Alessio Figalli

Fields Medal 2018

Courtesy of ETH, Zürich

Doctor Honoris Causa

of the

Universitat Politècnica de Catalunya

22nd November 2019

Alessio Figalli 1

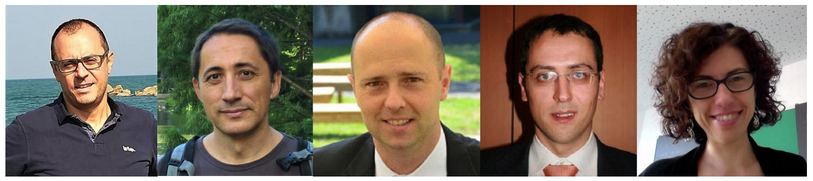
Main collaborators

Luigi **Ambrosio**, Scuola Normale Superiore di Pisa. *Fermat Prize* 2003. *Balzan Prize* 2019. Cédric **Villani**, Institut Henri Poincaré, Sorbone University, University of Lyon, Institut Camille-Jordan. ***EMS*** *Prize* 2008. *Fermat Prize* 2009. *Fields Medal* 2010. *Doob Prize* 2014. Luis A. **Caffarelli**, University of Texas at Austin. *Steele Prize* 2009. *Wolf Prize* 2012. *Shaw Prize* 2018. Jean **Bourgain** (1954-2018). *Fields Medal* 1994. *Shaw Prize* 2010. *Crafoord Prize* 2012. *Steele Prize* 2018.

David **Jerison**, Massachusetts Institute of Technology. *Bergman Prize* 2012. Juan L. **Vázquez**, Universidad Autónoma de Madrid. *Premio Nacional de Investigación Julio Rey Pastor* 2003. *Plenary speaker* at ICM-2006. *Medalla de la* ***RSME*** 2018. Albert **Fathi**, Georgia Institute of Technology. *Sophie Germain Prize* 2013. *Professeur émérite* à l'**ENS** de Lyon. Paul **Thierry**, Directeur de Recherche, Centre de Mathématiques Laurent Schwartz, École Polytechnique, Paris, France. Henrik **Shahgholian**, Royal Institute of Technology (**KTH**), Stockholm, Sweden.

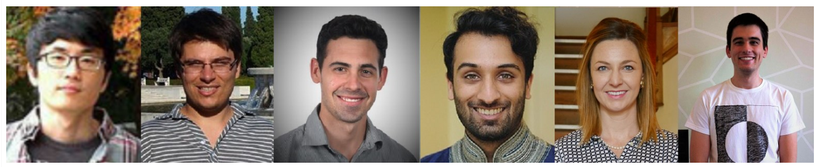
Francesco **Maggi**, University of Texas at Austin. *Miranda Prize* 2008. José A. **Carrillo** de la Plata, Imperial College London. *Von Mises Prize* 2006. *Wolfson Prize* 2012. Robert J. **McCann**, University of Toronto. *Coxeter-James Prize* 2005. *Jeffery-Williams Prize* 2017. Alice **Guionnet**, École Normale Supérieure de Lyon. *Loève Prize* 2009. *Pascal Medal* 2018. Kim **Young -Heon**, University of British Columbia.

Ludovic **Rifford**, Université Nice Sophia Antipolis. Director of the Centre International de Mathématiques Pures et Appliquées. *Eisenbud Professor at* ***MSRI*** 2013 (fall). Enrico **Valdinoci**, University of Western Australia. *Most cited mathematician according to his graduation year* (in all subjects and in Analysis). ***ERC*** *Starting Grant* 2011-2016. Aldo **Pratelli**, Università di Pisa. *Medal of the President of Italian Republic* for young researchers 2004. *Iapichino Prize* 2005. ***ERC*** *Starting Grant* 2010-2015. *Miranda Prize* 2011. *De Giorgi's Prize* 2015. Matteo **Bonforte**, Universidad Autónoma de Madrid. Álvaro **Pelayo**, University of California at San Diego. ***RSME*** *Rubio de Francia Prize* 2009. ***NSF*** *Faculty CAREER Award* 2016.



Marco **Di Francesco,** Dipartimento di Ingegneria e Scienze dell'Informazione e Matematica, Università degli Studi dell'Aquila. Dejan **Slepčev**, Department of Mathematical Sciences, Carnegie Mellon University. Giulio **Ciraolo**, Department of Mathematics Federigo Enriques, Università degli Studi di Milano. Nicola **Gigli**, Scuola Internazionale Superiore di Studi Avanzati. *De Giorgi's Prize* 2019. Laura Valentina **Spinolo**, **IMATI-CNR**, Pavia, Italy.

Guido **de Philippis**, Courant Institute, New York University. ***EMS*** *Prize* 2016. *Stampacchia Medal* 2018. Maria **Colombo, EPFL** Lausanne. *Iapichino Prize* 2016. *Miranda Prize* 2018. Begoña **Barrios,** Universidad de La Laguna, Tenerife, Spain. Xavier **Ros-Oton**, Universität Zürich. ***RSME*** *Rubio de Francia Prize* 2016. ***SeMA*** *Antonio Valle Prize* 2017. ***ERC*** *Starting Grant* 2018. *Fundación Princesa de Girona Prize* 2019. Joaquim **Serra**, **ETH** Zürich. ***SCM*** *Évariste Galois Prize* 2011. ***SCM*** *Josep Teixidó Prize* 2016. ***SeMA*** *Antonio Valle Prize* 2019. ***RSME*** *Rubio de Francia Prize* 2019.



Moon-Jin **Kang**, Sookmyung Women's University, Seoul, South Korea. Emanuel **Indrei**, Department of Mathematics, Purdue University. Connor **Mooney**, University of California at Irvine. *Firestone Medal for Excellence in Undergraduate Research*, Stanford University, 2011. Yash **Jhaveri**, Princeton Institute for Advanced Study. Robin **Neumayer**, Northwestern University. Xavier **Fernández-Real**, PhD student, **ETH** Zürich. *Whelan Prize in Mathematics* from Christ's College, University of Cambridge, 2015. ***SCM*** *Évariste Galois Prize* 2016. *National Award for Excellence in Academic Performance* 2013-14, First Prize, 2018.

Xavier **Cabré**, **ICREA** Research Professor, Department of Mathematics, Universitat Politècnica de Catalunya. *Kurt Friedrichs Prize*, New York University, 1995. Harrington Faculty Fellow, The University of Texas at Austin, 2001-02. *Fellow of the American Mathematical Society, inaugural class*, 2012.

STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC EQUATIONS ARE SMOOTH UP TO DIMENSION 9

XAVIER CABRÉ, ALESSIO FIGALLI, XAVIER ROS-OTON, AND JOAQUIM SERRA

ABSTRACT. In this paper we prove the following long-standing conjecture: *stable solutions to semilinear elliptic equations are bounded (and thus smooth) in dimension* $n \leq 9$.

This result, that was only known to be true for $n \leq 4$, is optimal: $\log(1/|x|^2)$ is a $W^{1,2}$ singular stable solution for $n \geq 10$.

> La physique mathématique et l'analyse pure ne sont pas seulement des puissances limitrophes, entretenant des rapports de bon voisinage : elles se pénètrent mutuellement et leur esprit est le même.
>
> Henri **Poincaré**, *La Valeur de la Science*.

Sources

- Alessio **Figalli** (**AF**) Web page (**WP**), particularly his **CV** (22.10.2019).
- Luis **Caffarelli** (**LC**). "Alessio Figalli: His Contributions to Geometry and Analysis". *Proceedings of the International Congress of Mathematicians*. World Scientific, 2019. Volume 1, 43-46.

> A recurring theme in Figalli's research is the interaction between the theory of *optimal transport* and other areas of mathematics. Optimal transport theory, although pioneered by G. Monge back in 1781, has emerged to paramount importance in many areas of mathematics only in the last two decades. One of the most valuable features of this theory is its *unifying power*, in the sense that key ideas and constructions in optimal transport have turned out to be useful in the most unexpected contexts. Figalli is currently one of the most authoritative experts in optimal transport and its many applications. (**LC**, p. 1 (43))

- Xavier **Ros-Oton** and Joaquim **Serra**. Private communications.
- MathSciNet.
- **FME**. Documentation on the **DHC** nomination of **AF**.

Topics

While the broad research area of **AF** is Partial Differential Equations and Calculus of Variations, he has landmark contributions in a large number of topics, like Regularity of solutions to elliptic PDE's and to free boundary problems; Isoperimetric and geometric inequalities; Minimal surfaces; ODE's and SDE's with non-smooth coefficients; Weak KAM theory; Random matrices.

Research Landmarks

Books are distinguished with *.

2007* **AF**. *Optimal Transportation and Action-Minimizing Measures*. PhD thesis, Supervised by Luigi **Ambrosio** and Cédric **Villani**. 251 pp. **WP**.

2009 **AF**. "Regularity of optimal transport maps (after **Ma-Trudinger-Wang** and **Loeper**)." Séminaire BOURBAKI 2008-2009, n° 1009. *Astérisque* **332**, SMF, 2010. **WP**.

2010* **AF** and Luigi **Ambrosio**. *Lecture notes on variational models for incompressible Euler equations*. **WP**.

2010 **AF**. "Review of Cédric **Villani**'s book *Optimal transport. Old and new* (Grundlehren der Mathematischen Wissenschaften 338. Springer-Verlag, 2009. xxii+973 pp)". *Bulletin of the AMS* (N.S.) **47**.4, 723-727.

2010 **AF** and Albert **Fathi**. "Optimal transportation on non-compact manifolds". *Israel Journal of Mathematics* **175**.1, 1-59.

2010 **AF**. "The optimal partial transport problem." *Archive for rational mechanics and analysis* **195**.2: 533-560.

2010 **AF**, and Ludovic **Rifford**. "Mass transportation on sub-Riemannian Manifolds". *Geometric and Functional Analysis* **20**.1, 124-159.

2010 **AF** and Kim **Young-Heon**. "Partial regularity of Brenier solutions of the Monge-Ampère equation". *Discrete and Continuous Dynamical Systems* **28**.2: 559-565.

2010 **AF**, Francesco **Maggi** and Aldo **Pratelli**: "A mass transportation approach to quantitative isoperimetric inequalities". *Inventiones Mathematicae* **182**.1, 167-211.

> … a sharp quantitative stability theorem for the Wulff inequality […] a most important mathematical result in our understanding of surface tension driven phase transitions, as it relates the microscopic structure of a given surface tension energy density to the macroscopic shape of the liquid/crystal observed at equilibrium. […] its sharp stability version allows one to describe in quantitative terms every low energy state. This is a result of clear physical importance, whose proof required several original and innovative mathematical ideas. (**LC**, p. 1-2 (4344))

2011* **AF**, William **Bench**, Carlo **De Franchis**, Laurent **Deproit**, Sébastien **Gilles**, Byunghyun **Oh**, Adrien **Tenne**, Kevin **Webster**. *Autour des inégalités isopérimétriques*. Éditions de l'École Polytechnique, Palaiseau. 124 p.

2011 **AF** and Francesco **Maggi**. "On the shape of liquid drops and crystals in the small mass regime". *Archive for Rational Mechanics and Analysis* **201**.1, 143-207.

2013 **AF** and Luis **Caffarelli**. "Regularity of solutions to the parabolic fractional obstacle problem". *Journal für die reine und angewandte Mathematik* (Crelles Journal) **680**: 191-233.

2013 **AF** and Guido **De Philippis**. "$W^{2,1}$ regularity for solutions of the Monge–Ampère equation". *Inventiones mathematicae* **192**.1: 55-69.

> … a fundamental, groundbreaking result on the second order Sobolev regularity of optimal transport maps and their link with the Monge-Ampère equation. (**LC**, p. 1 (43))

2014 **AF** and Guido **De Philippis**. "The Monge–Ampère equation and its link to optimal transportation". *Bulletin of the American Mathematical Society* **51**.4: 527-580.

2014 **AF**, Serena **Dipierro** and Enrico **Valdinoci**. "Strongly nonlocal dislocation dynamics in crystals". *Communications in Partial Differential Equations* **39**.12: 2351-2387.

2015 **AF** and Guido **de Philippis**: "Partial regularity for optimal transport maps". *Publications Mathématiques IHES* **121**.1: 81-112.

2015 **AF**, Gonzalo **Contreras** and Ludovic **Rifford**. "Generic hyperbolicity of Aubry sets on surfaces". *Inventiones Mathematicae* **200**.1, 201-261.

2016 **AF** and Alice **Guionnet**. "Universality in several-matrix models via approximate transport maps". *Acta Mathematica*. **217**.1, 81-176.

> Large random matrices arise as a natural model in diverse fields such as quantum mechanics, quantum chaos, telecommunications, finance, and statistics. The central mathematical question in this area is how much their asymptotic properties depend on the fine details of the model. […] About the universality of the fluctuations of the eigenvalues in several-matrix models, little was known except in very particular situations. […] developed a new approach to these questions by introducing particular approximate transport maps. […] This is a real breakthrough, which in addition, gives a firm mathematical grounding to the widely held belief coming from physics that a universality of local fluctuations holds, at least until some phase transition occurs. (**LC**, p.2-3 (44-45).

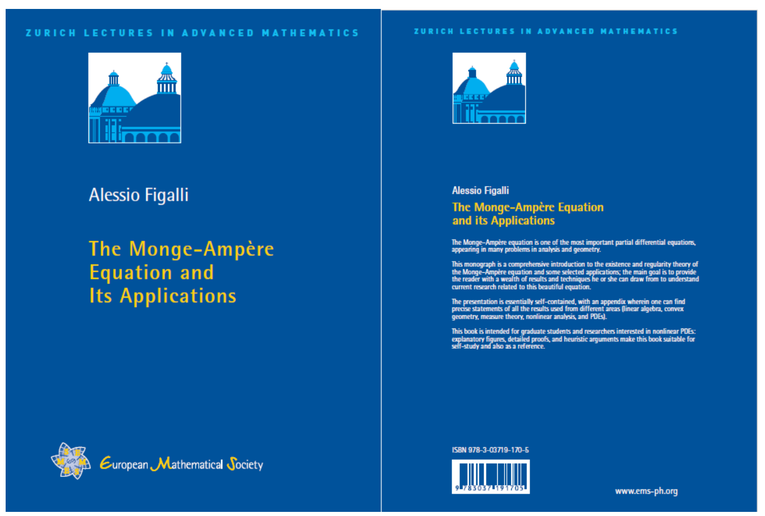
2016 **AF**, Diogo **Gomes** and Diego **Marcon**. "Weak KAM theory for a weakly coupled system of Hamilton–Jacobi equations". *Calculus of Variations and Partial Differential Equations* **55**.4: 55-79.

2017 **AF** and David **Jerison**. "Quantitative stability for the Brunn–Minkowski inequality". *Advances in Mathematics* **314**: 1-47.

> […] succeeded in the quite challenging task of combining tools from additive combinatorics, affine geometry, harmonic analysis and optimal transport to obtain the first quantitative stability result for the BM inequality in arbitrary dimension and on generic sets. […] quite impressive, both for the technical complexity, the wealth of original ideas involved, and the mathematical beauty of the question under consideration (**LC**, p. 2 (44)).

2017 **AF**, Brian **Krummel** and Xavier **Ros-Oton**. "On the regularity of the free boundary in the *p*-Laplacian obstacle problem". *Journal of Differential Equations* **263**.3: 1931-1945.

2017* **AF**. *The Monge–Ampère equation and its applications*. EMS, 2017. x+200 pp.



2018 **AF**, Begoña **Barrios** and Xavier **Ros-Oton**. "Free boundary regularity in the parabolic fractional obstacle problem". *Communications in Pure and Applied Mathematics* **71**.10, 2129-2159.

2018 **AF**, Begoña **Barrios** and Xavier **Ros-Oton**. "Global regularity for the free boundary in the obstacle problem for the fractional Laplacian". *American Journal of Mathematics* **140**.2, 415-447.

2018 **AF**. "Global existence for the semigeostrophic equations via Sobolev estimates for Monge-Ampere". *Partial Differential Equations and Geometric Measure Theory*. Springer, 2018. 1-42. These notes record and extend the lectures for the CIME Summer Course held by the author in Cetraro during the week of June 2–7, 2014. […] show how some recent developments in the theory of the Monge-Ampère equation play a crucial role in proving existence of global weak solutions to the semigeostrophic equations (Abstract).

2018* **AF**, Ireneo **Peral**, and Enrico **Valdinoci**. *Partial Differential Equations and Geometric Measure Theory*. Springer International Publishing, 2018.

2018 **AF**. "Regularity of interfaces in phase transitions via obstacle problems". *Proceedings of the International Congress of Mathematicians*. World Scientific, 2019. Volume 1, 225-247.

2019 **AF** and Joaquim **Serra**. "On the fine structure of the free boundary for the classical obstacle problem". *Inventiones mathematicae* 215.1: 311 -366.

> […] great acclaim greeted the work of Figalli, together with his co-author Joaquim Serra, who in 2017 gave a complete and definitive description of the free boundary. […] The new methods introduced in this work are having a wide impact (Allyn **Jackson**, *The Work of Alessio Figalli*, IMU, 2018).

2019 **AF** and Joaquim **Serra**. "On stable solutions for boundary reactions: a De Giorgi-type result in dimension 4 + 1". *Inventiones mathematicae* (Online July 2019, 26 p).

General Appraisals

"Figalli 's work is of the highest quality in terms of originality, innovation and impact both on mathematics per se as well as on its applications. He is clearly a driving force in the global mathematics community today. His approach to research is lively, dynamic and effective, and without a doubt will lead him to achieve many more stunning discoveries in years to come" (**LC**, 4 (46)).

"His work is also quantitatively striking. In twelve years, he has published more than 140 research papers, the vast majority of them in top journals, already with an impressive number of citations and collaborators. These numbers surely rank him among the most prolific scientists publishing in pure mathematics over the same period. A charismatic lecturer, he has given more than 200 invited lectures and research seminars, and he has visited many top mathematical institutions and departments" (Luigi **Ambrosio**).

"Alessio is an outstanding mathematician who has had a tremendous impact in several fields of partial differential equations and analysis. His work will have an everlasting influence in mathematics. […] Among his vast production in the area [Geometric and functional inequalities] another result I like a lot is in a joint paper with Jerison, which contains the first quantitative version of the Brunn-Minkowski inequality in arbitrary dimension and for generic sets. They use a striking combination of techniques from different areas to achieve their theorems" (Camillo **de Lellis**).

"Figalli's area of research is surrounded by formidable technical machinery that often proves difficult for outsiders to penetrate. A master of this machinery, Figalli has made the area more widely accessible through his outstanding expositions, which cut through technicalities and reveal the conceptual structure. His influence has also been amplified by his friendliness and generosity in sharing ideas with students and younger colleagues. These personal qualities combine with mathematical brilliance to make Alessio Figalli an ideal leader whose impact in mathematics has only just begun" (Allyn **Jackson**, *The Work of Alessio Figalli*, IMU, 2018).

Graduate Teaching

The following table is a synopsis of the graduate courses taught by **AF** since the term 2009-2010. Labels such as 12S and 12F denote the Spring and Fall of 2012, respectively. **UPS** stands for Université de Paris Sud.

09S	**OT**	**UPS**	12F	**TDE**	**MIT**	16S	**PDE**	**UTA**
11S	**PDE**	**UTA**	13S	**OT**	**UTA**	16S	**HJE**	**UTA**
11S	**GMT**	**UTA**	14S	**NLA**	**UTA**	16F	**FBP**	**ETH**
12S	**PDE**	**UTA**	14F	**MAE**	**ETH**	17S	**CV**	**ETH**
12S	**TCV**	**UTA**	15S	**PDE**	**UTA**	19F	**OT**	**ETH**

OT *Optimal Transport.* **PDE** *Partial Differential Equations II.* **GMT** *Geometric Measure Theory.* **TCV** (*Topics in the*) *Calculus of Variations.* **TDE** *Topics in Differential Equations.* **NLA** *Topics in Nonlinear Analysis.* **MAE** *Monge-Ampère Equations.* **HJE** *Hamilton-Jacobi Equations.* **FBP** *Free Boundary Problems.*

AF is an archetype for the research-based teaching, be it in graduate studies, as described above, or in the two-dozen invited graduate or research level courses delivered in conferences, workshops and Summer schools all over the world. A short sample:

April 2010

Optimal transport and applications, 2009-2010 Salomon **Bochner** Lectures in Mathematics (Houston, TX, USA).

May 2014

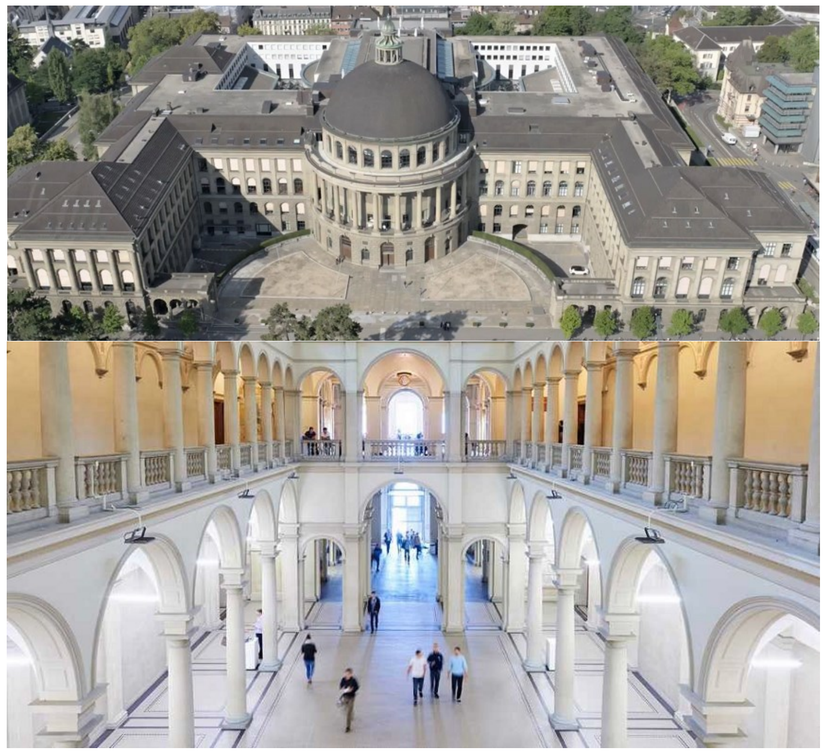
Stability results for geometric and functional inequalities, "**Nirenberg** Lectures in Geometric Analysis" at the **CRM** (Montreal, Canada).

March2015

The Monge-Ampère equation, "Thomas **Wolff** Memorial Lectures in Mathematics" at Caltech (Pasadena, CA, USA).

Selected Editorial Work

2011-------	**AIMS** Book Series, Applied Mathematics
2011-2018	Discrete and Continuous Dynamical Systems - Series A (Managing Editor 2012-2016)
2013-------	Analysis in Theory and Applications Analysis and PDE Journal de l'École polytechnique
2014-------	Duke Mathematical Journal
2015-------	Probability Theory and Related Fields
2016-2018	Communications in Contemporary Mathematics
2016-------	Archive for Rational Mechanics and Analysis

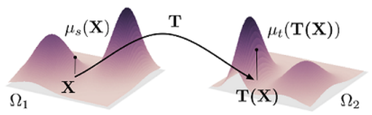


ETH, Zürich, houses the Department of Mathematics and FIM

Alessio Figalli 4 — The **Ros-Oton** survey — Work 3/3

What is optimal transport theory?

- It's the study of transportation and allocation of resources.
  The problem was formalized by the Gaspard Monge in 1781, and arises in a variety of settings (both in Mathematics and Applied Sciences).
- Given two distribution of masses, μ and ν, find the optimal way to transport μ onto ν by minimizing a cost $c(x,y)$.
- Transport map: $T : \Omega_1 \longrightarrow \Omega_2$ satisfying $T_\sharp \mu = \nu$.
- Find the map T minimizing total cost
$$\int_{\Omega_1} c(x, T(x)) d\mu(x)$$
- Most typical cost: $c(x,y) = |x-y|^2$



Optimal transport: Basic known results

- Existence: Even in $\mathbb{R}^n$ with very simple costs c, this is far from trivial!
- Theorem (Brenier '87): When $c(x,y) = |x-y|^2$, then:
  (i) there exists a unique solution T to the optimal transport problem.
  (ii) $T = \nabla u$ for some *convex* function u.
- If in addition T is a diffeomorfism, $\mu = f dx$ and $\nu = g dx$, then u solves the PDE
$$\det(D^2 u) = \frac{f}{g(\nabla u)}.$$
  This is called the **Monge-Ampere equation**. It's a fully nonlinear elliptic PDE.

KEY QUESTIONS

- Regularity: Is the optimal transport map T *smooth*?
- What happens for other costs? Or for the problem in a manifold?

The Monge-Ampere equation

$$\boxed{\det(D^2 u) = f > 0 \quad \text{in} \quad \Omega \subset \mathbb{R}^n}$$

- When u is convex, this is an **elliptic PDE**.
- It's a very nonlinear version of the simplest elliptic PDE: $\Delta u = f$ in $\Omega \subset \mathbb{R}^n$.
- The Monge-Ampere equation arises not only in optimal transport theory, but also in other PDE models, as well as in several problems in Analysis and Geometry.
- Minkowski problem (1897): Prescribing the Gaussian curvature of a manifold.
- One of the most important questions in PDE theory is that of **regularity**.
- The regularity theory for the Monge-Ampere equation is one of the most delicate topics in elliptic PDE.
- Classical theory: Minkowski, Alexandrov, Caffarelli, ...

Monge-Ampere equation: classical theory

$$\boxed{\det(D^2 u) = f \quad \text{in} \quad \Omega \subset \mathbb{R}^n}$$

- Existence and uniqueness of weak solutions: Alexandrov (1958).
- The regularity of such solutions is a *very* delicate problem.
- Best known results: [Caffarelli '90,'91,'92] if Ω is strictly convex, then
  - If $f \in C^{0,\alpha}$, then $D^2 u \in C^{0,\alpha}$
  - If $f \in C^0$, then $D^2 u \in L^p$ for all $p < \infty$
  - If f is merely bounded, then $\nabla u \in C^{0,\gamma}$ for some $\gamma > 0$
- The first two parts say "u is two derivaties more regular than f".
- However, the last part says nothing about $D^2 u$!

Monge-Ampere equation: Alessio Figalli's work

$$\boxed{\det(D^2 u) = f > 0 \quad \text{in} \quad \Omega \subset \mathbb{R}^n} \qquad (*)$$

- Motivated by the *semigeostrophic equations* (meteorology), as well as by optimal transport and geometry, an important open question was:
  If f is merely bounded, is it true that $D^2 u$ is in L^1?
- Counterexamples show that this would be sharp [Wang '95].

Theorem (DePhilippis-Figalli, Invent. Math.'13)

Let u be any weak solution to the Monge-Ampere eq. If Ω is strictly convex, then
$$f \text{ bounded} \implies D^2 u \in L^1$$

- This essentially completes the regularity theory for the Monge-Ampere eq (*).
  ...at least in convex domains Ω.

Optimal transport maps: classical theory

- When the cost is $c(x,y) = |x-y|^2$ in $\mathbb{R}^n$, and Ω_2 is convex, then the regularity theory for the Monge-Ampere equation can be applied, with $T = \nabla u$.
- What if Ω_2 is **not convex**?
  Counterexamples show that T may be discontinuous!
- What if we have a **different cost** $c(x,y)$, or we are in a manifold?
  Under a mysterious assumption ("*MTW condition*"), and a "c-convexity" assumption on the domain, regularity of optimal transport maps holds.
- The MTW condition is actually necessary! Also, it is quite restrictive (usually false).

Optimal transport maps: Alessio Figalli's work

- One can easily build examples where the MTW condition fails at *every* point.
- **Q**: When Ω_2 is **not convex**, or we have a **general cost**, or we are in a **manifold**... can one at least prove **partial regularity**?

Theorem (DePhilippis-Figalli, Publ. Mat. IHES'15)

Let $T : \Omega_1 \to \Omega_2$ be **any** *optimal transport map in $\mathbb{R}^n$ with* **any** *cost $c(x,y)$.*
Then, T is a smooth diffeomorfism, outside a closed set of zero measure.
Furthermore, the same happens in any Riemannian manifold M with cost $c = d^2$.

- This essentially settles the question of regularity for optimal transport maps!

Isoperimetric inequalities: classical theory

- Isoperimetric problem: Find the figure of largest volume, given the surface area.
$$\text{In } \mathbb{R}^n: \qquad \frac{|\partial\Omega|}{|\Omega|^{\frac{n-1}{n}}} \geq \frac{|\partial B_1|}{|B_1|^{\frac{n-1}{n}}}$$
  Similar problems arise in Physics, Geometry, Analysis, Probability, etc.
- Natural important problem: Quantitative version? (i.e., stability?)
- Theorem (Fusco-Maggi-Pratelli, Ann. Math.'05)
$$\frac{|\partial\Omega|}{|\Omega|^{\frac{n-1}{n}}} \geq \frac{|\partial B_1|}{|B_1|^{\frac{n-1}{n}}} + c_n [A(\Omega)]^2$$
  where $A(\Omega)$ is the "*asymmetry*" of the set.
- Proof: Quantitative symmetrization!
- Very powerful technique, but clearly specific to the *isotropic* case.

Isoperimetric inequalities: Alessio Figalli's work

- When studying crystals, phase transitions, etc., we need **anisotropic perimeters**. Corresponding isoperimetric inequality: Wulff inequality
- It is equivalent to the Brunn-Minkowski
$$|E+F|^{1/n} \geq |E|^{1/n} + |F|^{1/n}$$

Theorem (Figalli-Maggi-Pratelli, Invent. Math.'10)

For any convex sets $E, F \subset \mathbb{R}^n$,
$$|E+F|^{1/n} \geq \left(|E|^{1/n} + |F|^{1/n}\right)\left\{1 + c_n [A(E,F)]^2\right\}$$
where $A(E,F)$ is the relative asymmetry between two sets.

- Proof: via **optimal transport**!
- Very powerful technique, works in various different settings!

Random matrix theory: classical theory

- Problem: Understand the spectrum of large random matrices
- Wigner matrix ($N \times N$): i.i.d. entries with mean zero and covariance $1/N$.
- Theorem (Wigner'55) $\frac{1}{N}\sum \delta_{\lambda_i}$ converges in probability to $\frac{1}{2\pi}(4-x^2)_+^{1/2} dx$
  (That is, even if λ_i are random, as $N \to \infty$ they behave as deterministic!)
- **Q**: Can one understand the **fluctuations** of eigenvalues?
- The first results were for Gaussian models...
  ...but it was envisioned by Wigner that fluctuations should be **universal**, i.e., independent of the distribution of the entries.
- This deep fact was proved only recently in a series of remarkable papers (Erdös, Tao, Schlein, Yau, Vu, ...)

Random matrix theory: Alessio Figalli's work

- A related question is to understand **universality** of fluctuations for β-models, where the law of eigenvalues is
$$c_{V,N} \prod_{i<j} |\lambda_i - \lambda_j|^\beta e^{-N\sum_i V(\lambda_i)} d\lambda_1 \cdots d\lambda_N$$
  This is important in the study of several-matrix models (appearing in physics, random graphs, etc).
- $\beta = 1, 2, 4$ and $V(\lambda) = \lambda^2 \iff$ joint law of eigenvalues of Gaussian matrices.

Theorem (Figalli-Guionnet, Acta. Math.'16)

In β-models with "nice" interaction potentials V, the local fluctuations of the spectrum are **universal**, *that is, independent of the potential V.*

- Proof: **Optimal transport**!

Summary

- Alessio has solved some of the most important questions in optimal transport theory.
- Moreover, he has applied ideas from optimal transport in the context of isoperimetric inequalities and random matrices, obtaining striking new results.
- On top of this, he has actually worked in a variety of other settings!
  - Dynamical systems (weak KAM theory, generic hyperbolicity of Aubry sets)
  - Free boundary problems (Obstacle problems, Mumford-Shah)
  - Evolution equations (Vlasov-Poisson equation, Keller-Segel, nonlinear diffusions)
  - ...

Alessio Figalli 5

Academic Track. Mentoring

Mathematical education

2002-2006
Student of Mathematics, Scuola Normale Superiore di Pisa (**SNSP**)

- 2004
  Bachelor's degree. Degree thesis, upervised by Giovanni **Alberti**:
  Il problema de Berstein e una congettura di De Giorgi (In Italian).
- 2006
  Master's degree. Master's thesis, supervised by Luigi **Ambrosio**:
  Transporte ottimale su varietà non compatte (in English).

2006-2007

PhD student at the **SNSP** and the **École Normale Supérieure de Lyon**

- 2007 PhD thesis, Supervised by Luigi **Ambrosio** and Cédric **Villani**:
  Optimal Transportation and Action-Minimizing Measures.

2009

French habilitation (Habilitation à Diriger des Recherches):
Optimal transport, Euler equations, Mather and DiPerna-Lions theories.

Positions

2007-2008
Chargé de recherche CNRS, Universiité de Nice (Nice, France).

2008-2009
Hadamard Professor, École Polytechnique (Palaiseau, France).

2009-2010
Associate Professor and Harrington Faculty Fellow,
The University of Texas at Austin (**UTA**).

2010-2011 Associate Professor, **UTA.**

2011-2013 Full Professor, **UTA**.

2013-2016 Full Professor and R. L. Moore Chair, **UTA**.

2016----: Full Professor and Chair, ETH Zürich (Zurich, Switzerland).

2019----: FIM Director, ETH Zürich (Zurich, Switzerland).

PhD students

2009-2012
Eric **Baer**: *Symmetry properties of crystals and bounds from below on the temperature in compressible fluid dynamics*. **UTA**, 2012.

2009-2013
Emanuel **Indrei**:* *Optimal transport, free boundary regularity, and stability results for geometric and functional inequalities*. **UTA**, 2013.

2010-2013
Diego **Marcon Farias**: *Weak KAM and Aubry-Mather theories in an optimal switching setting*. Instituto Superior Técnico (Lisbon) and **UTA**, 2013. Coadvised with Diogo **Aguiar Gomes**.

2011-2012
Levon **Nurbekyan**: *Weak KAM theory on the d-infinite dimensional torus*. Instituto Superior Técnico (Lisbon), 2012. Coadvised with Diogo **Aguiar Gomes**.

2011-2016
Rohit **Jain**: MSC 35: *Partial differential equations*. **UTA**, 2016. Coadvised with Luis **Caffarelli**.*

2012-2015
Maria **Colombo**:* MSC 49: *Calculus of variations and optimal control*. **UTA** and **SNSP**, 2015. Coadvised with Luigi **Ambrosio**.*

2013-2017
Robin **Neumayer**:* MSC 49: *Calculus of variations and optimal control*. **UTA**, 2017. Coadviced with Francesco **Maggi**.*

2013-2017
Javier **Morales**: MSC 49: *Calculus of variations and optimal control*. **UTA**, 2017.

2014-2018
Yash **Jhaveri**:* *Regularity Results for Generated Jacobian Equations*. **UTA** and **ETH** Zürich, 2018.

2015----
Xavier **Fernández-Real**,* **UTA** and **ETH** Zürich.

2018----
Federico **Glaudo**, **ETH** Zürich.

2019---- Federico **Franceschini**, **ETH** Zürich.

Postdocs

2010-2011
Clayton **Bjorland**. **UTA**. Coadvised with Luis **Caffarelli**.*

2012
Filippo **Cagnetti**. **UTA**. Coadvised with Luis **Caffarelli**.*

2013
Shibing **Chen**. **MSRI**.

2014-2016
Xavier **Ros-Oton**.* **UTA.**

2014-2016
Begoña **Barrios**.* **UTA**.

2015-2016
Brian **Krummel**. **UTA**. Coadvised with Francesco **Maggi**.*

2016-2018
Connor **Mooney**.* **UTA** and **ETH** Zurich.

2016-2018
Joaquim **Serra**.* **ETH** Zurich.

2018----
Hardy **Chan**, **ETH** Zurich.

2019----
Yi **Zhang**, **ETH** Zurich.

* Included in Main collaborators (poster #1)

Do you have any specific plans for the FIM's future?

I want to make sure that everyone inside the mathematics community at ETH and the University of Zurich feels that the FIM represents every aspect of mathematics.

Interview with the new FIM Director Alessio Figalli
FIM, D-MATH News, 1.9.2019

Alessio Figalli 6

Honors

Prizes

2006-2007 Benedetto Sciarra Prize of the Scuola Normale Superiore of Pisa

2008 Giuseppe Borgia Prize of the Accademia Nazionale dei Lincei

Carlo Miranda Prize of the Accademia di Scienze Fisiche e Matematiche of Naples

2009-2010 Harrington Faculty Fellowship

2010 Gioacchino Iapichino Prize of the Accademia Nazionale dei Lincei.

Anile Prize of the Associazione Angelo Marcello Anile and the Consorzio Catania Ricerche.

2011-2012 Peccot-Vimont Prize and Cours Peccot of the Collège de France.

2012 EMS Prize (**6ecm**, Kraków, Poland)

For his outstanding contributions to the regularity theory of optimal transport maps, to quantitative geometric and functional inequalities and to partial solutions of the Mather and Mané conjectures in the theory of dynamical systems.

2015 Stampacchia Gold Medal of the Italian Mathematical Union

2017 Feltrinelli Prize of Accademia Nazionale dei Lincei.

2018 Fields Medal
(ICM-2018, Rio de Janeiro, Brazil)

For contributions to the theory of optimal transport, and its applications in partial differential equations, metric geometry, and probability.

Laudatio: "Figalli's work is of the highest quality in terms of originality, innovation and impact both on mathematics per se as well as on its applications. [He is] bound to be one of the most influential mathematicians of his generation." (Luis **Caffarelli**).

2019 Gili Agostinelli Prize of the Accademia delle Scienze di Torino.

Awards

2017 O'Donnell Award in Science of The Academy for Medicine, Engineering, & Science of Texas (TAMEST)

2018 Doctorate *Honoris Causa* of the Université Côte d'Azur

Knight of the Order of Merit of the Italian Republic

Academies

2017 Fellow and Honorary Member of the European Academy of Sciences

2018 Foreign Member of the Royal Spanish Academy of Sciences

2019 Foreign Member of the Academy of Sciences of Bologna
Member of Academia Europaea

Grants

2010-2013
Analytical and geometrical problems in calculus of variations and partial differential equations (NSF Grant DMS-0969962)

2013-2018
Regularity and stability results in variational problems
(NSF Grant DMS-1262411)

2014-2017
Vectorial and geometric problems in the calculus of variations
(NSF Grant DMS-1361122. FRG, Collaborative Research)

2017-2022
Regularity and Stability in Partial Differential Equations
(ERC Grant)

Selected invited lectures

2009/06 Bourbaki seminar (IHP, Paris, France)

2009-2010 Salomon Bochner Lectures in Mathematics (Houston, TX, USA)

2012/07 European Congress of Mathematics (6**ecm**, Kraków, Poland)

2013/06 **INdAM** Day (Palermo, Italy)

2013/12 **SIAM** Conference on Analysis of Partial Differential Equations* (Lake Buena Vista, Florida, USA)

2014/04 **AMS** Spring Central Regional Meeting* (Texas Tech University, Lubbock, TX, USA)

2014/05 1st Nirenberg Lectures in Geometric Analysis (CRM, Montreal, Canada)

2014/07 XV International Conference on Hyperbolic Problems* (Rio de Janeiro, Brazil)

2014/09 International Congress of Mathematicians (Seoul, Korea)

2014/Fall *Nachdiplom-Vorlesungen* (ETH Zürich, Zurich, Switzerland)

2015/03 Thomas Wolff Memorial Lectures in Mathematics (Caltech, Pasadena, CA, USA)

2015/09 XX Congress of the Italian Mathematical Union* (Siena, Italy)

2016/11 Leonardo Da Vinci Lectures (Milan, Italy)

2017/06 23rd Rolf Nevanlinna Colloquium (ETH Zürich, Zurich, Switzerland)

2018/02 Harold J. Gay Lecture (Worcester, MA, USA)

2018/05 Rouse Ball Lecture (University of Cambridge, Cambridge, UK)

2018/06 Bourbaki seminar (IHP, Paris, France)

2018/08 International Congress of Mathematicians* (Rio de Janeiro, Brazil):
Regularity of interfaces in phase transitions via obstacle problems

2018/10 John von Neumann Lecture (Münster University, Münster, Germany)

2019/02 Joseph D'Atri Memorial Lectures (Rutgers University, New Brunswick, NJ, USA)

PIMS-UBC Math Distinguished Colloquium (University of British Columbia, Vancouver, Canada)

2019/07 **Equadiff 2019*** (Leiden, Netherlands)

2019/09 Dynamics, Equations and Applications (**DEA 2019**)* (Krakow, Poland)

* Plenary speaker

Alessio Figalli 7

Biographics

Childhood

AF was born in Rome in 1984. His father, Gennaro **Figalli**, was a professor of engineering. His mother, Giuseppina **Carola**, was a high school professor of Latin and Greek.

AF at five in the Padre **Lais** school.
Courtesy of Alessio Figalli.

"As a kid Figalli liked to play soccer, watch cartoons, and hang out with his friends — and, he recalls, he always made the rational decision to get his homework done first, so that he could fully enjoy himself. 'For me it was always a balance between how good a grade I could get and how much time I had to spend to get such a grade,' he said. 'I was always an optimizer, I wanted the best for the least effort.' " [Q]

"Figalli liked math from an early age. He regarded it as an easy subject, something he was good at without having to work hard, and it took him a while to pursue the subject with any zeal." [Q]

Youth

"I liked mathematics as a kid —because it was easy for me. I was thinking about becoming an engineer. Then I had to decide at the age of 13 which high school I would like to attend. There are many different types in Italy, but the main ones that prepare you for university are the science and humanist grammar schools. You learn Greek and Latin, philosophy and so on. I chose this direction. In Italy there is always the idea that the humanistic Gymnasium offers the broadest education and that with this training you can study afterwards whatever you want." [Z]

In his third year in high school, Antonio **Corbo**, a mathematician working in the same university as his father, suggested **AF** to participate in the Mathematical Olympiad. This led him to the realization that there were mathematics problems whose solution required inventiveness, and his aptness and joy in solving them was a moving revelation.

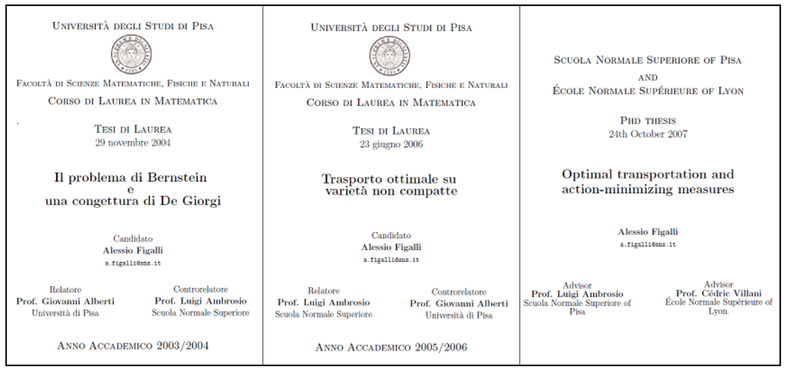
"At the Mathematics Olympiad, I met other teens who loved math. All these kids really wanted to study at the Scuola Normale Superiore in Pisa, which offers a high level of education. Anyone who manages to win one of the coveted scholarships does not have to pay anything. Living, eating, studying —this is all free. I wanted that too." [Z]

"I crammed on mathematics and physics on my own and succeeded in the entrance exam. The first year at the Scuola Normale was tough. I did not even know how to calculate a derivative, while my colleagues were much further ahead than myself; they had already gone over partial derivatives in the first lesson. But in the second year my distance to my fellow students became much smaller." [Z]

"He learned quickly and caught up to his peers within a year. At the start of his second year he began to work on a highly technical paper that Luigi **Ambrosio** had recently written. **Ambrosio** expected that the novice student would struggle to get anywhere with it." [Q]

But "Alessio came to me less than one week after and I realized he understood everything." This step set **AF** course on mathematical research: one year later he completed his undergraduate degree and in the next three years he earned the Master's and PhD degrees, all with full honors.

"Alessio is unbelievably quick. Quick on the essential issues, and quick on isolating an important point" (David **Jerison**, **MIT**).



UNIVERSITÀ DEGLI STUDI DI PISA
FACOLTÀ DI SCIENZE MATEMATICHE, FISICHE E NATURALI
CORSO DI LAUREA IN MATEMATICA
TESI DI LAUREA
29 novembre 2004
Il problema di Bernstein e una congettura di De Giorgi
Candidato: Alessio Figalli
Relatore: Prof. Giovanni Alberti, Università di Pisa
Controrelatore: Prof. Luigi Ambrosio, Scuola Normale Superiore
ANNO ACCADEMICO 2003/2004

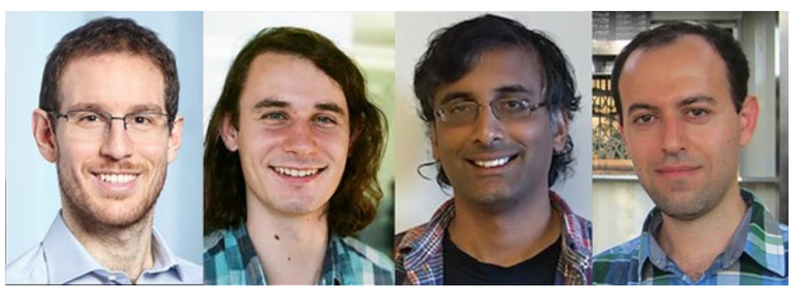
UNIVERSITÀ DEGLI STUDI DI PISA
FACOLTÀ DI SCIENZE MATEMATICHE, FISICHE E NATURALI
CORSO DI LAUREA IN MATEMATICA
TESI DI LAUREA
23 giugno 2006
Trasporto ottimale su varietà non compatte
Candidato: Alessio Figalli
Relatore: Prof. Luigi Ambrosio, Scuola Normale Superiore
Controrelatore: Prof. Giovanni Alberti, Università di Pisa
ANNO ACCADEMICO 2005/2006

SCUOLA NORMALE SUPERIORE OF PISA
AND
ÉCOLE NORMALE SUPÉRIEURE OF LYON
PHD THESIS
24th October 2007
Optimal transportation and action-minimizing measures
Alessio Figalli
Advisor: Prof. Luigi Ambrosio, Scuola Normale Superiore of Pisa
Advisor: Prof. Cédric Villani, École Normale Supérieure of Lyon

Maturity

As witnessed in the preceding posters, **AF**'s work in the decade after his landmark PhD is astounding on all counts. His leading role in the global mathematical landscape has increased steadily along the years, with a conspicuous boost after receiving the Fields Medal.

One of the effects of this prominence is the solicitation by the media for all sorts of interactions. We end by quoting a few thoughts culled from the interview [Z].



Fields Medallists 2918: **AF** (**ETH** Zürich), Peter **Scholze** (Universität Bonn), Akshay **Venkatesh** (**IAS**), Caucher **Birkar** (University of Cambridge).

What is the most important tool for your work? The computer or rather paper and pencil?

Paper and pencil! My research is done on paper. Of course, computers are practical because they allow us to immediately read the publications of other researchers and facilitate communication. But when I really think about a problem, I turn off the computer. Then I can concentrate better.

When and where do you get the best ideas?

That's a long process. You spend hours, days, months with the problem you want to solve. Sometimes you are lucky, and things move quickly forward, but most of the time you get stuck somewhere. Some of my key ideas came to mind when I came back to my desk late at night after having a beer with friends. But sometimes it just takes many hours at the desk.

What do you do to escape the pressure of not getting stuck?

I think every mathematician has his own strategies to deal with it. Mine is to work on several projects at the same time. So I can put away what's just not working, and come back to it later. The beauty of the brain is that it works in the background. For example, I found the solution for one of my most important works in 2012 [regularity for solutions of the Monge–Ampère equation]. But I started thinking about the problem in 2005. Had I been working on just that one problem in the seven years in between, I would probably be crazy by now —and I would not have a job because I could not produce any scientific results. Instead, I have dedicated myself to many other problems in between and have always come back to this one.

[Q] Kevin **Hartnet**: "A Traveller Who Finds Stability in the Natural World". *Quanta Magazine*, August 1st, 2018.

[Z] **Neue Zürcher Zeitung** interview by Helga **Rietz**, 1.8.2018.

Acknowledgements

We are grateful to Xavier **Ros-Oton**, Joaquim **Serra**, and Xavier **Cabré** for the positive feedback we got from them during the preparation of these posters.

JAUME FRANCH BULLICH

SEBASTIÀ XAMBÓ-DESCAMPS