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Assignatura:

Estudiant/a: [redacted]

Data:

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Problem 1

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a) $\{\varepsilon\}^{dev} := \{\varepsilon^e\}_i - \{\varepsilon^e\}_j$

Note: I am not sure this is the definition of deviatoric strain.

$\varepsilon^{dev} = \varepsilon - \frac{tr(\varepsilon)}{3} I$

$\sigma^e = n \cdot (\{\varepsilon^e\}_i - \{\varepsilon^e\}_j)$

$$\int_V \varepsilon(v)^h : \sigma^e = \int_V \varepsilon(v)^h : n(\{\varepsilon^e\}_i - \{\varepsilon^e\}_j)\, dv =$$

$$n \cdot \int_V \varepsilon(v)^h : (\{\varepsilon^e\}_i - \{\varepsilon^e\}_j) = n \cdot \int_V \varepsilon(v)^h : (B_i v_i - B_j v_j)\, dv$$

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b) Dimension of K^e 8 nodes x 3 $\frac{\text{displacement}}{\text{node}}$ = 24 $\Rightarrow$ $K^e = 24 \times 24$

Each node is in 8 elements so the maximum rank is ~~3~~.

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Problem 2

a) 5

$$M^{-1} = \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix} \cdot \frac{6}{ph} = \frac{2}{ph} \cdot \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$M^{-1} \cdot K = \frac{E}{h} \cdot \frac{2}{p \cdot h} \cdot \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{2E}{p \cdot h^2} \cdot \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} =$$

$$= \frac{6 \cdot E}{p \cdot h^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\lambda_i = \text{eigen}(M^{-1} K) \qquad \lambda_1 = 0 \qquad \lambda_2 = \frac{12E}{p h^2}$$

$$\omega_i = \sqrt{\lambda_i} \qquad \omega_1 = 0 \qquad \omega_2 = \sqrt{\frac{12 \cdot E}{p \cdot h^2}} = \frac{2}{h} \cdot \sqrt{\frac{3 \cdot E}{p}}$$

5

b)

$$\underline{M}^L = \frac{hp}{6} \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \frac{hp}{2} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underline{M}^{L^{-1}} = \frac{2}{hp} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M^{L^{-1}} \cdot K = \frac{2 \cdot E}{p \cdot h^2} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\lambda_i = \text{eigen}(M^{L^{-1}} K) \qquad \lambda_1 = 0 \qquad \lambda_2 = \frac{4E}{p \cdot h^2}$$

$$\omega_i = \sqrt{\lambda_i} \qquad \omega_1 = 0 \qquad \omega_2 = \sqrt{\frac{4 \cdot E}{p \cdot h^2}} = \frac{2}{h} \cdot \sqrt{\frac{E}{p}}$$

c) 5

$$\omega_{max} = \frac{2}{h} \cdot \sqrt{\frac{3E}{\rho}} \qquad \omega_{max}^{L} = \frac{2}{h} \cdot \sqrt{\frac{E}{\rho}}$$

$$\omega_{max} > \omega_{max}^{L}$$

$$\Delta t_{crit} < \Delta t_{crit}^{L}$$

The Δt_{crit} is the maximum timestep we can use in the solver algorithm. As the lumped one is bigger, this mean there are timesteps 't_i' such that $\Delta t_{crit} < t_i < \Delta t^{L}_{crit}$ wich is stable in the lumped system and unstable in the normal one. So the lumped system is better from an stability point of view

d) 5

Fixing all the other variables:

- $h' < h$ $\quad \frac{\omega_{max}'}{\omega_{max}} = \frac{h}{h'} \Rightarrow \omega'_{max} = \omega_{max} \cdot \frac{h}{h'} \Rightarrow \omega'_{max} > \omega_{max}$
- $E' < E$ $\quad \omega_{max}' = \frac{\sqrt{E'}}{\sqrt{E}} \cdot \omega_{max} \Rightarrow \omega_{max}' < \omega_{max}$
- $\rho' < \rho$ $\quad \omega_{max}' = \sqrt{\frac{\rho}{\rho'}} \cdot \omega_{max} \Rightarrow \omega_{max}' > \omega_{max}$

By the same argument of the previus exercice reducing E would increase stability and reducing h and ρ would reduce stability. Morover we have seen that a proportional change in h is worser than in ρ.

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Problem 3

Von Mises:

$$f(\sigma) = \sqrt{3 J_2} - \sigma_y$$

$$\sigma' := \sigma - \sigma_m \cdot I \qquad \sigma_m = \tfrac{1}{3} \cdot I_1$$

$$I_1 = tr(\sigma) = 0 \quad \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sigma^2 = \sigma \cdot \sigma = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \tau^2 & 0 & 0 \\ 0 & \tau^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$tr(\sigma^2) = 2\tau^2$$

$$I_2 = \frac{1}{2} \cdot \left[(tr(\sigma))^2 - tr(\sigma^2) \right] = -\tau^2$$

$$J_2 = \frac{1}{2} \cdot I_1^2 - I_2 = \tau^2$$

$$\sigma_{eq} = \sqrt{3 \cdot J_2} = \sqrt{3} \cdot |\tau|$$

Imposing the yield condition

$$f(\sigma) = 0 \Rightarrow \sqrt{3} \cdot |\tau_{max}| - \sigma_y = 0$$

$$\boxed{|\tau_{max}| = \frac{\sigma_y}{\sqrt{3}}}$$

✓

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Probem 4

a) The Reynolds number (Re) has increased 100 times:

100

$$Re_2 = \frac{L \cdot V_2}{\nu} = \frac{L \cdot 100 \cdot V_1}{\nu} = Re_1 \cdot 100$$

As we increase the Reynolds number the turbolences and the problem complexity increase due to the non-linear convection term.

This complexity is the cause that our Newton-Raphson method fails to find a valid solution.

Several things can be done to get a solution, most of the time we ~~would~~ will need to use more than one:

a) Change the mesh: Adding more nodes to the mesh can help to overcome the complexity of the problem. Also addapting the mesh to the problem will have a similar efect. ✓

b) Iterate to get a better solution: ~~By~~ Increasing the inflow velocity with multiple steeps and ~~...~~ feeding the next initial solution with the last solution can help the solver to converge faster. ✓

c) Set a bigger ~~...~~ step limit in the Newton-Raphson algorithm: This can help the algorithm to converge in some cases.

b) 100

We can obtain the solution from the first problem solution.

In this case the Reynolds number (Re_3):

$$Re_3 = \frac{L_3 \cdot V_3}{\nu} = \frac{L_1}{100} \cdot \frac{100 V_1}{\nu} = \frac{L_1 \cdot V_1}{\nu} = Re_1$$

So our solution will have the same non-dimensional solution as in the first computation.

$$\tilde{v} = \frac{v_1}{V_1} = \frac{v_3}{V_3} \Rightarrow \boxed{v_3 = v_1 \cdot \frac{V_3}{V_1} = v_1 \cdot 100}$$

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Problem 5

a/ $\varnothing$ $\nabla p \cdot n$

$$\begin{bmatrix} \frac{\partial p}{\partial t} \\ \frac{\partial q}{\partial t} \end{bmatrix} - c \cdot \begin{bmatrix} \frac{\partial U_2}{\partial x} \\ \frac{\partial U_1}{\partial x} \end{bmatrix} = 0$$

$$\frac{\partial p}{\partial t} - c \frac{\partial U_2}{\partial x} = 0 \; ; \; \frac{\partial p}{\partial t} = c \frac{\partial U_2}{\partial x}$$

Flux splitting for $n=-1$: $-c \cdot \frac{\partial U_2}{\partial x}$

Flux splitting for $n=1$: $c \frac{\partial U_2}{\partial x}$

$$\tilde{F}_{-n}(U^{out}, U) = -\tilde{F}_n(U, U^{out})$$

$$-n \cdot \nabla p = -n \cdot \nabla p$$

b/