Total: 8.5

Silent Block

[redacted], [redacted], [redacted] and [redacted]

April 2019

In order to study the dynamical response of a squared body subjected to oscillatory loads. One of the potential applications is the analysis of mechanical isolators for vibrating machines, so called silent blocks. Figure 1 shows the squared domain analysed, and the oscillatory boundary load being applied, with frequency $\bar{\omega}$. The points at the upper and lower part of the body can not move horizontally (x direction), and the load is applied only vertically (y direction).

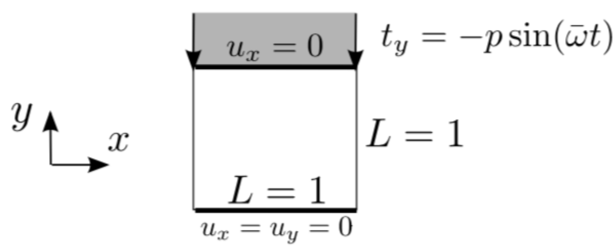
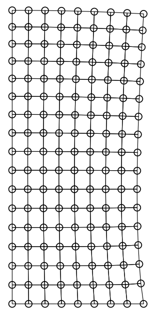


Figure 1: Schematic of the analysed domain and boundary condition

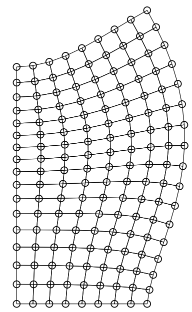
In order to numerically study the response of the isolator, assume that the body is made of a material with Young modulus $E = 100$, Poisson ratio$\nu = 0.3$, and density $\rho = 1$. Then, answer the following questions:

a) **Perform a two-dimensional modal analysis of the domain shown in Figure 1 and with the boundary conditions indicated. Consider exploiting the symmetry of the problem. Give the four lowest eigen-frequencies and eigen-modes using a 16×16 mesh for the whole domain.**

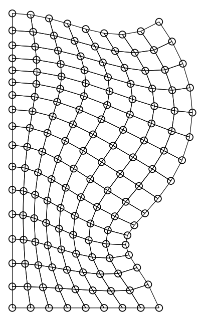
Solution:
We have use the code cantilever in Matlab, with a square mesh, where we have simulate the silent block with the initial condition given above. In order to simplify the problem we take advantage of the symmetry of



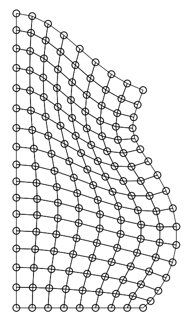
(a) Frequency = 2.52 Hz



(b) Frequency = 5.13 Hz



(c) Frequency = 6,31 Hz



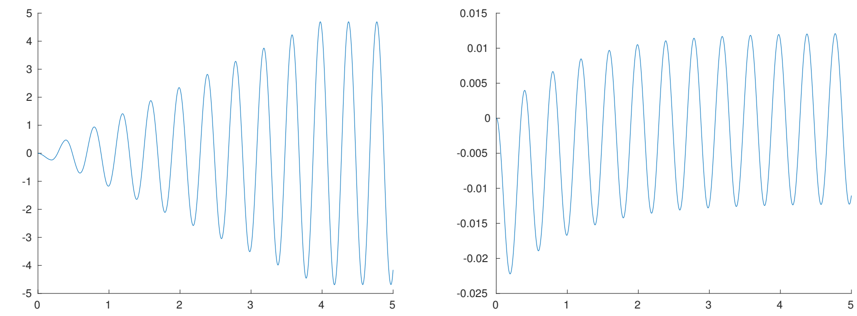
(d) Frequency = 7,14 Hz

Figure 2: First four eigen modes with each eigen frenquency

the problem, i.e. we just take into account the half part of block, then our block is define in $x \in [0, 0.5]$ and $y \in [0, 1]$. In fig. 2 we can see the deformation of the block for each eigenfrequency, the slowest frequency usually controls the behaviour in the real case. Why?

b) **According to the results in a), can the silent block exhibit resonance for some applied $\bar{\omega}$? If yes, give the lowest value of $\bar{\omega}$ for which the resonance phenomena may arise, and state what may physically mitigate this resonance phenomena.**

We can see that the input frequency is of $\bar{w} = 2rad/s$ which is slower than the slowest eigenfrequency $2.517Hz$, or $15.8rad/s$. Therefore we won't have resonances, however if we increase $\bar{w}$ we will start seeing resonances when $\bar{w} \simeq 15.8rad/s$. If we do this in our simulation we can see that the deformation explodes as it can be seen in fig. 3a. Changing the material to move the eigenfrequencies higher or adding damping can mitigate the resonance. We can see how adding damping helps mitigate this in fig. 3b



(a) Undamped (b) Damped

Figure 3: Deformation of the middle node at the top of the element

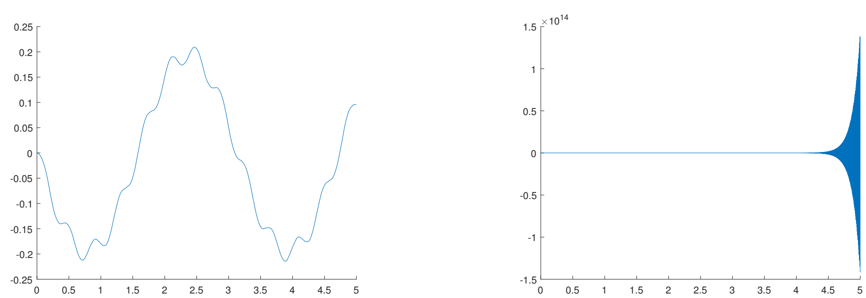
Which damping do you use?

c) **Use the explicit centered differences algortihm in order to simulate the first 5 seconds with $p = 1$ and $\bar{\omega} = 2rad/s$. Use the largest time-step hmax that you can detect that gives stable results. How does this value comply with the Courant criteria?**

Solution:
In our model the displacement will be linear combinations of the eigenmodes found in section "a". Therefore the largest frequency that can appear in our solution is the largest eigenfrequency $\omega_{max} = 673.45881 rad/s$.

By the Nyquist or Courant criteria we need to be able to sample at least twice as fast as the fastest frequency appearing in the solution. So our sampling rate h must be at most the half of the period, i.e. $h \leq \frac{2}{\omega_{max}}$.

The maximum h we may have theoretically is $h_{crit} = \frac{2}{\omega_{max}} = 0,002969744$. As we can observe in fig. 4 a very similar frontier exist in the practical case which is around $h_{\max} = 0.002965$.



(a) $h = 0.00296$ (b) $h = 0.00297$

Figure 4: Displacement of the top middle node

d) **Use the HHT algorithm with $(\alpha, \beta, \gamma) = (0, 0.25, 0.5)$ and the time-step 2hmax found in c). How does the result and the computational cost compare with the results in c)?**

Solution:

The central difference algorithm is a explicit and second order algorithm. Therefore the computational cost that we would expected is low, the central difference is stable for values of $h \leq \frac{2}{\omega_{max}}$. However, the Newmark algorithm for $\gamma = 0.5$ and $\beta = 0.25$ is implicit, though is unconditionally stable. Therefore choosing an appropriate time step size depends on solution accuracy, algorithm stability, solver convergence and computational cost.
It is interesting to analyze how the election of the mesh it will affect the value of h_{crit}, since $h \leq \frac{2}{\omega_{max}}$ then h_{crit} it is inversely proportional to ω_{max}, so in figure fig. 5 it is shown the relation between the ω_{max} $(y-axis)$ and the number of divisions in the squared body$(x-axis)$, as the number of divisions increases the ω_{max} increases so the h_{crit} decreases, so as the mesh is refined the time step needed to converge it will decrease, so the running time will be higher.

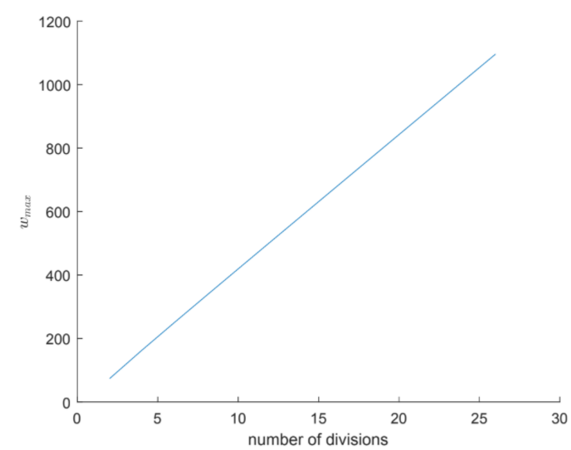


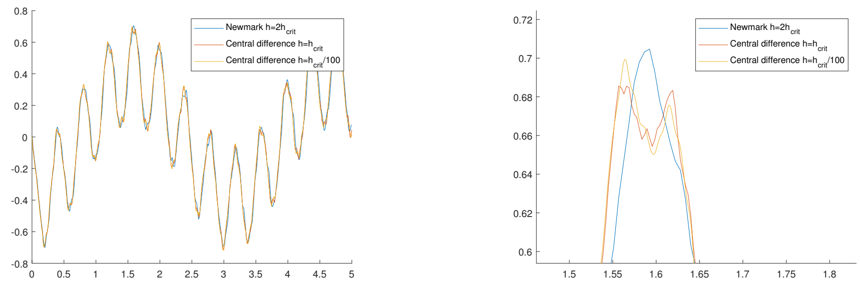
Figure 5: Relation between ω_{max} $(y-axis)$ and the number of divisions in the squared body$(x-axis)$.

If we use for Newmark algorithm for a timestep equal to $h = 2h_{\mathbf{crit}}$ and $(\alpha, \beta, \gamma) = (0, 0.25, 0.5)$ we have that the average time that our algorithm need to converge is $1,42s$, note that we have use a corrector-predictor algorithm to solve Newmark. On the other hand if we use the central difference method (implemented from the pseudo-code in the course slides)

Converge or finish?

for the value $h_{\text{crit}} = 0.0029$ calculated in c) then it took $1.74s$. fig. 6a shows the different solutions for the two cases mentioned before and a solution which we call "real" solution since we set the time step at $\Delta t = \frac{h_{\max}}{100}$. We can see that both solution are really close to this "real" solution. However in fig. 6b which shows a zoom of the previous one we can see that the central difference is more accurate.

Is the explicit algorithm exploited? (use of lumped, no system inversion....)



What if the same time-step is used?

(a) Full view

(b) Zoomed on one peak

Figure 6: Evolution of the velocity of the top middle node for different algorithms

e) **Plot the evolution of the elastic, kinetic, and total energy, denoted by E,K and T, for the case p = 0, and using the Newmark algorithm with $(\alpha, \beta, \gamma) = (0, 0.25, 0.5)$ and $(\alpha, \beta, \gamma) = (-0.1, 0.3025, 0.6)$. Comment on the different results.**

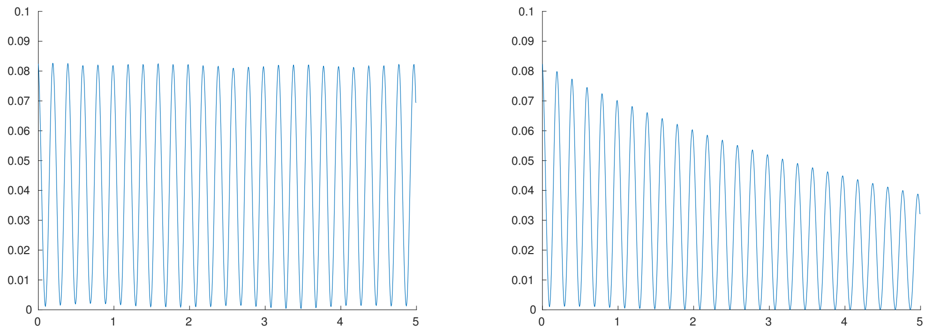
Solution:

In this case we set the applied loads $p = 0$. Therefore in other to see how is the evolution of the energy we will assume initial velocity parallel to y-axis different from zero, with magnitude equal to each element heights.

As we have an initial velocity, the kinetic energy in fig. 7a has an initial value different from zero but then it will get traded for potential energy fig. 8a. This trade of energy occurs at a frequency of around $5Hz$ which is similar to the double the frequency of the first mode. This makes sense because when subjected to an initial velocity, the dominant mode will prevail. Since we have that the kinetic energy is defined by $K = \frac{1}{2}mv^2$, if v oscillates with a frequency w, K will oscillate with a frequency $2w$. This is due to the trig identities: $cos^2(wt) = \frac{1+cos(2wt)}{2}$.

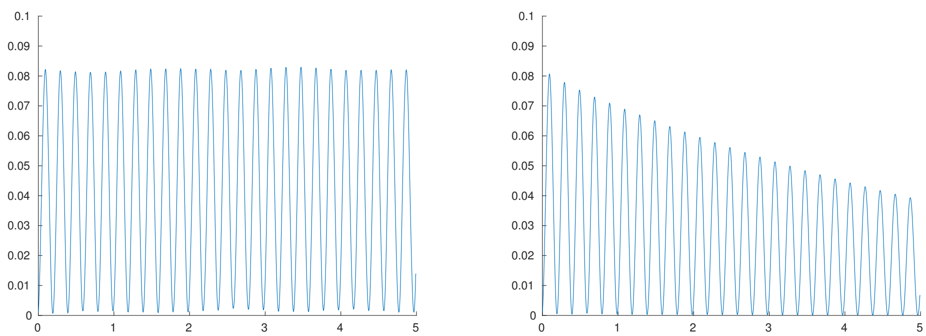
Yes

Newmark algorithm is a special case of HHT-α algorithm for the case of $\alpha = 0$. As we have seen in class when we have $\alpha \neq 0$ we introduce a numerical dissipation without accuracy lost, this is the reasons that in the first case (Newmark) we have that the total energy in fig. 9a is conserved. However since we have added this numerical dissipation for $\alpha \neq 0$ then we see that the total energy in fig. 9b decreases as we expected from this damping.



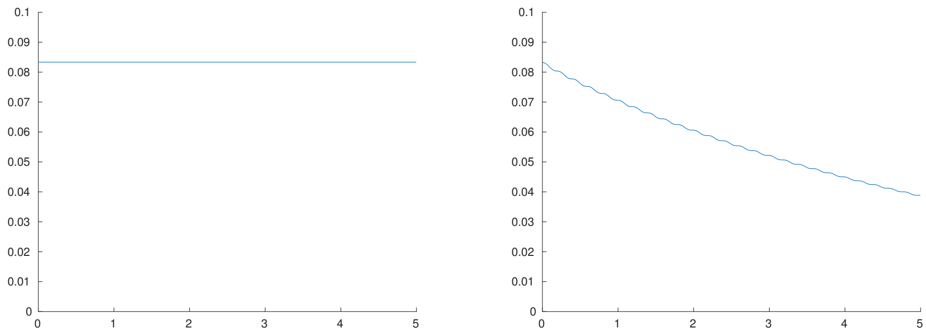
(a) $(\alpha, \beta, \gamma) = (0, 0.25, 0.5)$ (b) $(\alpha, \beta, \gamma) = (-0.1, 0.3025, 0.6)$

Figure 7: Evolution of kinetic energy



(a) $(\alpha, \beta, \gamma) = (0, 0.25, 0.5)$ (b) $(\alpha, \beta, \gamma) = (-0.1, 0.3025, 0.6)$

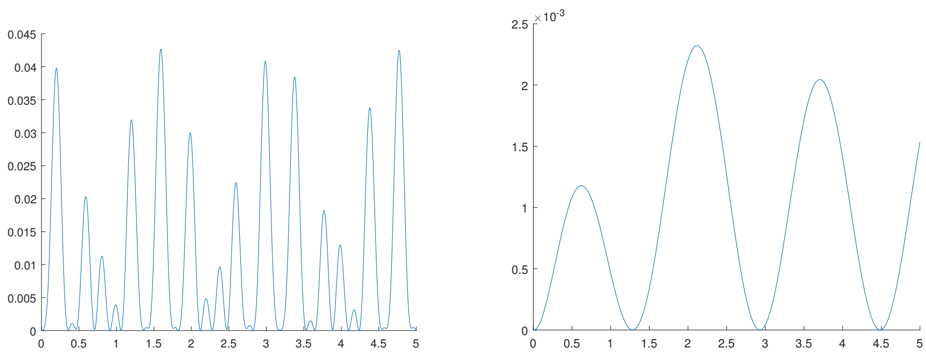
Figure 8: Evolution of deformation energy



(a) $(\alpha, \beta, \gamma) = (0, 0.25, 0.5)$ (b) $(\alpha, \beta, \gamma) = (-0.1, 0.3025, 0.6)$

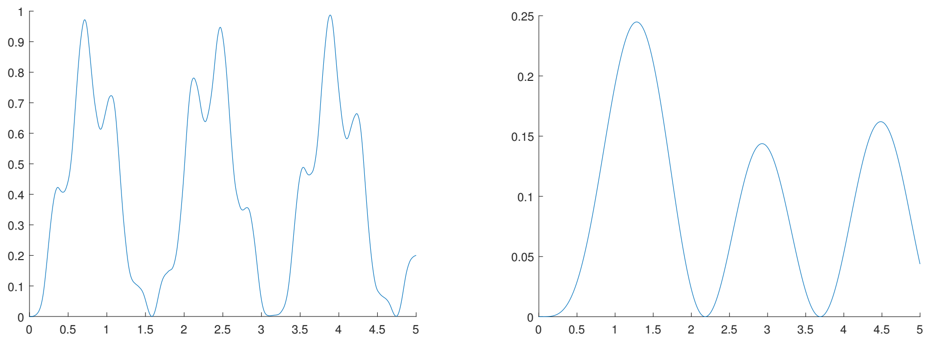
Figure 9: Evolution of total energy

f) **Use now** $p = 1$ **and** $\bar{\omega} = 2rad/s$**. Plot the evolution of E,K and T using no damping** $(C = 0)$ **and** $C = M + K$**. Use the HHT algorithm with the trapezoidal rule,** $(\alpha, \beta, \gamma) = (0, 0.25, 0.5)$**. Interpret the results.**



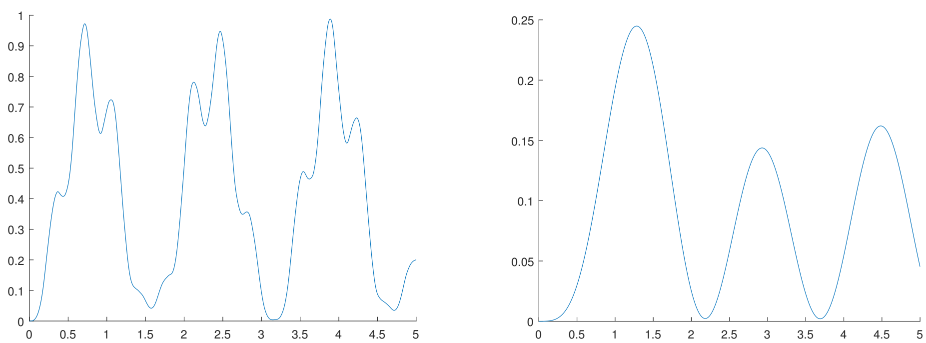
(a) Undamped (b) Damped

Figure 10: Evolution of kinetic energy



(a) Undamped (b) Damped

Figure 11: Evolution of elastic energy



(a) Undamped (b) Damped

Figure 12: Evolution of total energy

As we can see in figs. 10 to 12 the smallest frequencies are the first one to be dissipated. This is the reason why the damping plots are smoother.

Also we can see in the damped simulation the amount of total energy is less that in the undamped one due to the energy the damping dissipated energy.

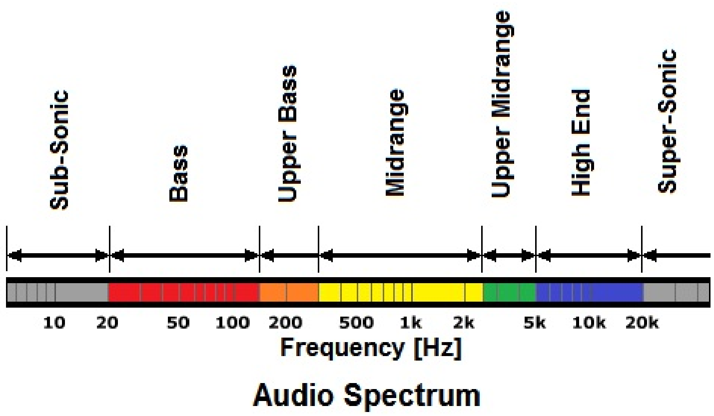


Figure 13: Audio Frequency Spectrum

A material with this damping would dissipated the fastest frequencies that are in range of human sensibility leaving only the sub-sonic ones.

Reyleigh damping: dissipation of largest and lowest freq.