalues}(M^{-1}K) = \left\{ \begin{matrix} 0 \\ \frac{12E}{h^2\rho} \end{matrix} \right\}$ ✓

Thus, $\boxed{\omega_1 = 0}$, $\boxed{\omega_2 = \sqrt{\frac{12E}{h^2\rho}} = \frac{2}{h}\sqrt{\frac{3E}{\rho}}}$

b) The lumped mass matrix is: $M^L = \frac{h\rho}{6}\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

Proceeding the same as before:

$$(M^L)^{-1}K = \frac{2E}{h^2\rho}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \Rightarrow \omega^2 = \left\{ \begin{matrix} 0 \\ \frac{4E}{h^2\rho} \end{matrix} \right\}$$

Thus, $\boxed{\omega_1 = 0, \quad \omega_2 = \frac{2}{h}\sqrt{\frac{E}{\rho}}}$ ✓

c) The central differences scheme is stable for

$$h \leq \frac{2}{\omega_{max}} = \Delta t_{crit}$$

$$\Delta t_{crit}^{M} = \frac{2}{\omega_{max}^{M}} = \frac{2}{\frac{2}{h}\sqrt{\frac{3E}{\rho}}} = \frac{1}{\sqrt{3}} \cdot h\sqrt{\frac{\rho}{E}}$$

$$\Delta t_{crit}^{M^L} = \frac{2}{\omega_{max}^{M^L}} = \frac{2}{\frac{2}{h}\sqrt{\frac{E}{\rho}}} = h\sqrt{\frac{\rho}{E}}$$

So we have $\boxed{\Delta t_{crit}^{M} = \frac{1}{\sqrt{3}} \Delta t_{crit}^{M^L}}$

Thus, the better one will be the lumped one, since its Δt_{crit} is bigger.

d) If we reduce h or ρ, Δt_{crit} will be lower (less stability)

If we reduce E, Δt_{crit} will be larger (more stability)

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EXERCISE 3

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We know that for Von-Mises plasticity, the yield function is $f(\sigma) = \sqrt{3J_2} - \sigma_y$, where

$$J_2 = \frac{1}{6}\left((\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2 \right), \quad \sigma_i \text{ eigenval. of } \sigma.$$

So, eigenvalues $(\sigma) = \{0, \tau, -\tau\}$

Thus, $J_2 = \frac{1}{6}\left(\tau^2 + \tau^2 + 4\tau^2 \right) = \tau^2$

$\Rightarrow f(\sigma) = \tau\sqrt{3} - \sigma_y$

✓

We know that $f(\sigma) \leq 0$, so $\tau \leq \dfrac{\sigma_y}{\sqrt{3}}$

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EXERCISE 4

a) $Re_1 = \frac{V_1 L_1}{\nu_1}$, $Re_2 = \frac{V_2 L_2}{\nu_2}$

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Since $\nu_1 = \nu_2$ and $L_1 = L_2$, we get:

$$\frac{V_1}{Re_1} = \frac{V_2}{Re_2} = \frac{100 V_1}{Re_2} \Rightarrow \boxed{Re_2 = 100\, Re_1}$$

Since the Reynolds number in the second computation is much ~~more~~ bigger than in the first one, the algorithm is not able to converge. → why?

To solve this problem, we could do intermediate calculations (calculating with $Re < Re_2$) and using these solutions as initial approximation for the next calculation.
(+ refined mesh)

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b) Computing again the Reynolds number (using $\nu_1 = \nu_2$):

$$\frac{V_1 L_1}{Re_1} = \frac{V_2 L_2}{Re_2} = \frac{100 V_1 \cdot \frac{1}{100} L_1}{Re_2} = \frac{V_1 L_1}{Re_2} \Rightarrow$$

$$\Rightarrow Re_1 = Re_2$$

That means that we do not need to do the computation twice and we can obtain the solution from the first computation.

If we use the non-dimensionalized Navier-Stokes equations, the velocity fields in the domain will be the same. $\bar{v}_2 = (?)\,\bar{v}_1$

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EXERCISE 5

a) (100)

$$F_n = n \cdot F = -c \cdot n \begin{pmatrix} U_2 \\ U_1 \end{pmatrix} = \underbrace{-c \cdot n \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{A_n} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$$

$$A_n = X D X^{-1}, \quad \text{with } D = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot (-cn)$$

$$X = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

Then, we can split the contributions as:

$$\left.\begin{aligned} A_n^+ &= X \cdot \begin{pmatrix} 0 & 0 \\ 0 & c \end{pmatrix} X^{-1} \\ A_n^- &= X \begin{pmatrix} -c & 0 \\ 0 & 0 \end{pmatrix} X^{-1} \end{aligned}\right\} \text{ for } n = 1$$

$$\left.\begin{aligned} A_n^+ &= X \begin{pmatrix} c & 0 \\ 0 & 0 \end{pmatrix} X^{-1} \\ A_n^- &= X \begin{pmatrix} 0 & 0 \\ 0 & -c \end{pmatrix} X^{-1} \end{aligned}\right\} \text{ for } n = -1$$

And thus, the incoming and outgoing fluxes are:

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$$n=1\begin{cases} F_n^+ = A_n^+ \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \frac{1}{2}c \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \\ F_n^- = A_n^- \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = -\frac{1}{2}c \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \end{cases}$$

$$n=-1\begin{cases} F_n^+ = A_n^+ \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \frac{1}{2}c \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \\ F_n^- = A_n^- \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = -\frac{1}{2}c \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \end{cases}$$

As we can see, $\left.\begin{cases} F^+_{n\,(n=1)} = -F^-_{n\,(n=-1)} \\ F^-_{n\,(n=1)} = -F^+_{n\,(n=-1)} \end{cases}\right\} \Rightarrow$

$\Rightarrow$ the formulas are conservative

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b) Recall we have $\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = 0 \Rightarrow$

$$\Rightarrow -\frac{\partial U}{\partial t} = \frac{\partial F}{\partial x}$$

We want to define it over every element:

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$$\underbrace{-\int_{V_e} \frac{\partial u}{\partial t}}_{\shortparallel} = \underbrace{\int_{V_e} \frac{\partial F}{\partial x}\, dV_e}_{\shortparallel}$$

$$|V_e| \cdot \frac{du_e}{dt} \qquad \underset{(n=1)}{F(x_{e+1/2})} \overset{+}{-} \underset{(n=-1)}{F(x_{e-1/2})}$$

So, using Euler's method, the final formula is:

$$U_e^{n+1} = U_e^{n} - \frac{\Delta t}{\underset{|V_e|}{h}} \left[\underset{n=1}{\tilde{F}}(\underset{(u_e,\,u_{e+1})}{x_{e+1/2}}) - \underset{n=-1}{F}(\underset{(u_e,\,u_{e-1})}{x_{e-1/2}}) \right]$$

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