First Rate Scientist or Second Rate Mathematician

30 August 2023

Outline

- NASA Carbon Overwrapped Pressure Vessel (COPV) Study
  - Background
  - Experimental Designs
  - Analyses
  - Incorporating the Physics
- Jet Turbine Data

NASA Strand and Vessel Testing

- NASA's Engineering Safety Center (NESC) project to assess safety of Composite Overwrapped Pressure Vessels (COPVs)
- COPVs
  - Transport gasses under high pressure
  - Metal Liner
  - Wrapped by a Series of Carbon Strands
- Research Question: **Determine Reliability of COPVs at Use Conditions for the Expected Mission Life**
  - Primary Focus on Strands
  - Secondary Focus on Relationship to Vessels
  - Strands Less Expensive to Test
- https://www.nasa.gov/offices/nesc/home/Feature_COPVs_Jan-2012.html

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NASA Strand and Vessel Testing

- Analyses Use Classic Weibull Model

$$\mathrm{R}(t_i) = e^{-\left(\frac{t_i}{t_{ref}} SR^{\rho}\right)^{\beta}}$$

- Observed Life Time: t_i
- SR: Stress Ratio, ratio of stress level to strength scale parameter
- Critical Parameters:
  - ρ: Sensitivity to Stress Ratio
  - β: Shape parameter for time to Failure
  - t_{ref}: Reference time to Failure when SR=1

NASA Strand Study

- Previous Strand Test
  - Relevant strand study conducted at a national lab
  - 57 strands at high loads for 10 years
  - Net information learned:
    - Strands either fail very early or
    - Last more than 10 years
  - Limited information based on 10 years of study!

- Estimates of Critical Parameters for Planning

NASA Strand Study

- Team’s Initial Concept
  - Much larger study that the original 10 year study
  - Censor very early
    - Reduces time
    - Allows for the larger study in a practical amount of time

- Proceed in phases

- Have detailed data records to track any problems

NASA Strand Study

Experimental Phases

- Phase A – During "shake-out" of tests rigs
- Phase B – "Gold Standard" Experiment for Strands
- Phase C – "Proof" Study
- In Parallel: Vessel Studies (Opportunistic)

Phase A

- Conducted During Shake-Out of Equipment
  - Small study (although bigger than the national lab study!)
  - Statistical goal: Determine if the parameters from the national lab study are valid as the basis for planning the larger study!
  - Note: Phase A gave the team an opportunity to re-plan the larger experiment, if necessary!

Phase B

- "Gold Standard" Experiment
  - Planned time required: 1 year
  - Used 4 "blocks" of almost equal numbers of strands
    - Allowed the team to correct for time effects
    - Allowed the team to mitigate problems, especially early
  - Study assumed the "classic" Weibull model
  - Size of the experiment assured ability to assess model

Block	SR	Number	Proportion
1	0.80	176	0.718
	0.85	50	0.204
	0.90	19	0.078
	Sum	245	1.00
2	0.80	170	0.708
	0.85	50	0.208
	0.90	20	0.083
	Sum	240	1.00
3	0.80	174	0.710
	0.85	51	0.208
	0.90	20	0.082
	Sum	245	1.00
4	0.80	176	0.718
	0.85	49	0.200
	0.90	20	0.082
	Sum	245	1.00

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Observations

- Phase A: Surprisingly Similar to Initial Study

- Phase B:
  - Serious problem occurred with the gripping in the first block
  - Serious conversations with possibility of replacing!
  - Other three blocks well behaved and by themselves produced better than the planned precision for the estimates

- Final Decision: Drop the First Block

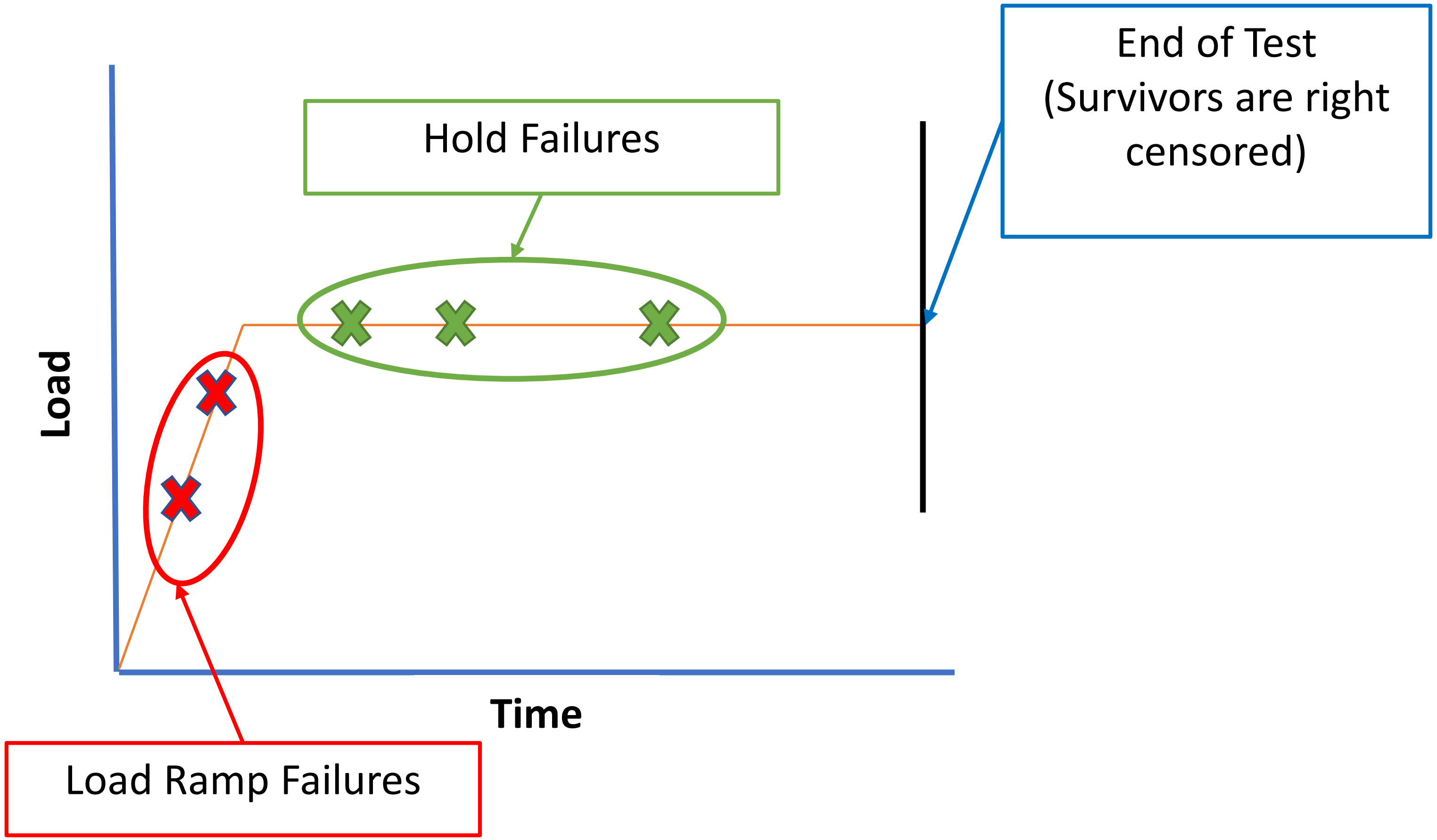
NASA Strand Study: Benefits

- Phase A:
  - Opportunity to Confirm Initial Study Parameter Estimates
  - Allowed opportunity to revise the experimental protocol if the estimates were significantly different

- Phase B:
  - Allowed opportunity to model changes in time over the year.
  - Mitigated the problem with the first block!
  - Provided simple mechanism for replacing the first block if needed!

Description of Stress Rupture Test

- Stress Rupture
  - Failures occur after a period of time where there is no increase in load
- Failures are needed to determine reliability
- Must extrapolate from where test is performed versus where reliability predictions are made
- Test strands at higher loads and then extrapolate
- Need a model to make predictions



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Classic Stress Rupture Model: Weibull

- Classic Weibull Survival Function

$$S(t_i) = P(T > t_i) = e^{-\left(\frac{t_i}{t_{ref}} SR^{\rho}\right)^{\beta}}$$

Note: $\eta = t_{ref} SR^{-\rho}$

- Observed Life Time: t_i
- SR: Stress Ratio, ratio of stress level to strength scale parameter
- Critical Parameters:
  - ρ: controls the relationship between the failure time and stress ratio (SR)
  - β: Shape parameter for time to Failure
  - t_{ref}: Reference time to Failure

Classic Stress Rupture Model: SEV

- Re-expressed Survival Function

$$S(t_i) = e^{-\left(\frac{t_i}{t_{ref}} SR^{\rho}\right)^{\beta}} = e^{-e^{\beta(\log t_i - \theta + \rho \ln(SR))}}$$

where $\theta = \log(t_{ref})$ and $\mu = \log(\eta) = \theta - \rho \ln(SR)$

Now working with a linear model, similar to simple linear regression

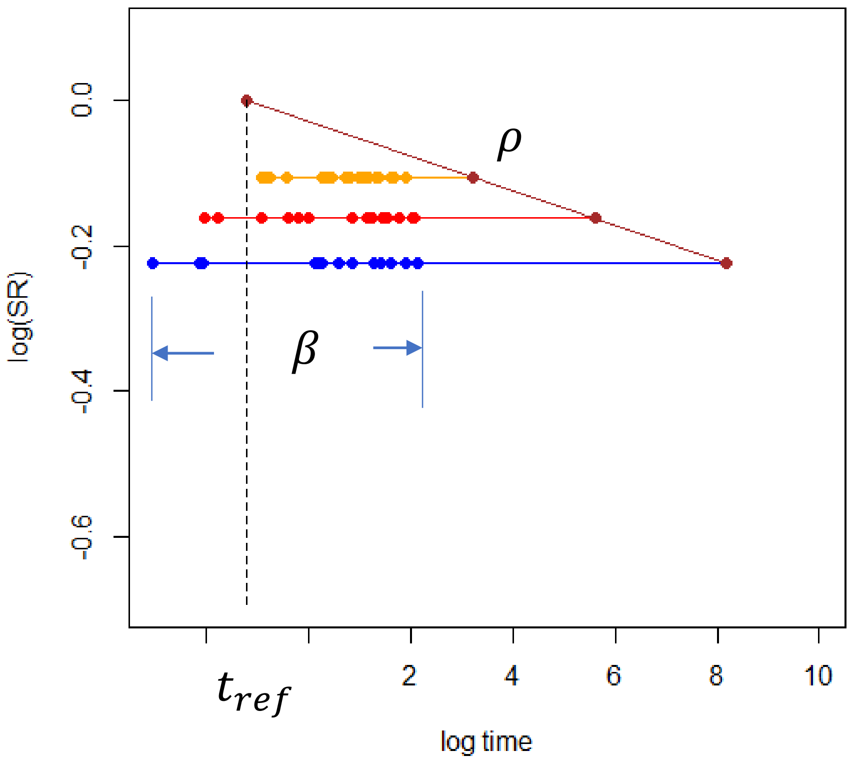
- Scaled Residuals
  - $z_i = \beta e_i = \beta(\log t_i - \mu) = \beta(\log t_i - \theta + \rho \ln(SR))$
  - Used for predictions of the log probability for specific observations

True Structure of the Stress Rupture Model

$$\mu = \log(\eta) = \theta - \rho \ln(SR)$$

- ρ: controls the relationship between the failure time and stress ratio (SR)
- β: Shape parameter for time to Failure
- t_{ref}: Reference time to Failure

Log Stress Ratio versus Log Time



Stress Rupture model explains the behavior of the items *on hold*.

- Weibull regression gives us estimates for ρ, β and t_{ref}

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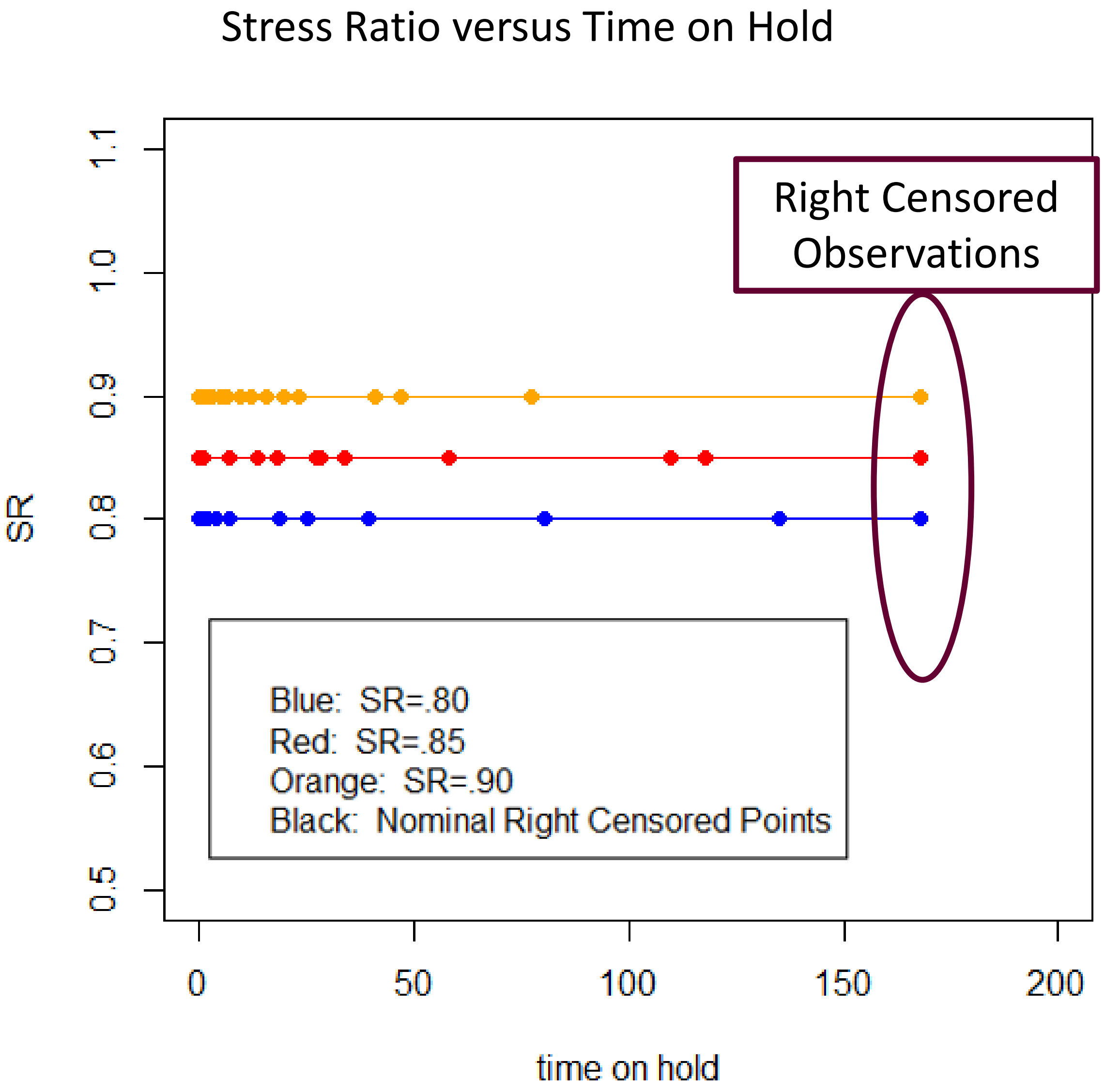
"Full Model"

- Separate individual models to each stress ratio
  - Two parameters for the SR=1 data: $\eta_{.80}$ and $\alpha_{.80}$
  - Two parameters for the SR=2 data: $\eta_{.85}$ and $\alpha_{.85}$
  - Two parameters for the SR=3 data: $\eta_{.90}$ and $\alpha_{.90}$
- Largest possible Weibull model for the data
  - Has the largest log-likelihood
- Will compare to the Full Model to subset models to determine whether the improvement in log-likelihood justifies the extra parameters



Proper Analysis:

- Model the data that have achieved the target load as defined by the experimental protocol (no ramp data)

- Defines that the time at the sustained constant load begins the moment the test item achieves the target load

- Assumes a Weibull distribution to describe the time to failure under the sustained constant load

- Experimental protocol uses right-censoring at a nominal time

Proper Analysis of Full Model

- Ramp and Hold data are modeled separately
  - Three parameters to explain the hold data: ρ, β, and $\theta = \log t_{ref}$
- Model assumes

$$\alpha_{.80} = \alpha_{.85} = \alpha_{.90} = \beta$$

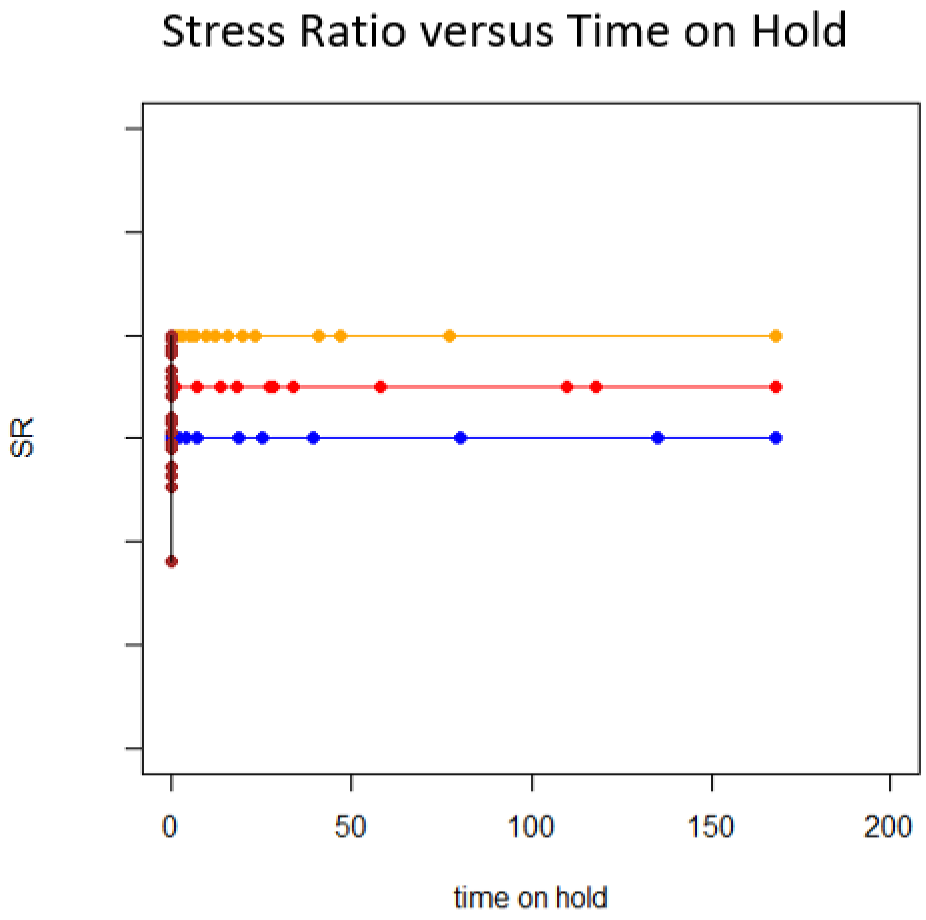
Comparisons

Model:	Fit-to-Hold	Full Model
Number of Observations:	708	708
True Log-Likelihood:	−306.411	−305.900
Log-Likelihood Statistic:	612.822	611.800
AIC:	618.822	623.800

- The p-value associated with the χ^2 based on the difference in the log-likelihood statistics is 0.7959
  - The three extra parameters in the full model are not significant
- Smallest AIC value for Fit-to-Hold (adjustment for parameters)

Adaptations to Include Ramp Failures

- Rigorous Approach
  - Add two additional parameters for a Weibull Distribution fit to only the ramp data along with the Fit-to-Hold Analysis
    - Two parameters for the ramp data: η and α
    - Three parameters to explain the hold data: ρ, β, and $\theta = \log t_{ref}$



- Left Censored Analysis
  - Assume that ALL data follow the same failure mechanism
  - Left censor all ramp failures and some early stress rupture failures
    - Three parameters to explain the ramp and hold data: ρ, β, and $\theta = \log t_{ref}$

Comparisons

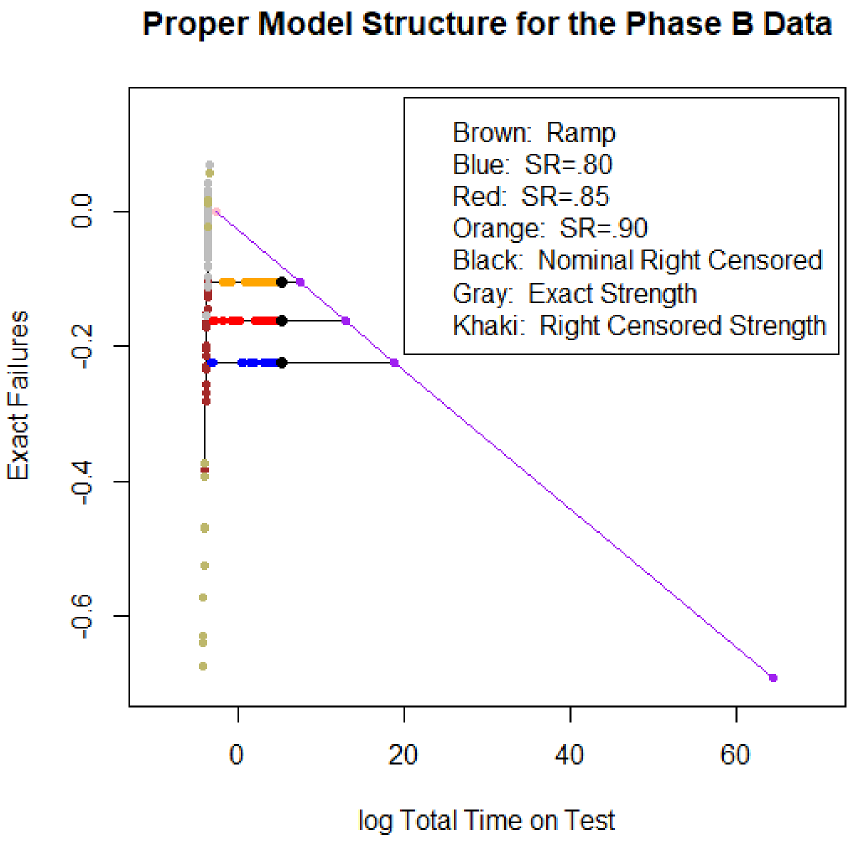
	Rigorous	Left-Censored	Full Model
Overall log $\mathcal{L}$:	-251.103	-390.779	-250.592
Log-Like Stat:	502.206	781.558	501.184
AIC:	512.206	789.558	517.184
Ramp log $\mathcal{L}$:	55.308		55.308
Hold log $\mathcal{L}$:	-306.4114		-305.900

- All fit statistics indicate that the rigorous model is the superior fit to the data compared to the Left-Censored approach
  - Smallest AIC value (512.206) and log-likelihood statistic (502.206) and the largest overall log-likelihood value (-251.103)The three extra parameters in the full model are not significant
- The probability that the left censored analysis explains the data at least as well as the rigorous model is 1.03946 E-62
- Counter-intuitive to penalize the maximum likelihood fit to the data with left censoring especially when we know the precise time these items failed on the ramp

Physics “Infused” Approach

- Approaches presented to this point: All empirical!
- Stress-Strain controls tensile strength and stress rupture.
  - Ramp: Rapid Pull
  - Stress Rupture: Redistribution of Load as Fibers Fail (Slow)
- Physics: Failure when strain exceeds threshold.
- Task: Can we illustrate with our experimental results.

Structure of the Data



Key Points

- Failure on Hold Requires Test Item to Survive Ramp
- Estimate Effective/Equivalent Load by Probability Item Fails on Hold
  - $S_R(SR_t)$: Survivor Probability that Item Achieves Target Stress Ratio
  - $S_h(t)$: Survivor Probability for Item Fails on Hold
  - $P(t \leq t_h)$ Probability Item Fails at Time t_h

$$P(t \leq t_h) = S_R(SR_t)+S_R(SR_t) * [1 - S_h(t)]$$

$$= 1 - S_R(SR_T) * S_H(t_h)$$

Estimating the Effective Load

- Weibull

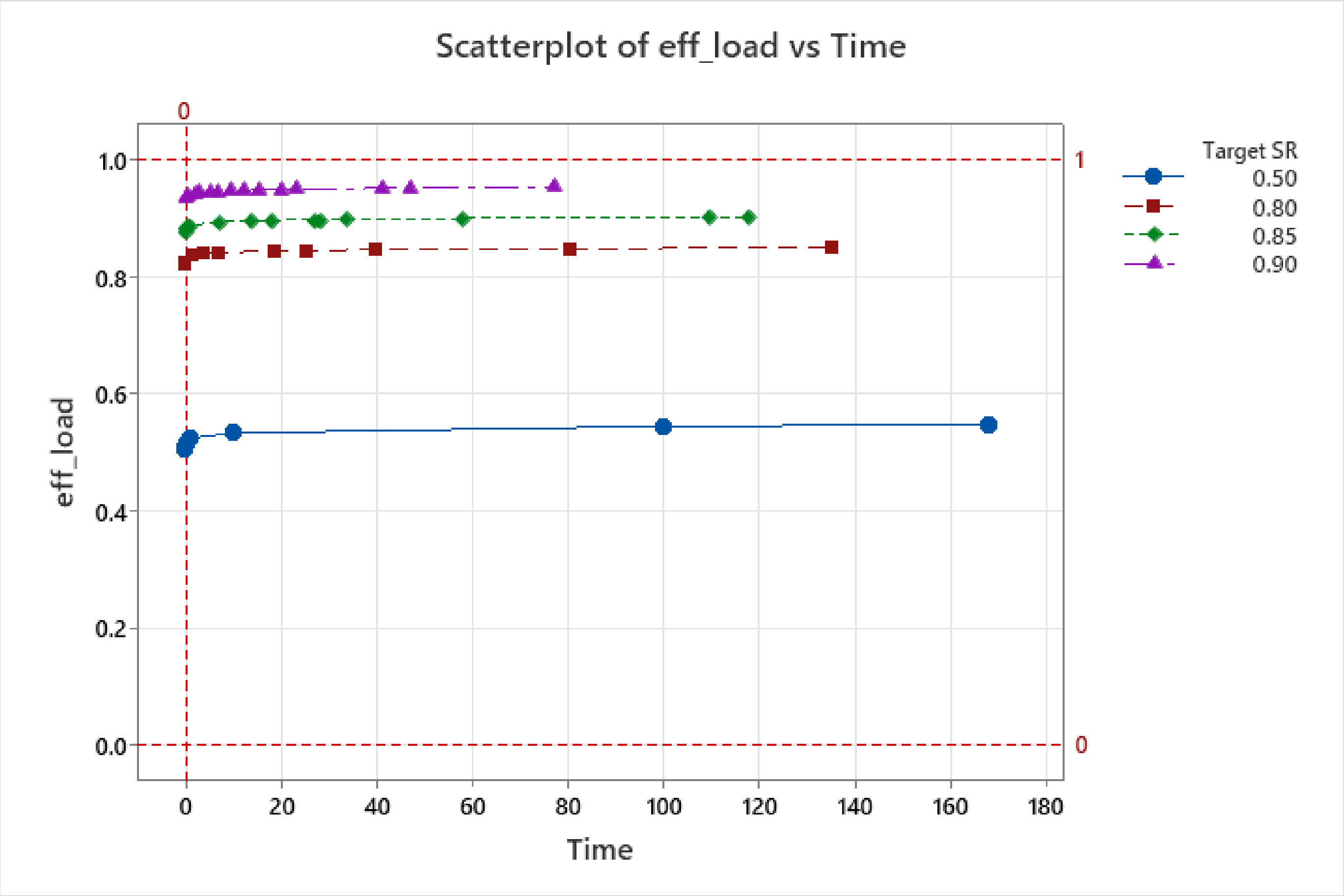
$$L_0 = \{-\ln[1 - P(t < t_h)]\}^{1/\alpha}$$

- Log-Normal

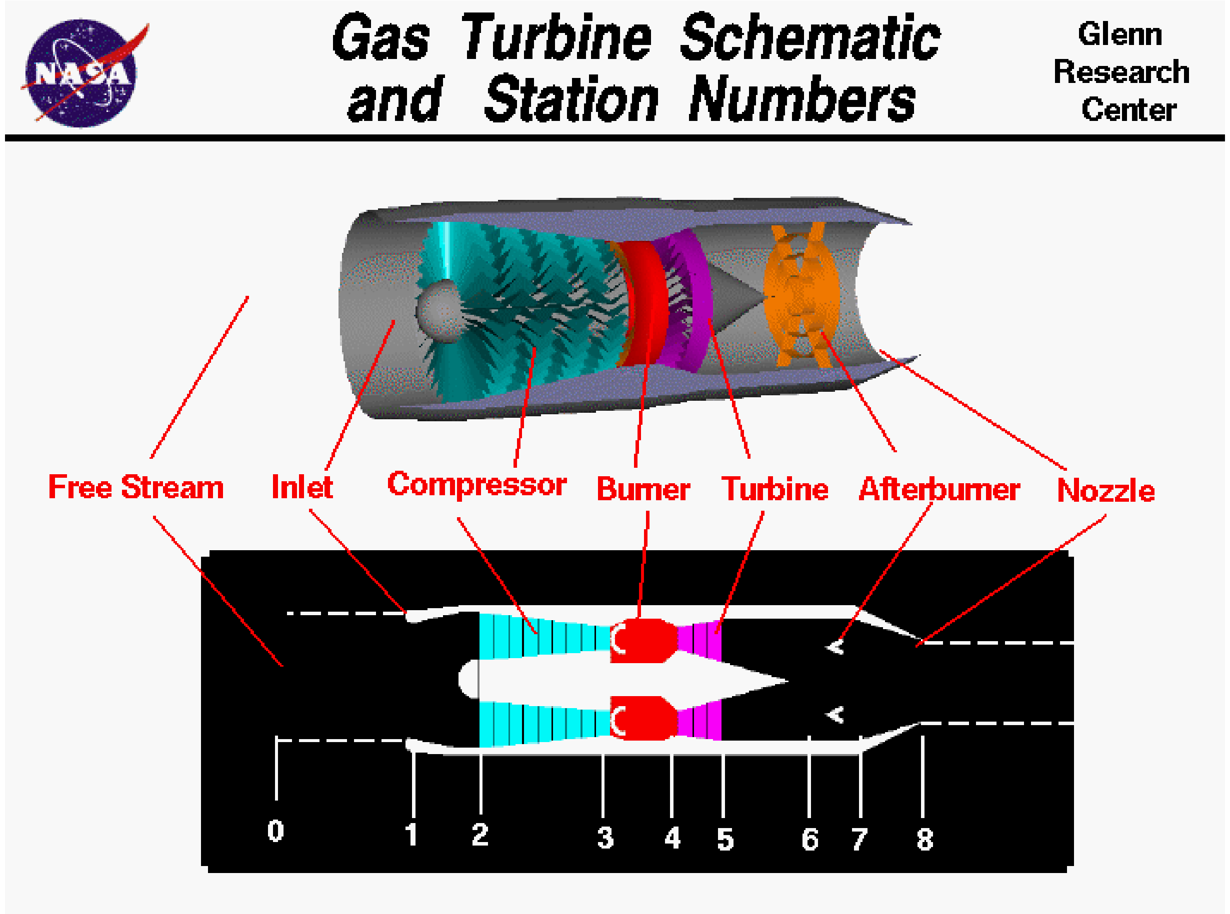
$$L_0 = \exp\{\mu + \sigma \cdot F^{-1}[P(t < t_h)]\}$$

  - μ: Mean for the Log-Normal Distribution
  - σ: Standard Deviation

Curve: Strands



Case Study: Structure of Jet Turbine Engine



Opportunity Presented by Industry 4.0

- For a Given Critical Quality Characteristic, y:
  - Very Serious Economic Consequences If Not under “Control”
  - Large Amount of High Quality Data over Time
  - Typical Behavior over Time Is Non-Linear
- Frequently, Large Number of Ancillary Variables, $x_1, x_2, \cdots, x_m$
  - Highly Correlated with y
  - Also, Large Amount of High Quality Data
  - Proper Modeling Defines the Effect of the x’s on y
  - These Effects Are the Observed Manifestations of the System of Causes
- Challenge: Building Proper Set of Models

Understanding the Science: Thermodynamics

- Underlying Thermodynamics:

$$T = {T_{20}}^{\theta_1} \left(\frac{P_{30}}{P_{20}} \right)^{\theta_2}$$

$$\log T = \theta_1 \log T_{20} + \theta_2 \log \left(\frac{P_{30}}{P_{20}} \right)$$

$$\hat{T} = \exp \left[\hat{\theta}_1 \log T_{20} + \theta_2 \log \left(\frac{P_{30}}{P_{20}} \right) \right]$$

- Define the “Thermo Residual”:

$$e_{th} = T - \hat{T}$$

Monitoring Procedure: First Step

- Critical Quality Characteristic: TGT.
- Obtain a Training Data Set.
- Estimate θ_T and ω_T Using the Model

$$\log TGT = \theta_T \log T_{atm} + \omega_t \log\left(\frac{P_{30}}{P_{20}}\right) + \epsilon$$

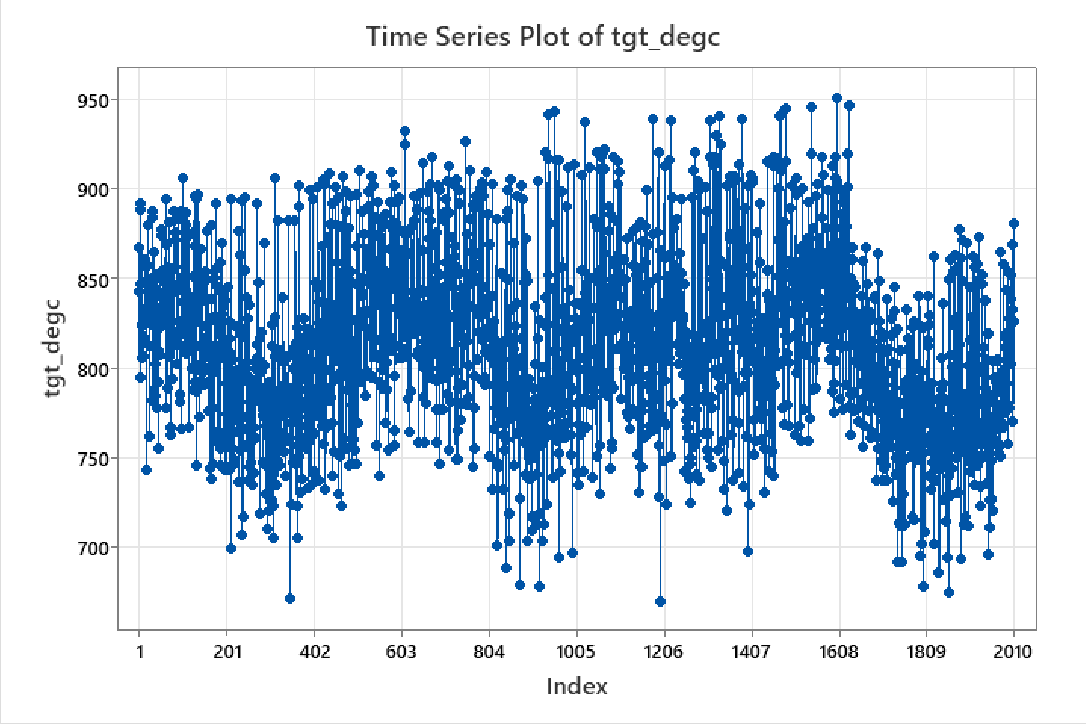
- Resulting Residuals Explain the First Variance Component.
- Variance of These Residuals Reflect Basic Thermodynamics

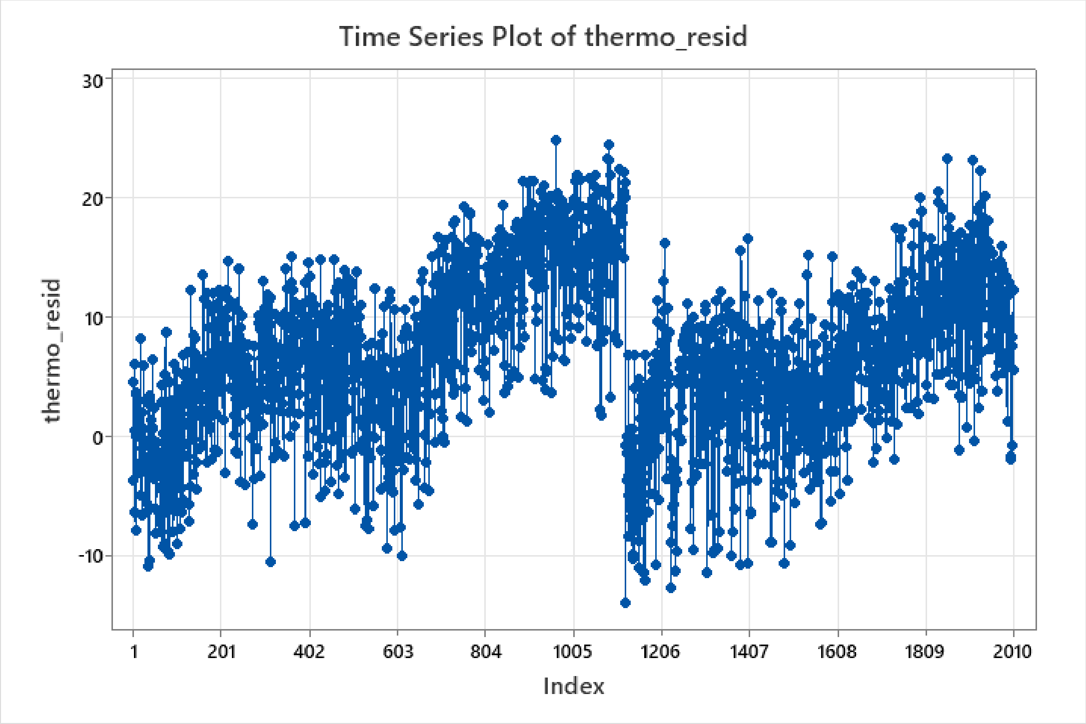
Monitoring Procedure: Second Step

- Thermo Residuals Use Only:
  - T_{atm}
  - Pressure Ratio: P_{30}/P_{20}
- There > 40 Other Candidate Variables to Explain the Behavior!
- Critical Issues for Selecting Models:
  - Centered, Scaled Variables!
  - Good Model Selection Approaches
- Primary Variables: Second Variance Component across Engines
- Identified Best Model: 15 Predictors

Monitoring Procedure: First Step

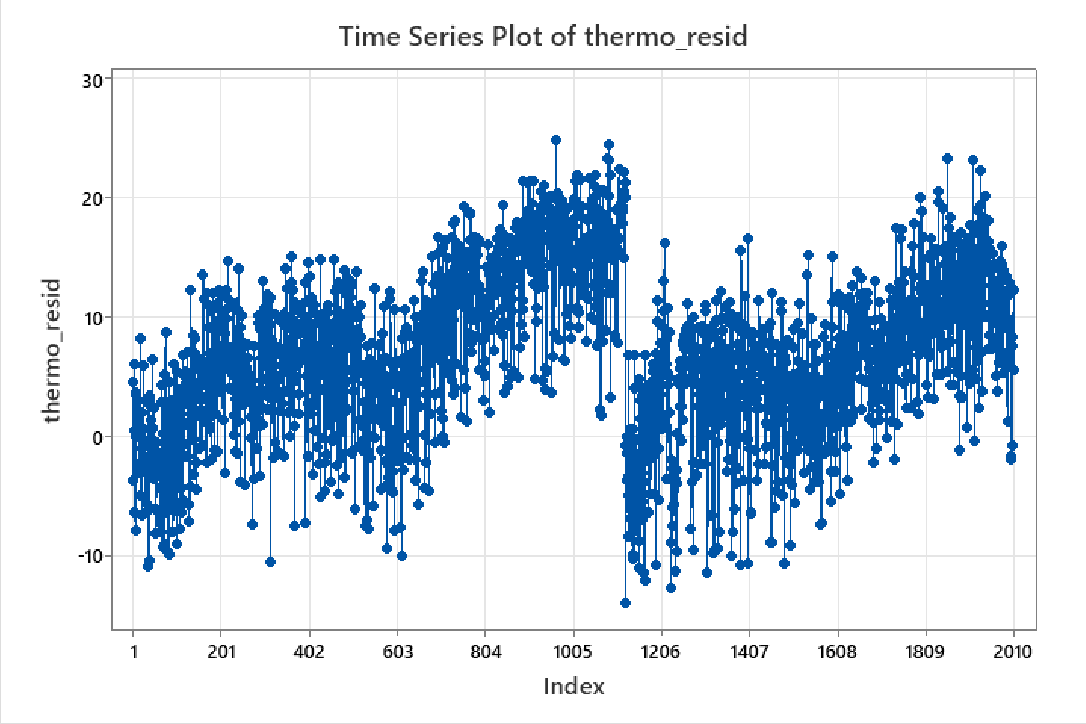
Notice Change in Scale!

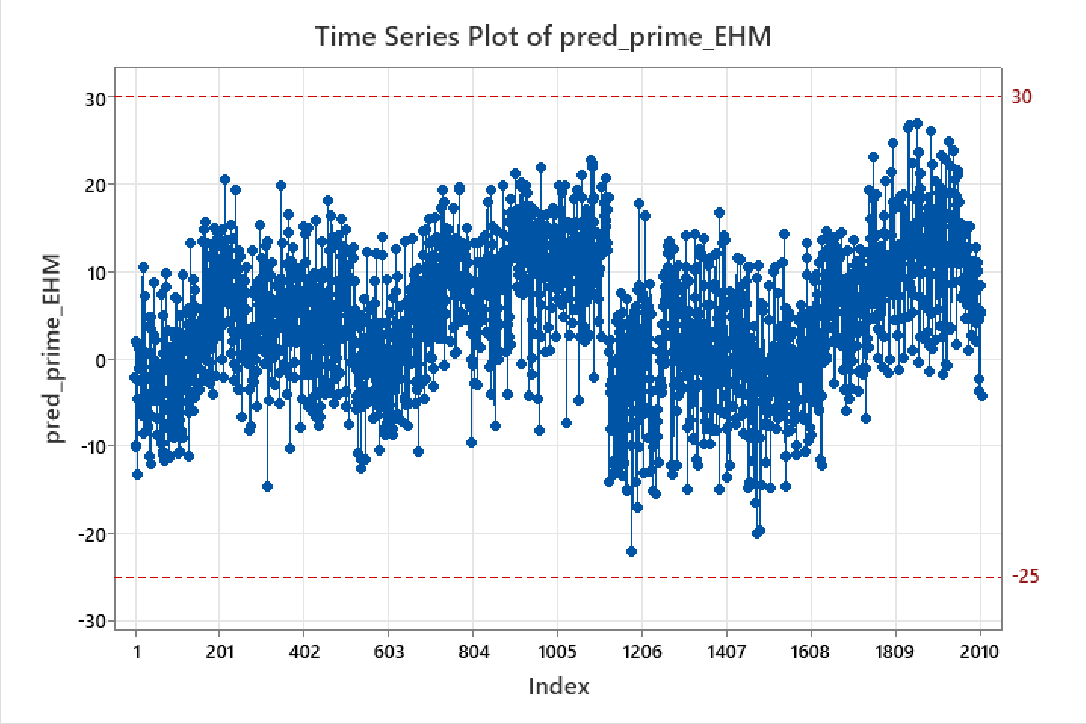




Monitoring Procedure: Second Step

Notice Similarity in Patterns!





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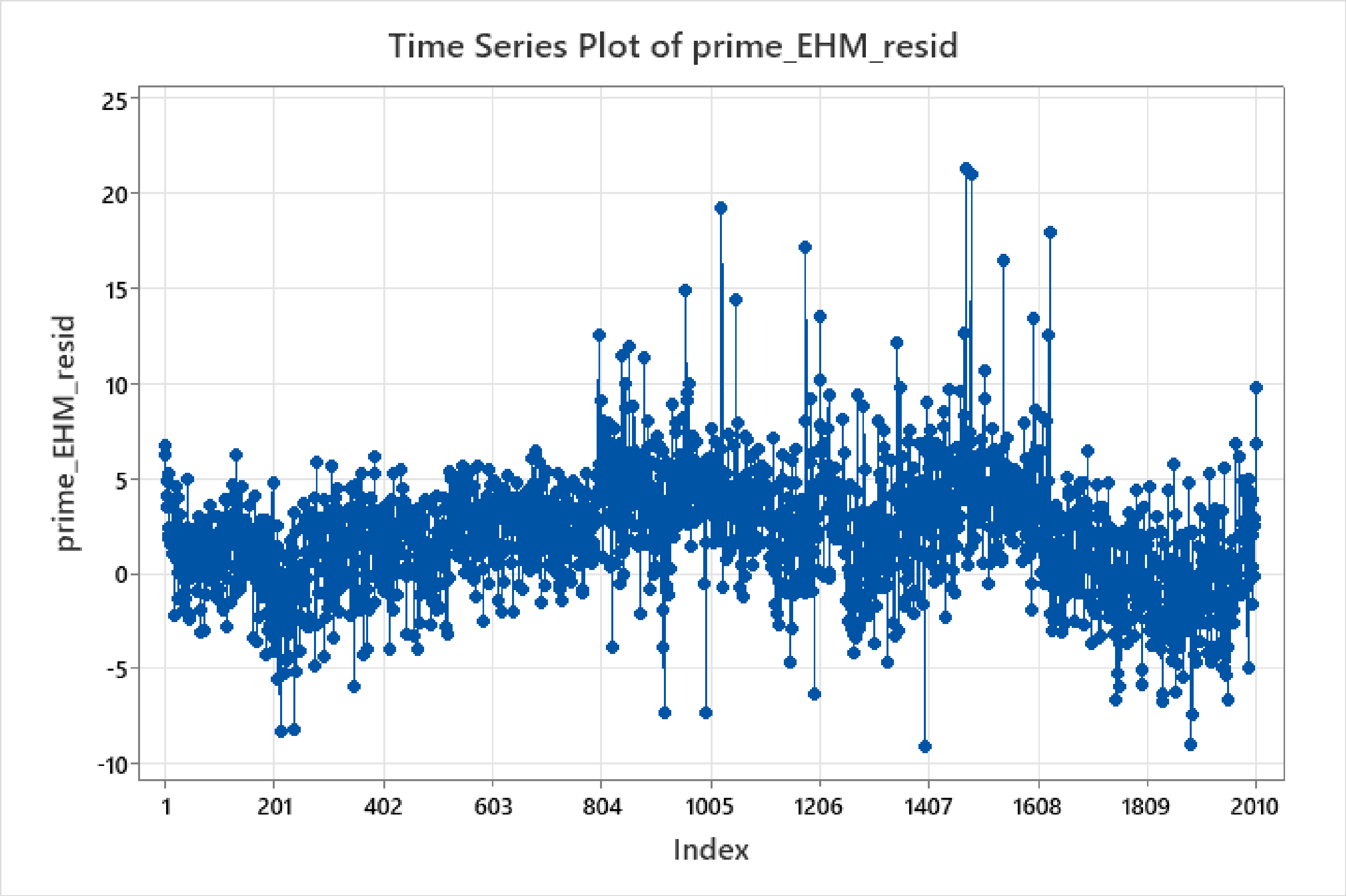

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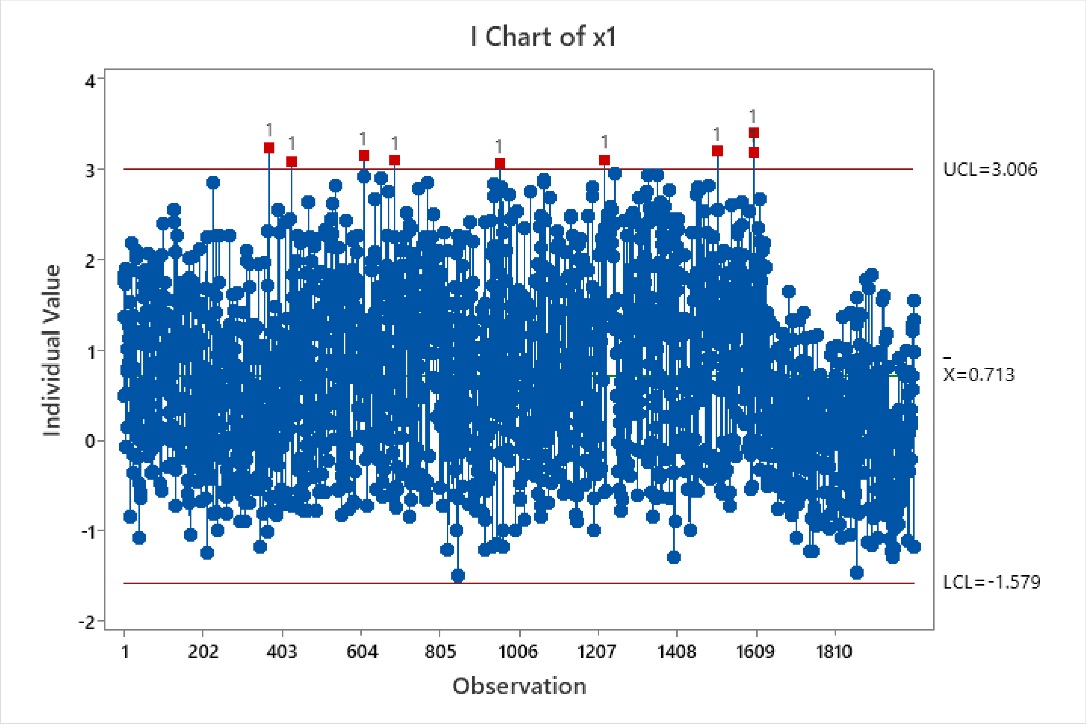
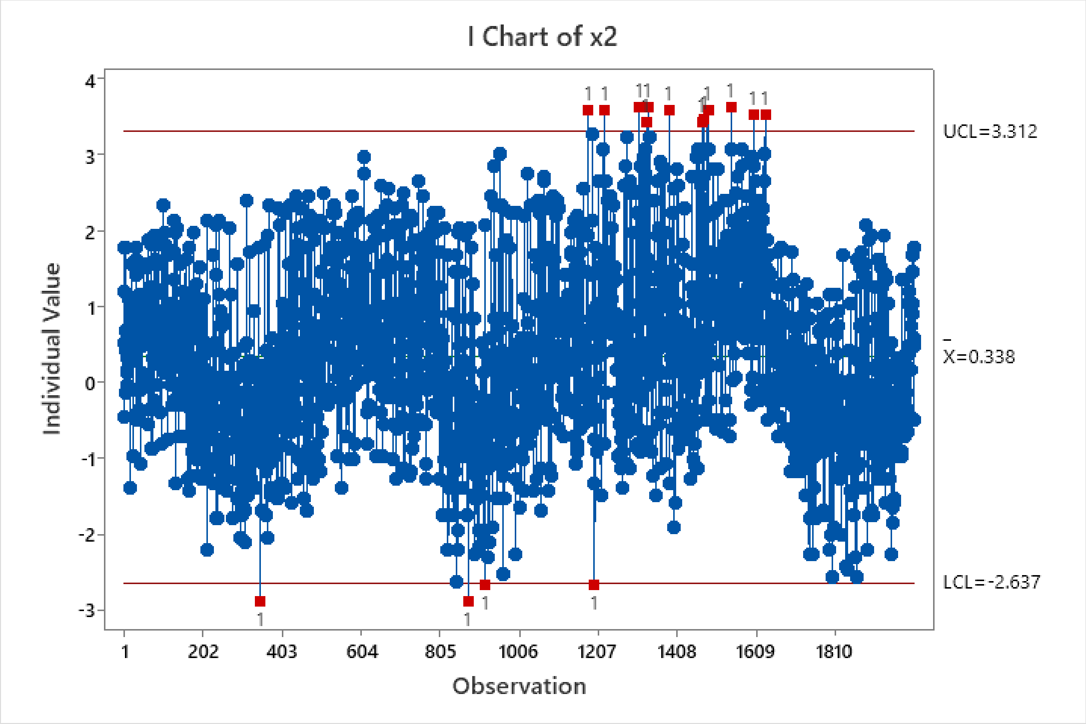
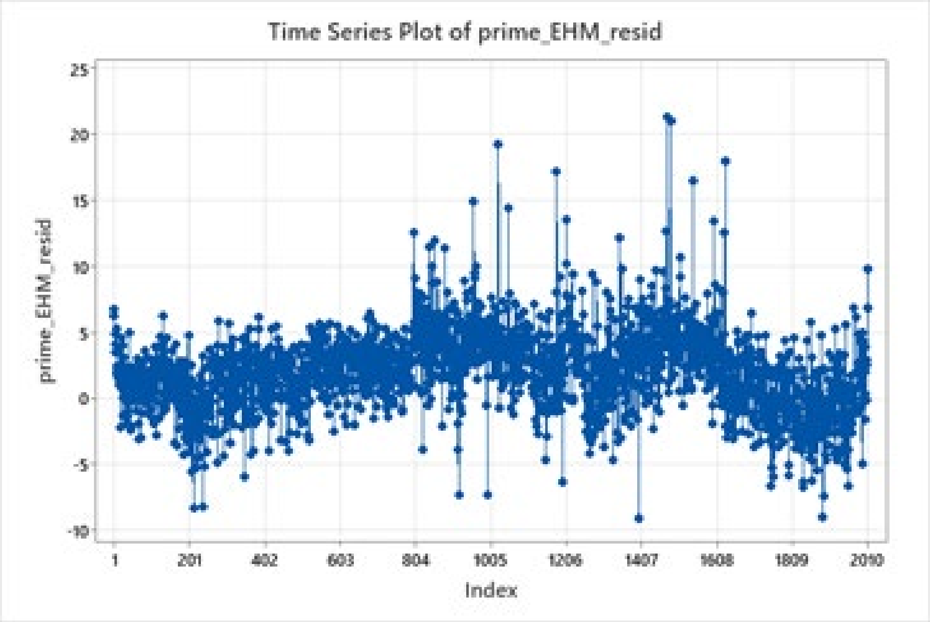
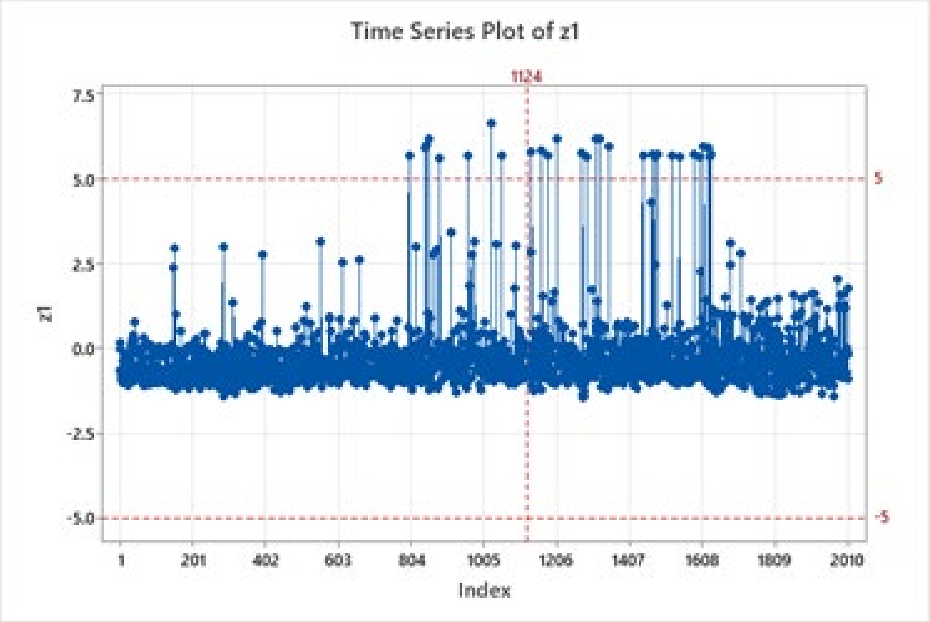




Third Layer: Variables Not "Important" for All Engines





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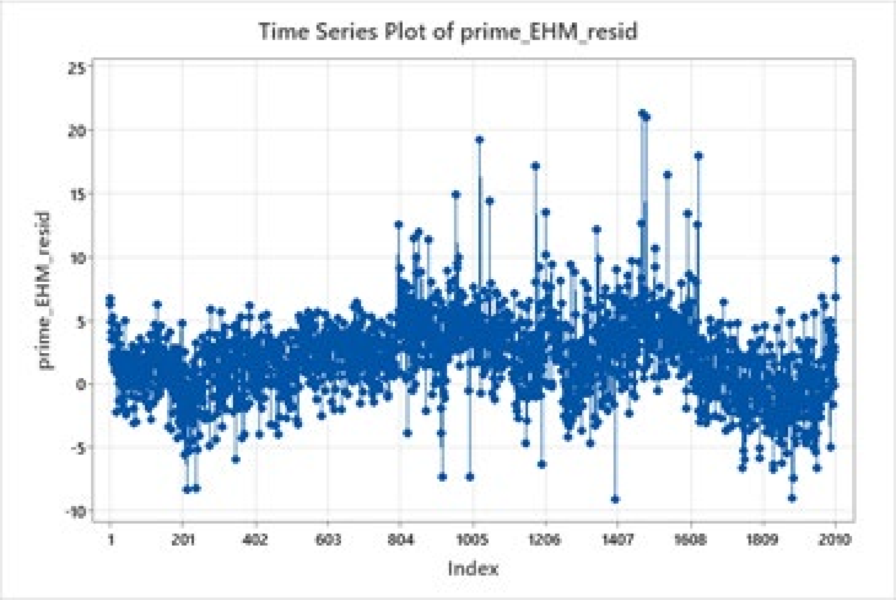

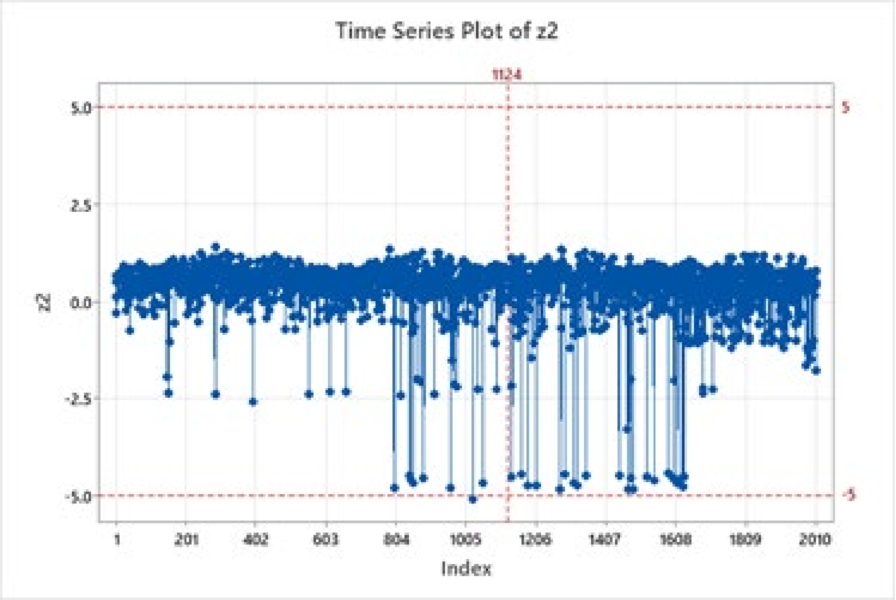

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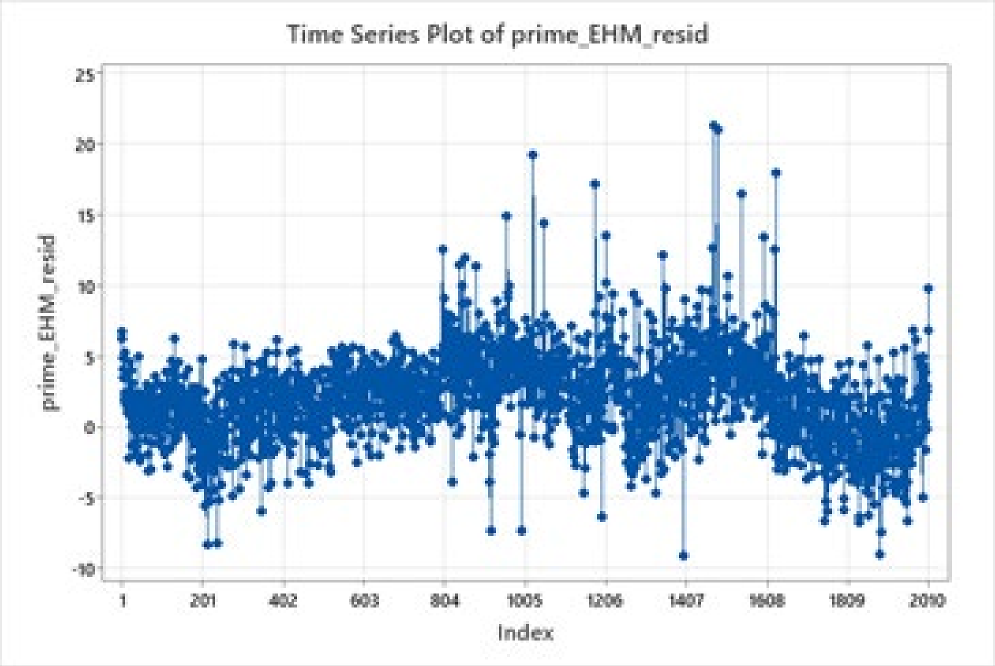
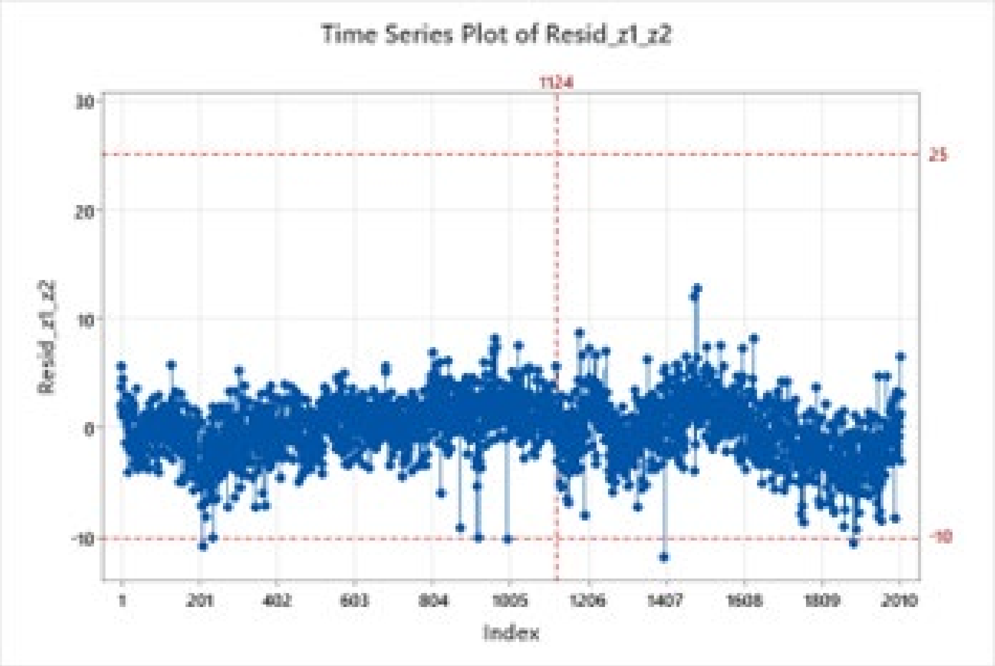




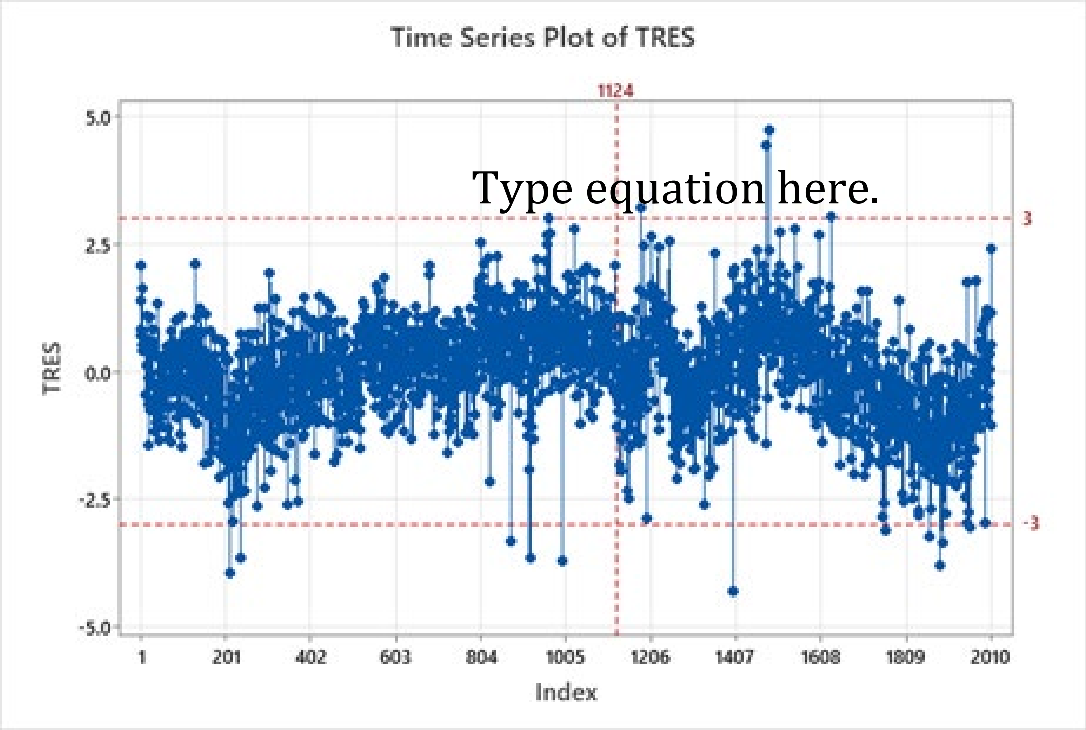
Third Layer: Variables Not "Important" for All Engines

Basic Linear Models Theory:

Proper Residuals for Checking Assumptions Are Externally Studentized Residuals.

Common Diagnostic: Time Plot of Residuals.

No Brainer: Add Limits



Bonferroni: No Observation Is Significant!