Assignatura: Computational mechanics

Estudiant/a: [redacted]

Data: 04/05/2019

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(1) We have $\sigma = \eta \, \varepsilon^{\boxed{dev}}$!

(a) The constitutive law: $\underline{\underline{\sigma}} = \underline{\underline{f}}(\underline{\underline{\varepsilon}})$ (relation between $\underline{\underline{\sigma}}$ and $\underline{\underline{\varepsilon}}$)

$$\left.\begin{array}{l} \underline{\underline{\varepsilon}} = \nabla^S \underline{u} = \frac{1}{2}\left(\nabla u + (\nabla u)^T\right) \\ \varepsilon_{ij} = \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{1}{2} \end{array}\right\} \text{Small strain tensor}$$

In our case $f(\underline{\underline{\varepsilon}}) = \underline{\underline{\sigma}} = \eta \underline{\underline{\varepsilon}}$

$$\nabla^S \underline{\underline{u}}^h \underline{\underline{\sigma}}^h = \{\varepsilon^h\}^T \{\underline{\underline{\sigma}}^h\} = \left\{ \begin{array}{c} \frac{\partial u_x^h}{\partial x} \\ \frac{\partial u_y^h}{\partial y} \\ \frac{\partial u_y^h}{\partial z} \\ \frac{\partial u_x}{\partial y} + \frac{\partial u_y^h}{\partial x} \\ \frac{\partial u_x^h}{\partial z} + \frac{\partial u_z^h}{\partial x} \\ \frac{\partial u_y^h}{\partial z} + \frac{\partial u_z^h}{\partial y} \end{array} \right\}^T \left\{ \begin{array}{c} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{array} \right\}$$

$$\sum v_i \left[\int \rho \frac{d^2 u_j}{dt^2} N_j(x) N_i(x) \, d\Omega + \int B_i^T \{\underline{\underline{\sigma}}\} \, d\Omega = \int \rho b N_i + \int t N_i \, dS \right] \quad \forall v_i$$

$$\Longrightarrow \int \rho N_i(x) N_j(x) \, d\Omega \, \frac{\partial^2 u_j}{\partial t^2} + \underbrace{\int \underline{\underline{B}}^T \underline{\underline{C}} \, \underline{\underline{B}}_j \, d\Omega}_{K_{ij}^e} \, u_j = f_i$$

Linear elasticity

$$K_{ij}^e = \int_\Omega B^T C B_j \, d\Omega$$

(b) $$\{\varepsilon(u^h)\} = \sum_{j=0}^{8} \begin{vmatrix} \partial_x N_j & 0 & 0 \\ 0 & \partial_y N_j & 0 \\ 0 & 0 & \partial_z N_j \\ & \vdots & \end{vmatrix}_{8\times 3} \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix} = \sum_{i=1}^{8} v_i^T B_i^T \underline{\underline{\sigma}}$$

We have $3\times(n+1) = 3(8+1) = 27$ equations $\cancel{\mathrm{Dim}(k) = 27}$.

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2) We have $K = \frac{E}{h}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ $\quad M = \frac{h\rho}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

(a) We know that the eigen-frequencies ω_i are

$$\omega_i^2 = \lambda_i \quad \text{where} \quad M^{-1}Ku = \lambda u$$

Compute $M^{-1}K$:

$$M^{-1} = \frac{6}{3h\rho}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \frac{2}{h\rho}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$M^{-1}K = \frac{2}{h\rho}\frac{E}{h}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{2E}{\rho h^2}\begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix}$$

Now, find eigenvalues: ~~for~~

$$\begin{vmatrix} x-3 & -3 \\ -3 & x-3 \end{vmatrix} = (x-3)^2 - 9 = 0 \iff \boxed{\begin{matrix} x=0 \\ x=6 \end{matrix}}$$

Therefore $\omega_1 = \sqrt{0 \cdot \frac{2E}{\rho h^2}} \Longrightarrow \boxed{\omega_1 = 0}$

$$\omega_2 = \sqrt{6 \cdot \frac{2E}{\rho h^2}} \Longrightarrow \boxed{\omega_2 = \frac{1}{h}\sqrt{\frac{12E}{\rho}}}$$

(b) Now we have $\underline{\underline{M}}^L = \frac{h\rho}{6}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Compute $(M^L)^{-1}K$:

$$(M^L)^{-1} = \frac{2}{3h\rho}\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$(M^L)^{-1}K = \frac{2E}{3h^2\rho}\begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix}$$

The eigen-values are $\begin{cases} x = 0 \\ x = 6 \end{cases}$

But now, we have:

$$\omega_1 = \sqrt{0 \cdot \frac{2E}{3h^2\rho}} = 0 \implies \boxed{\omega_1 = 0}$$

$$\omega_2 = \sqrt{6 \cdot \frac{2E}{3h^2\rho}} \implies \boxed{\omega_2 = \frac{2}{h}\sqrt{\frac{E}{\rho}}}$$ ✓

0

(c) We want Δt_{crit} minimum, because we want a minimum critical time-step difference. No!

In case (a), we have $\omega_{2(a)} = \frac{1}{h}\sqrt{12\frac{E}{\rho}} = \frac{2}{h}\sqrt{\frac{3E}{\rho}}$

In case (b) we have $\omega_{2(b)} = \frac{2}{h}\sqrt{\frac{E}{\rho}}$

$\implies \omega_{2(a)} > \omega_{2(b)}$ and in our case we want the maximum because $\frac{2}{\omega_{max}}$ be minimum.

[Therefore, the M^L is better. ✓]

(d) If h is less, ω_2 will increase ? $\implies \rho$ is going to affect to the stability.

✗

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(3) We have a shear stress $\sigma = \begin{pmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

And the material follows a Von Mises plasticity criteria.

This implies that $f(\sigma) = \sqrt{3J_2} - \sigma_y$ ✓

And we know that when $f(\sigma) < 0$ we have elasticity and when $f(\sigma) = 0$ we have plasticity.

We have to compute J_2.

$$J_2 = \frac{1}{2}\sigma' : \sigma' = \frac{1}{2}\sigma : \sigma - \frac{3}{2}\sigma_m^2 = \frac{1}{3}I_1^2 - I_2 = \tau^2$$

Where $I_1 = tr(\underline{\sigma})$ and $I_2 = tr(\underline{\sigma}^2)$

$I_1 = 0 \quad I_2 = 2\tau^2$ ✗

$$\Rightarrow J_2 = \frac{1}{3} \cdot 0 - 2\tau \Rightarrow J_2 = \cancel{+}2\tau^2$$

Now, $f(\sigma) = \sqrt{-2\tau \cdot 3} - \sigma_y = \sqrt{-6\tau} - \sigma_y$

We want $f(\sigma) = 0 \Rightarrow \sqrt{-6\tau} = \sigma_y$

$$\Leftrightarrow -6\tau = \sigma_y^2 \Rightarrow \boxed{\tau = \frac{-\sigma_y^2}{6}}$$

This is the maximum shear stress τ.

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(4)

(a) The scheme of the problem is:

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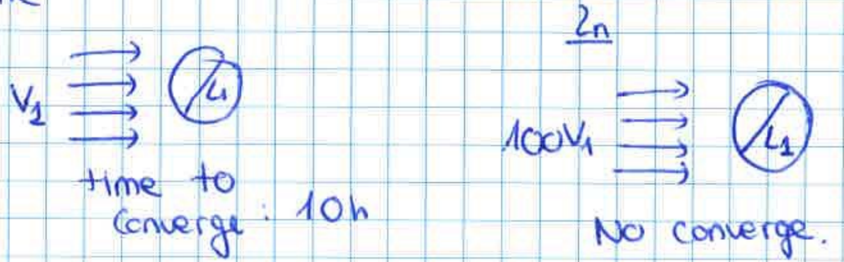
1st time



In the first time you have a Reynolds number is $Re_1 = \frac{L_1 V_1}{\nu}$ and in the second case: $Re_2 = 100 Re_1$

The error in the second case may be because the other conditions (except the velocity) are the same.
A fine enough mesh (or adapted to features) is necessary for convergence of a non-linear solver, because we are in Navier-Stokes equation. ✓

The solution is take a mesh with more nodes.
(+ better initial guess)

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(b) Now we have $L_2 = L_1/100$ and $V_2 = 100 V_1$

Now, $Re_1 = \frac{L_1 V_1}{\nu}$ and $Re_2 = \frac{L_2 V_2}{\nu} = \frac{100 L_1 \, V_1/100}{\nu} = \frac{L_1 V_1}{\nu}$

$\Rightarrow Re_1 = Re_2$

In this case we don't have a problem with the computation because the Reynolds number is equal. We can obtain the solution with previous ones.

$$\bar{v}_2 = (?)\,\bar{v}_1$$