GUIA DOCENT

25/26

Curs Sophie Germain

Master's degree in Advanced Mathematics and Mathematical Engineering (MAMME)

Sophie Germain
01/04/1776-27/06/1831
Curs 2025-2026

Facultat de Matemàtiques i Estadística

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Facultat de Matemàtiques i Estadística

TABLE OF CONTENTS

Master's degree in Advanced Mathematics and Mathematical Engineering (MAMME)

SCHOOL OF MATHEMATICS AND STATISTICS (FME)

The **master's degree in Advanced Mathematics and Mathematical Engineering (MAMME)** (**master's degree website**) is a master's programme in mathematics offered at the School of Mathematics and Statistics (FME).

The courses offered in MAMME allow our students to design their curriculum with two different orientations: a pure mathematics curriculum (oriented to research in fundamental mathematics) or an applied mathematics curriculum (preparing them for applied mathematics research and for interdisciplinary teamwork, in collaboration with engineers, physicists, biologists, economists, etc.).

The curriculum comprises a total of 60 ECTS credits, divided into 45 credits for courses and 15 for the master's thesis. It is intended to be completed in one academic year. In addition, MAMME offers the possibility of enrolling for up to 22.5 ECTS credits in other master's degrees in mathematics or statistics, or in other UPC master's programmes, opening the path for an interdisciplinary curriculum based on selected courses in master's degrees in engineering and applied sciences. See the MAMME focus proposals at http://mamme.masters.upc.edu/en.

GENERAL DETAILS

Duration and start date
1 academic year, 60 ECTS credits. Starting September and February

Timetable and delivery
Afternoons. Face-to-face

Fees and grants
Approximate fees for the master's degree, **excluding other costs** (does not include non-teaching academic fees and issuing of the degree certificate):
€1,162 (€2,700 for non-EU residents).
More information about fees and payment options
More information about grants and loans

Language of instruction
English

Information on language use in the classroom and students' language rights.

Location

School of Mathematics and Statistics (Barcelona)

Official degree
Recorded in the Ministry of Education's degree register

ADMISSION

General requirements

Academic requirements for admission to master's degrees

Specific requirements

This master's degree is aimed at students with good abstract reasoning, an interest in problem solving, strong work habits and a liking for mathematics.

A scientific background is required, with basic mathematical foundations. For this reason, a bachelor's degree in mathematics, statistics, physics, engineering, economics or science is recommended. This list is non-exhaustive, and all applications will be reviewed on an individual basis.

Admission criteria

The following elements will be taken into consideration during the evaluation process: the academic record, the CV, a statement of purpose and, if deemed necessary, a personal interview and recommendation letters.

Places

30 (September) and 3 (February)

Pre-enrolment

Pre-enrolment for this master's degree is currently **closed**. Use the "Request information" form to ask for information on **upcoming pre-enrolment periods**.

How to pre-enrol

Enrolment

How to enrol

Legalisation of foreign documents

All documents issued in non-EU countries must be legalised and bear the corresponding apostille.

CURRICULUM

Subjects	ECTS credits	Type
FIRST SEMESTER		
Commutative Algebra	7.5	Optional
Discrete and Algorithmic Geometry	7.5	Optional
Graph Theory	7.5	Optional
Mathematical Modelling with Partial Differential Equations	7.5	Optional
Mathematical Models in Biology	7.5	Optional
Non-Commutative Algebra	7.5	Optional
Number Theory	7.5	Optional
Numerical Methods for Dynamical Systems	7.5	Optional
Numerical Methods for Partial Differential Equations	7.5	Optional
Quantitative and Qualitative Methods in Dynamical Systems	7.5	Optional
SECOND SEMESTER		
Advanced Course in Partial Differential Equations	7.5	Optional
Algebraic Geometry	7.5	Optional

Subjects	ECTS credits	Type
Codes and Cryptography	7.5	Optional
Combinatorics	7.5	Optional
Computational Mechanics	7.5	Optional
Differentiable Manifolds	7.5	Optional
Hamiltonian Systems	7.5	Optional
Machine Learning	7.5	Optional
Seminar on Algebra, Geometry and Discrete Mathematics	3	Optional
Seminar on Analysis, Differential Equations and Modelling	3	Optional
Master's Thesis	15	Project

PROFESSIONAL OPPORTUNITIES

Professional opportunities

Some of the career prospects of master's degree graduates are academic research (by pursuing a PhD in mathematics, science or engineering, for instance), mathematical modelling in industry, finance, statistics and applied research (biomedical research centres, computer vision, etc.).

Competencies

Generic competencies

Generic competencies are the skills that graduates acquire regardless of the specific course or field of study. The generic competencies established by the UPC are capacity for innovation and entrepreneurship, sustainability and social commitment, knowledge of a foreign language (preferably English), teamwork and proper use of information resources.

Specific competencies

On completing this master's degree, students will be able to:

1. (Research). Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. (Modelling). Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. (Calculus). Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. (Critical assessment). Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.
5. (Teaching). Teach mathematics at university level.

QUALITY ACCREDITATION

Check the degree’s main quality indicators in the University Studies in Catalonia portal of the Catalan University Quality Assurance Agency. Find information on topics such as degree evaluation results, student satisfaction and graduate employment data.

Further information

ORGANISATION: ACADEMIC CALENDAR AND REGULATIONS

UPC school

School of Mathematics and Statistics (FME)

Academic coordinator

Jordi Saludes Closa

Academic calendar

General academic calendar for bachelor’s, master’s and doctoral degrees courses

Academic regulations

Academic regulations for master's degree courses at the UPC

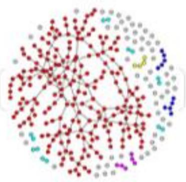
MASTER'S DEGREE WEBSITE

September 2025. UPC. Universitat Politècnica de Catalunya · BarcelonaTech

FOCUS PROPOSALS

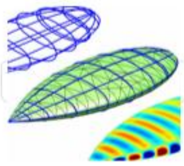
The following is a **NON-EXHAUSTIVE** list of focus proposals that may be considered for the selection of courses in MAMME, including both MAMME courses and courses in other master programs. They are **just examples** for selection of courses, and they will not be mentioned in the master's degree certificate. Students are encouraged to design their own curriculum with total freedom.

In any case, recall that a minimum of 22.5 ECTS in MAMME courses is mandatory. Registration in non-MAMME courses requires the approval of the director of MAMME and of the director of the other master.



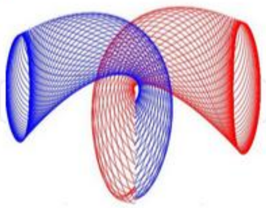
Discrete Mathematics

Partial Differential Equations and Analysis



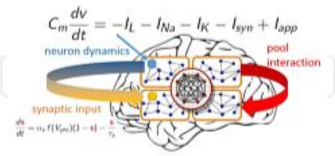
Mathematical and Computational Modelling

Optimization and Operation Research

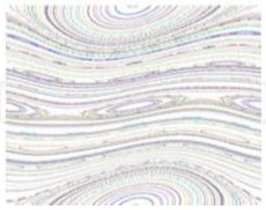


Dynamical Systems and Applications to Celestial Mechanics

Algebra, Geometry and Number Theory



Modelling and analysis in Biomedical Science



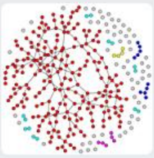
Geometry and its applications

Mathematical Physics

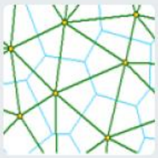
Focus on Discrete Mathematics

Discrete Mathematics has had a strong development from the second half of the XXth century fostered by the development of computers and communication technologies. The main topics include algorithms, coding theory, combinatorics, cryptography, discrete and computational geometry, finite geometry, game theory, graph theory, logic, operation research an random structures. Besides the wealth of problems which have become central in the development of contemporary mathematics, discrete mathematics holds a strong connection with applications in Bioinformatics, Computer Graphics, Information Theory, Networks or Theoretical Computer Science, as well as with other areas of mathematics like Algebra, Analysis, Number Theory or Topology.

The UPC gathers one of the strongest research groups in Spain in the area with a broad international projection providing a sound training. Most of the former students of the master have found job opportunities in industry and in academics by pursuing a PhD in UPC or in prestigious universities in Europe, the USA or Canada.



Random graphs, the basic model for random structures



Triangulations, a basic tool for computational geometry

Cryptography, one of the key applications of discrete mathematics

Students interested in focusing on Discrete Mathematics are invited to select 45 ECTS from this list:

Combinatorics	7.5 ECTS	English	MAMME
Graph Theory	7.5 ECTS	English	MAMME
Codes an Cryptography	7.5 ECTS	English	MAMME
Discrete and Algorithmic Geometry	7.5 ECTS	English	MAMME
Optimización Entera y Combinatoria	5 ECTS	Spanish	Máster Univ. en Estadística e Investigación Operativa, UPC-UB
Algorithmic Methods for Mathematical Models	6 ECTS	English	Master in Innovation and Research in Informatics, UPC
Computational Complexity	6 ECTS	English	Master in Innovation and Research in Informatics, UPC
Combinatorial Set Theory	6 ECTS	English	Master in Pure and Applied Logic, UB-UPC

Registration to non-MAMME courses requires the approval of the director of the corresponding master program.

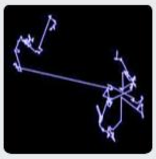
Recall that a minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

Focus on partial Differential Equations and Analysis

Partial Differential Equations (PDEs) play a central role in physics, chemistry, biology, industry, mathematical finance, and image processing. Their analysis often requires deep mathematical techniques, which makes PDEs to at the heart of both historical and recent developments in analysis, geometry, and probability. Because of this and their applications, PDEs is a very active area of mathematics, the one with the largest number of publications.

Pattern formation with reaction-diffusion systems of PDEs

Free boundaries and PDEs: the Stefan problem for melting ice



Lévy flights and PDEs in finance, biological invasions...

Students interested in focusing on PDEs and Analysis are invited to select 45 ECTS from this list and the suggestions below:

Partial Differential Equations	6 ECTS	English	Master in Advanced Mathematics, UB
Functional Analysis and PDEs	7.5 ECTS	English	MAMME
Mathematical Modeling with PDEs	7.5 ECTS	English	MAMME
Numerical Methods for PDEs	7.5 ECTS	English	MAMME
Stochastic Calculus	6 ECTS	English	Master in Advanced Mathematics, UB
Complex Analysis	6 ECTS	English	Master in Advanced Mathematics, UB

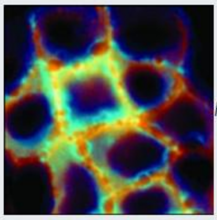
A minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

Other appropriate courses (depending on the student interests) with connections to PDEs are:
Quantitative and Qualitative Methods in Dynamical Systems (Q1 MAMME), Hamiltonian Systems (Q2 MAMME), and courses within the Barcelona universities masters offer in Mathematical Finance, Mathematical Biology, Image Processing, Functional Analysis, or Differential Geometry.

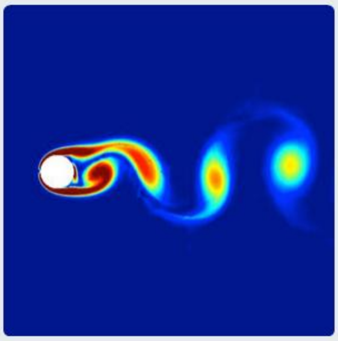
The UPC master course "Functional Analysis and PDEs" (currently "Advanced Course in Partial Differential Equations") and the UB master course "Partial Differential Equations" cover different topics of the theory of PDEs. Their contents do not overlap (or intersect), but both courses complement each other. At the same time, their contents do not overlap with a possible first undergraduate course in PDEs that the student may have taken.

Focus on Mathematical and Computational Modelling with PDEs

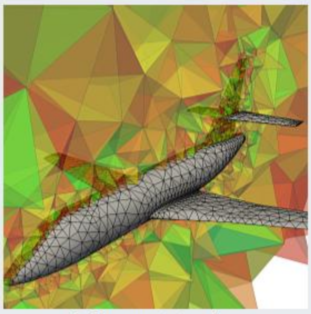
Mathematical and computational modelling with Partial Differential Equations (PDEs) is nowadays an essential tool for analysing, understanding and predicting phenomena in physics, biology, engineering, economics, social sciences and related fields. The applications cover a wide spectrum ranging from the modelling of the aerodynamical behaviour of an airfoil, to the simulation of the impact of a tsunami in a coastal area, or the study of fracture in epithelial cell sheets.



Hydraulic fracture during epithelial stretching



Flow past a cilindre. Numerical solution of Navier-Stokes eq.



Mesh for 3D Finite Element computations

Students interested in focusing on modeling with PDEs are invited to select 45 ECTS from this list:

Mathematical Modelling with PDEs	7.5 ECTS	English	MAMME
Numerical Methods for PDEs	7.5 ECTS	English	MAMME
Computational Mechanics	7.5 ECTS	English	MAMME
Advanced Course in PDEs	7.5 ECTS	English	MAMME
Advanced Fluid Mechanics**	5 ECTS	English	Master in Numerical Methods in Engineering, UPC
Finite Elements in Fluids**	5 ECTS	English	Master in Numerical Methods in Engineering, UPC
Advanced Discretization Methods**	5 ECTS	English	Master in Numerical Methods in Engineering, UPC
Numerical Modelling*	9 ECTS	English	Master en Enginyeria de Camins, Canals i Ports, UPC

Recall that a minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

(*) "Numerical Modeling" is recommended to students that do not have a solid background in numerical methods and programming. Registration to this course requires the approval of the director of the corresponding master.

Focus on Optimization and Operation Research for Efficient Decision Making

Efficient decision making based on quantitative results is essential for success in business and management. Operations Research (also known as “Management Sciences” or “Analytics”) is a discipline that deals with the application of advanced analytical methods to help make better decisions. Project planning, network optimization, facility location, routing, supply chain management, scheduling, among others, are real problems tackled by Operation Research. Industrial sectors that benefit from Operation Research range from airlines (scheduling, tariff policy), to hospitals (scheduling), to electric utilities (production, trading) and logistics (route scheduling).

Travelling salesman problem solution

Traffic simulation system

Students interested in focusing on Optimization and Operation Research should select 45 ECTS from this list:

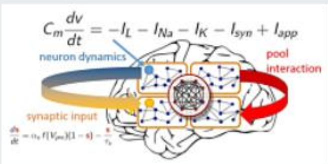
Continuous Optimization	5 ECTS	Spanish	MESIO UPC-UB
Optimization in Energy Systems and Markets	5 ECTS	Spanish	MESIO UPC-UB
Stochastic Optimization	5 ECTS	English	MESIO UPC-UB
Large Scale Optimization	5 ECTS	English	MESIO UPC-UB
Integer and Combinatorial Optimization*	5 ECTS	Spanish	MESIO UPC-UB
Statistical Data Protection*	5 ECTS	English	MESIO UPC-UB
Graph Theory*	7.5 ECTS	English	MAMME
Combinatorics*	7.5 ECTS	English	MAMME
Mathematical Models in Biology	7.5 ECTS	English	MAMME
Numerical Methods for Dynamical Systems	7.5 ECTS	English	MAMME
Numerical Methods for Partial Differential Equations	7.5 ECTS	English	MAMME

Recall that a minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

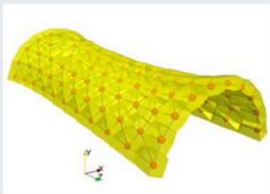
(*) These courses are proposed to students willing to get a deeper focus on discrete and combinatorial optimization.

Focus on Modelling and Analysis in Biomedical Sciences

Research in biomedical sciences increasingly involves mathematical modelling as a support to validate theories, to test computational replicas, to manage biomedical data and to deal with new challenges that are hard to explore either clinically or experimentally. All these goals require scientists with the solid basis provided in standard mathematical undergraduate programs, but also equipped with advanced mathematical and computational tools, as well as a practical spirit, to serve at the interface of biology, medicine, mathematics and computation. Fortunately, while the MAMME program gives the opportunity of acquiring a basic knowledge of mathematical models in biology and advanced mathematical/computational tools, other master's programs at UPC, allow offering a complete training to prepare our students in this stimulating interdisciplinary area. Students interested in joining this area through the MAMME will receive advice from the master's coordination to tailor their curriculum according to different scopes in mathematical modelling of biomedical sciences. We aim at giving a broad training in the mathematical modelling of medically significant biological problems and, additionally, endow their careers with an initial subfocus in some specific problems. The list of courses below represent the wide offer at the UPC to tailor specific profiles (the student has to select 45 ECTS from it), which can be also complemented with problem-oriented master's theses (for example, study of phylogenetic trees, mathematical and computational neuroscience, electro-mechanical models in cardiac physiology, mathematical epidemiology,...), eventually co-advised with partners in biomedical labs. Researchers involved in the area offer their advice to adapt the curriculum to each student’s background and interests.



Brain dynamics: modelling and analysis at different levels, with different tools (differential equations, graphs, statistics,...)



Simulation of curved cellular monolayers with computational mechanics

Mathematical models in biology	7.5 ECTS	English	MAMME
Numerical Methods for dynamical systems	7.5 ECTS	English	MAMME
Mathematical Modeling with PDEs	7.5 ECTS	English	MAMME
Numerical Methods for PDEs	7.5 ECTS	English	MAMME
Computational Mechanics	7.5 ECTS	English	MAMME
Qualitative and quantitative methods in dynamical systems	7.5 ECTS	English	MAMME
Graph theory	7.5 ECTS	English	MAMME
Inferencia estadística avanzada	5 ECTS	Spanish	MESIO UPC-UB
Fundamentos de bioinformática	5 ECTS	Spanish	MESIO UPC-UB
Multivariate data analysis	5 ECTS	Spanish-English	MESIO UPC-UB
Probability and stochastic processes	5 ECTS	English	MESIO UPC-UB
Time series	5 ECTS	Spanish-English	MESIO UPC-UB
Numerical Modeling*	9 ECTS	English	Màster en Enginyeria de Camins, Canals i Ports, UPC
Técnicas básicas en neurociencia**	2.5 ECTS		Màster official en neurociencia, UB-UPF-UDL-URV
Biología Celular y Molecular de la Neurona**	3 ECTS		Màster official en neurociencia, UB-UPF-UDL-URV
Diseño y análisis de datos en neurociencia cognitive**	2.5 ECTS		Màster official en neurociencia, UB-UPF-UDL-URV
Neurociencia computacional**	2.5 ECTS		Màster official en neurociencia, UB-UPF-UDL-URV

We remind that a minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

(*) "Numerical Modeling" is recommended to students that do not have a solid background in numerical methods and programming. Registration to this course requires the approval of the director of the corresponding master.

(**) These courses are proposed to students willing to get a deeper focus neuroscience. Registration to these courses requires the approval of the director of the corresponding master.

Focus on Algebra, Geometry and Number Theory

Mathematics departments at UPC gather several research groups specialized in Number Theory, Algebraic Geometry, Differential Geometry and commutative and non-commutative Algebra. All of them collaborate closely with other researchers of the Universitat de Barcelona and the Universitat Autònoma de Barcelona, and with research groups of some of the most prestigious universities around the world. Many young researchers began their scientific careers by coursing the Master and/or doing the PhD in one of the UPC groups: Ariadna, Biel, Carlos, Enrique, Francesc, Francesc, Marc, María, Martí, Pere-Daniel, Santi, Víctor, Xevi, among others.
A knowledge of some basics in Algebra, Geometry and Number Theory is also very useful for people aimed to work in applications of Mathematics to Cryptography, Coding Theory, Discrete Mathematics, Control Theory, Mathematical Physics, Algorithmics, Biosciences, etc.

Students interested in focusing on Algebra, Geometry and Number Theory are invited to select 45 ECTS from this list:

Commutative Algebra*	MAMME
Non-Commutative Algebra*	MAMME
Differentiable Manifolds*	MAMME
Number Theory*	MAMME
Algebraic Geometry*	MAMME
Geometry and Topology of Varieties	Master in Advanced Mathematics, UB
Algebraic Curves**	Master in Advanced Mathematics, UB
Computational Algebra	Master in Advanced Mathematics, UB
Geometrical Methods in Number Theory	Master in Advanced Mathematics, UB
Local Algebra	Master in Advanced Mathematics, UB

A minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

(*) Please check at the Study Program web page if this course is offered in the academic year you are interested in.

(**) This course is not recommended to students who have followed the course "Geometría Algebraica" of *Grau en Matemàtiques* at FME.

Focus on Dynamical Systems and Applications to Celestial Mechanics

Dynamical Systems provide a powerful mathematical background to explore a great variety of models involving natural and social sciences, physics, chemistry, ecology, conomics, neuroscience, astrodynamics among other fields. As a consequence Dynamical Systems theory has become an important and attractive branch of mathematics to students in many disciplines.

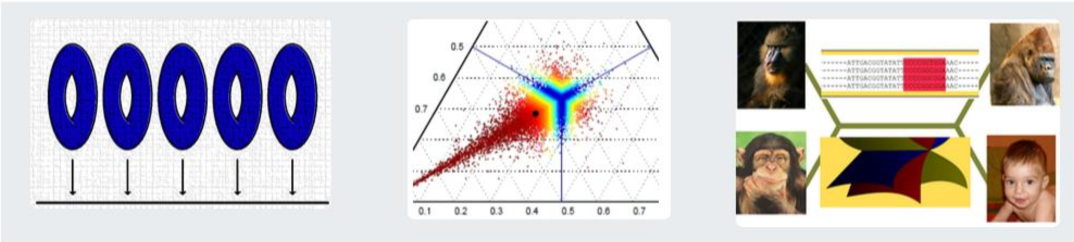
The courses proposed below aim at adquiring a basic and transversal knowledge of both the theory of Dynamical Systems as well as computational tools. Along the courses several applications are considered (see the course on 'Mathematical methods in Biology') but special emphasis is focused on Celestial Mechanics.

Other complementary courses from the Master at the Universitat de Barcelona are also given.

Qualitative and quantitative methods in dynamical systems	7.5 ECTS	English	MAMME
Numerical methods for dynamical systems	7.5 ECTS	English	MAMME
Hamiltonian systems	7.5 ECTS	English	MAMME
Mathematical models in biology	7.5 ECTS	English	MAMME
Advanced course in partial differential equations	7.5 ECTS	English	MAMME
Dynamical systems	6 ECTS	English	Master's degree in advanced and professional mathematics (UB)
Simulation methods	6 ECTS	English	Master's degree in advanced and professional mathematics (UB)

Focus on Geometry and its Applications

Geometry is a multifaceted research field which is at the crossroad of other topics such as Mathematical Physics and Applied Mathematics. The different branches of Geometry include Algebraic Geometry and its applications to Phylogenetics and Robotics, Algebraic Topology and its applications to Computational Topology, Differential Geometry and its applications to Mathematical Physics and Control Theory.



Students interested in focusing on Geometry and its applications are invited to select 45 ECTS from this list:

Students interested in focusing on Geometry and its applications are invited to select 45 ECTS from this list:

Commutative Algebra (7,5 ECTS)	MAMME
Non-Commutative Algebra (7,5 ECTS)	MAMME
Differentiable Manifolds (7,5 ECTS)	MAMME
Algebraic Geometry (7,5 ECTS)	MAMME
Geometry and Topology of Varieties (6 ECTS)	Master in Advanced Mathematics, UB
Algebraic Curves (6 ECTS)	Master in Advanced Mathematics, UB
Computational Algebra (6 ECTS)	Master in Advanced Mathematics, UB
Local Algebra (6 ECTS)	Master in Advanced Mathematics, UB

A minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

Focus on Mathematical Physics

The Focus on *Mathematical Physics* concentrates on the interplay between mathematics and physics, exploring the deep and intricate connections that bridge these two fundamental disciplines. This focus illustrates how mathematical theories can provide precise and predictive frameworks for understanding physical phenomena, while physical insights, in turn, inspire the development of new mathematical concepts and techniques.



Students interested in focusing on Physics and Mathematics are invited to select 45 ECTS from this list:

Differentiable Manifolds	7.5 ECTS	MAMME
Hamiltonian Systems	7.5 ECTS	MAMME
Quantitative and Qualitative Methods in Dynamical Systems	7.5 ECTS	MAMME
Graph theory	7.5 ECTS	MAMME
Computational Mechanics	7.5 ECTS	MAMME
Mathematical Modelling with PDEs	7.5 ECTS	MAMME
Numerical methods for dynamical systems	7.5 ECTS	MAMME
Codes and Cryptography	7.5 ECTS	MAMME
Numerical Methods for PDEs	7.5 ECTS	MAMME
Critical Phenomena and complexity	5 ECTS	MEPH
Quantum Matter	5 ECTS	MEPH
Computational Astrophysics	4 ECTS	MEPH
Complexity in Biological systems	4 ECTS	MEPH
Stochastic Methods for Optimization & Simulation	4 ECTS	MEPH
Materials Science of Drugs	4 ECTS	MEPH
Atomic and molecular physics	4 ECTS	MEPH

A minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

Double degrees

Attached there is the list of double degrees in association with MAMME

MAMME-IIT (IIT, USA)
MAMME-INP Grenoble (Grenoble, France)

Master's thesis

All students are required to write and defend a master thesis, usually during the second term of the academic year.

It may be carried out at a research group in UPC (see the research groups associated to the PhD program in Applied Mathematics at UPC), at a research group from another university, at a research center or at a company.

A list of proposals for master thesis can be found at the FME intranet at Borsa de Projectes. Even though the list includes many proposals, other researchers not included there will also be willing to supervise your work. Thus, if you are interested in a particular area, you can contact the coordinator or the course closer to it, or the master's director or a member of the academic committee for guidance.

Regulations, calendar and templates

The calendar and the academic regulations for the master's thesis can be found at the FME web page.

Templates for the document of the master thesis can be downloaded here:

- LaTeX template
- Cover page
- Master Thesis evaluation

Master thesis defence

The master thesis is defended publically and evaluated by 3 members of UPC. The usual session works as follow:

1) The defence of the master thesis done by the candidate (around 25 minutes). The defence must be done preferably in English.

2) A round of questions (around 20 minutes) from the committee.

3) A round of questions (if any) from people in the audience with a master degree.

Forthcoming defenses

A list of forthcoming presentations of master's thesis can be found at this link.

Index

Course guide
34963 - ACPDE - Advanced Course in Partial Differential Equations

Last modified: 29/05/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: OLLI OSKARI SAARI

Others:

PRIOR SKILLS

Basic knowledge of Partial Differential Equations (undergraduate level).
Basic knowledge of Mathematical Analysis (undergraduate level).
Basic knowledge of Functional Analysis (undergraduate level).

REQUIREMENTS

Undergraduate courses in Partial Differential Equations and in Mathematical Analysis.
It is helpful an undergraduate course in Functional Analysis.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Classes will combine theoretical aspects and proofs with resolution of concrete problems and exercises.
Further reading from the bibliography will be given often.

LEARNING OBJECTIVES OF THE SUBJECT

This course is intended to be an introduction to modern methods for solving elliptic partial differential equations. However, some insights to classical solutions to parabolic and hyperbolic equations will also be given. The objectives of the course are:
- understand the classical methods to solve the transport, wave, heat, Laplace, and Poisson equations,
- understand the role of Sobolev norms and compact embeddings to solve PDE and find spectral decompositions,
- learn the modern methods to solve elliptic PDE.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Classical methods in PDEs

Description:
[This topic will only be treated in the exercises sessions.] Classical solutions to the transport, wave, heat, Laplace, and Poisson equations. Maximum principles, Green's functions, separation of variables, energy methods, probabilistic interpretation.

Full-or-part-time: 46h
Theory classes: 15h
Self study : 31h

Hilbert space techniques

Description:
Orthogonal projections, Riesz-Fréchet representation theorem, Lax-Milgram theorem.

Full-or-part-time: 25h
Theory classes: 8h
Self study : 17h

Sobolev spaces

Description:
Mollifiers, Fréchet-Kolmogorov theorem, distributions, Sobolev norms, Poincaré inequality, compact embeddings, approximation by smooth functions, traces.

Full-or-part-time: 29h
Theory classes: 9h
Self study : 20h

Weak formulation and the weak maximum principle

Description:
Weak solutions via Hilbert space techniques and interpretation, comparison principles in the weak formulation.

Full-or-part-time: 25h
Theory classes: 8h
Self study : 17h

Regularity theory

Description:
Boundedness of weak solutions, Sobolev-Gagliardo-Nirenberg inequality, regularity in Sobolev spaces, the translation method, bootstrap technique.

Full-or-part-time: 25h 30m
Theory classes: 8h
Self study : 17h 30m

Eigenvalues

Description:
Spectral decompositions, applications to (time dependent) evolution equations, Rayleigh quotient, description of the first eigenvalue for the Dirichlet problem on a bounded domain.

Full-or-part-time: 17h
Theory classes: 6h
Self study : 11h

Nonlinear problems

Description:
Calculus of variations, monotone iteration method, obstacle problems.

Full-or-part-time: 20h
Theory classes: 6h
Self study : 14h

GRADING SYSTEM

The evaluation of the course is based on:
- the resolution of problems proposed in class (40%),
- a midterm exam (20%),
- a final comprehensive exam (40%).
The active participation during the course will be a requirement for the evaluation of the final exam.

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BIBLIOGRAPHY

Basic:

- Chipot, M. Elliptic equations : an introductory course [on line]. Basel: Birkhäuser Basel, 2009 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-7643-9982-5. ISBN 9783764399825.
- Evans, Lawrence C. Partial differential equations. Providence, Rhode Island: American Mathematical Society, 1998. ISBN 0821807722.
- Salsa, Sandro. Partial differential equations in action : from modelling to theory [on line]. Milan: Springer, 2008 [Consultation: 10/07/2023]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=3062992. ISBN 9788847007512.
- Brézis, H. Análisis funcional : teoría y aplicaciones. Madrid: Alianza, 1984. ISBN 8420680885.

Complementary:

- Folland, G. B. Introduction to partial differential equations. 2nd ed. Princeton (N.J.): Princeton University Press, 1995. ISBN 0691043612.
- Struwe, Michael. Variational methods : applications to nonlinear partial differential equations and hamiltonian systems [on line]. 2nd rev. and substantially expanded ed. Berlin: Springer, 1996 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-540-74013-1. ISBN 3540520228.
- Gilbarg, David; Trudinger, Neil S. Elliptic partial differential equations of second order. 2nd ed., rev. third printing. Berlin: Springer-Verlag, 1998. ISBN 354013025X.
- Necas, Jindrich. Introduction to the theory of nonlinear elliptic equations. Chichester: John Wiley & Sons, 1986. ISBN 0471908940.

Course guide
34952 - AG - Algebraic Geometry

Last modified: 29/05/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: MARIA ALBERICH CARRAMIÑANA

Others:

PRIOR SKILLS

Aquaintance with mathematical computations, both by hand and with a computer, and mathematical reasoning, including proofs.

REQUIREMENTS

Basic abstract Algebra, Topology and Differential Geometry.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Approximately 50% of class time will be dedicated to interactive master classes, in which the lecturer will discuss course topics and propose small challenges and questions to solve. The other half of the class time will be structured as a problem-solving session, in which students will solve problems from a proposed list on the blackboard, based on the course syllabus, and their solutions will be discussed by the class.

LEARNING OBJECTIVES OF THE SUBJECT

The main objective of the course is to introduce students to local algebraic geometry, with a focus on plane curve singularities. It aims to provide insight into the singularity theory of plane curves and the geometric theory of valuations of the ring of convergent series of two variables over the complex numbers. The course will demonstrate that singular points of algebraic curves in the complex plane is a meeting point for various areas of mathematics.

The course will heavily rely on examples, emphasizing the geometric significance of the subject. The specific topics for the final projects will be determined based on the students' interests.

STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

Total learning time: 187.5 h

CONTENTS

Chapter 1: Algebraic and analytic varieties

Description:
Algebraic preliminaries. Affine algebraic varieties, Nullstellensatz. Analytic varieties, Teorema de Rückert.

Full-or-part-time: 11h 30m
Theory classes: 5h
Self study : 6h 30m

Chapter 2: Parametrizing branches of plane curves

Description:
Newton-Puiseux algorithm, Weierstrass preparation and division theorems, Hensel's lemma. Intersection multiplicity: Halphen's formula. Testing analytic reducibility.

Full-or-part-time: 12h 30m
Theory classes: 6h
Self study : 6h 30m

Chapter 3: Infinitely near points and resolutions of singularities

Description:
Proximity, Enriques diagrams, dual graph. Resolutions of singularities. Rings in successive neighbourhoods. The order of singularity.

Full-or-part-time: 23h
Theory classes: 10h
Self study : 13h

Chapter 4: Topological classification of singularities

Description:
Topological classification of plane curves: equisingularity, semigroup of values, complete equisingularity invariants. Milnor number. Topological conic structure of a singularity. Topology of the singularity link. The Milnor fibration.

Full-or-part-time: 23h
Theory classes: 10h
Self study : 13h

Chapter 5: Constructions on the resolution tree

Description:
Homology of a blow-up. The exceptional divisor of a curve. Functions on the resolution tree. Complete ideals. Multiplier ideals and jumping numbers. Topological zeta function.

Full-or-part-time: 18h
Theory classes: 8h
Self study : 10h

Chapter 6: Analytic classification of plane curves

Description:
Jacobian ideal and its semimodule of values. Tjurina algebra and Tjurina number. Unfoldings of equations. Deformations of curves: versal and miniversal deformations. Zariski's moduli space. Teissier's monomial curve.

Full-or-part-time: 18h
Theory classes: 8h
Self study : 10h

Chapter 7: Valuations and complete ideals

Description:
Classification of valuations. Zariski decomposition of complete ideals.

Full-or-part-time: 18h
Theory classes: 8h
Self study : 10h

Chapter 8: Final projects

Description:
The final essays of the course on the topics chosen by the students will be presented by the students themselves and commented by the course lecturers.

Full-or-part-time: 25h
Theory classes: 5h
Self study : 20h

GRADING SYSTEM

Students who solve a sufficient number of problems on the blackboard during the problem-solving class will pass the course. If they wish to improve their grade from a passing grade to a higher score, they will be assigned a final project. The final project will involve studying, writing an essay and delivering a lecture on an additional topic towards the end of the course.

Students who have not actively participated enough in the problem-solving class, or still wish to improve their grade even after completing the problem class and final project, will be required to take a final exam lasting approximately 4 hours.

EXAMINATION RULES.

The problem list for participation in the problem-solving class will be published at the beginning of each course unit. Students are expected to prepare these problems in advance at home.

The topics for optional final projects aimed at increasing grades will be proposed around Easter. Students will be responsible for preparing the lecture and the essay of the final project independently at home.

Students who choose to take the final exam will be required to do so without any notes, books, or other materials whatsoever.

BIBLIOGRAPHY

Basic:
- Casas Alvero, Eduardo. Singularities of plane curves. Cambridge: Cambridge University Press, 2000. ISBN 0521789591.
- Fischer, Gerd. Plane algebraic curves. American Mathematical Society, cop. 2001. ISBN 0821821229.
- Ghys, E. A Singular mathematical promenade. Lyon: CNRS-ENS Éditions, 2017. ISBN 9782847889390.
- Wall, C. T. C. Singular points of plane curves. Cambridge, UK ; New York: Cambridge University Press, cop. 2004. ISBN 0521547741.
- Zariski, Oscar; Linchtin, Ben; Teissier, Bernard. The Moduli problem for plane branches. American Mathematical Society, 2006. ISBN 9780821829837.

Complementary:
- Brieskorn, Egbert; Knörrer, Horst. Plane algebraic curves : translated by John Stillvell [on line]. Springerlink, 2012 [Consultation: 15/06/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-0348-0493-6. ISBN 9783034804929.
- Casas-Alvero, Eduardo. Algebraic curves, the brill and noether way. Springer, ISBN 9783030290153.
- Chenciner, Alain. Courbes algébriques planes [on line]. Berlin: Springer, 2008 [Consultation: 07/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-540-33708-9. ISBN 9783540337072.
- Greuel, Gert-Martin; Lossen, Christoph; Shustin, Eugenii. Singular algebraic curves : with an appendix by Oleg Viro. Springer, ISBN 9783030033491.
- Kollar, János. Lectures on resolution of singularities. Princeton University Press, ISBN 9780691129235.

Course guide
34954 - CC - Codes and Cryptography

Last modified: 29/05/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: SIMEON MICHAEL BALL MARKS

Others:

PRIOR SKILLS

Basic probability, basic number theory and linear algebra

REQUIREMENTS

Undergraduate mathematics

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

The course is divided in two parts: codes and cryptography. Each part consists of 26 h of ordinary classes, including theory and problem sessions.

LEARNING OBJECTIVES OF THE SUBJECT

This course aims to give a solid understanding of the uses of mathematics in Information technologies and modern communications. The course focuses on the reliable and efficient transmission and storage of the information. Both the mathematical foundations and the description of the most importants cryptographic protocols and coding systems are given in the course.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Introduction

Description:
The problem of communication. Information theory, Coding theory and Cryptographic theory

Full-or-part-time: 6h 15m
Theory classes: 2h
Self study : 4h 15m

Information and Entropy

Description:
Uncertainty or information. Entropy. Mutual information

Full-or-part-time: 18h 45m
Theory classes: 6h
Self study : 12h 45m

Source codes without memory

Description:
Codes. Average length. Huffman codes. Extensions of a source. Theory of an noiseless communication. Notes of compression.

Full-or-part-time: 12h 30m
Theory classes: 4h
Self study : 8h 30m

Channel coding

Description:
Discrete channels without memory. Symmetric channels. Shannon's theorem.

Full-or-part-time: 18h 45m
Theory classes: 6h
Self study : 12h 45m

Block codes

Description:
Hamming's distance. Detection and correction of errors. Bounds. Linear codes.

Full-or-part-time: 18h 45m
Theory classes: 6h
Self study : 12h 45m

Cyclic codes

Description:
Cyclic codes. Generator and control matrices. Factorization of x^n-1. Roots of a cyclic code. BCH codes. Primitive Reed-Solomon codes. Meggit's decoder.

Full-or-part-time: 18h 45m
Theory classes: 6h
Self study : 12h 45m

Introduction to modern cryptography

Description:
The setting: secure storage and symmetric key encryption. Turing machines and complexity classes. Security definitions. Adversarial models. Reductionist security proofs.

Full-or-part-time: 15h 37m
Theory classes: 5h
Self study : 10h 37m

Symmetric key cryptography

Description:
Symmetric key encryption. Pseudorandom generators. Block ciphers. Message authentication codes.

Full-or-part-time: 15h 38m
Theory classes: 5h
Self study : 10h 38m

Public key encryption

Description:
Definitions and security notions. One way functions. Probabilistic encryption. Main constructions. Homomorphic encryption. Chosen ciphertext security.

Full-or-part-time: 15h 37m
Theory classes: 5h
Self study : 10h 37m

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Digital signatures

Description:
Security definitions. RSA and Schnorr signatures.

Full-or-part-time: 15h 38m
Theory classes: 5h
Self study : 10h 38m

Proofs of knowledge and other cryptographic protocols

Description:
Ring signatures. Distributed signatures. Identity and attribute based protocols.

Full-or-part-time: 15h 37m
Theory classes: 5h
Self study : 10h 37m

Multiparty computation

Description:
Secret sharing schemes. Unconditionally and computationally secure multiparty computation.

Full-or-part-time: 15h 38m
Theory classes: 5h
Self study : 10h 38m

GRADING SYSTEM

Exam of coding part (50%) and exam of crypto part (50%). If the average is less than 5 out of 10, there is a chance to pass the subject in a final exam.

EXAMINATION RULES.

All the subjects are important. To pass the course it is required to fulfill all the items.

BIBLIOGRAPHY

Basic:
- Delfs, Hans; Knebl, Helmut. Introduction to cryptography : principles and applications. 2nd ed. Berlin: Springer, 2007. ISBN 9783540492436.
- Katz, Jonathan; Lindell, Yehuda. Introduction to modern cryptography : principles and protocols. Boca Raton: Chapman & Hall, 2008. ISBN 9781584885511.
- Ball, Simeon. A Course in algebraic error-correcting codes [on line]. Birkhauser, 2020 [Consultation: 07/07/2023]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=6194949. ISBN 9783030411534.
- Jones, Gareth A.; Jones, J. Mary. Information and coding theory. Springer, 2000. ISBN 9781447103615.

Complementary:
- Huffman, W. Cary; Pless, Vera. Fundamentals of error-correcting codes. Cambridge: Cambridge University Press, 2003. ISBN 0521782805.
- Justesen, Jorn; Hoholdt, Tom. A Course in error-correcting codes. Zürich: European Mathematical Society, 2004. ISBN 3037190019.

- Welsh, Dominic. Codes and cryptography. Oxford: Oxford university Press, 1988. ISBN 0198532881.
- Xambó Descamps, Sebastián. Block error-correcting codes : a computational primer. Berlin: Springer, 2003. ISBN 3540003959.
- Goldreich, Oded. Foundations of cryptography : basic tools. New York: Cambridge University Press, 2001. ISBN 0521791723.
- Goldreich, Oded. Foundations of cryptography : basic applications. New York: Cambridge University Press, 2004. ISBN 9780521830843.

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Course guide
34955 - COMB - Combinatorics

Last modified: 23/06/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: JUAN JOSÉ RUE PERNA

Others:

PRIOR SKILLS

Basic calculus and linear algebra. Notions of probability.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.
MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

There will be a lecture each week, followed by a problem session.

LEARNING OBJECTIVES OF THE SUBJECT

To use algebraic, probabilistic and analytic methods for studying combinatorial structures. The main topics of study are: partially ordered sets, extremal set theory,finite geometries, Ramsey theory and enumerative combinatorics.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Partially ordered sets and extremal set theory

Description:
Sperner's theorem. LYM inequalities. Bollobás's theorem. Erdos-Ko-Rado Theorem. Dilworth's theorem. Applications of Dilworth Theorem. Lattices and distributive lattices. The 4 functions theorem and applications

Full-or-part-time: 24h 40m
Practical classes: 4h
Laboratory classes: 4h
Self study : 16h 40m

Ramsey theory

Description:
Theorems of Ramsey and Hales-Jewett. Theorems of Schur, Van der Waerden and Rado.

Full-or-part-time: 31h 40m
Theory classes: 5h
Laboratory classes: 5h
Self study : 21h 40m

Probabilistic methods in combinatorics

Description:
First and second moment. Lovász Local Lemma and entropy methods. Applications: Permanents, transversals, hypergraph coloring. Monotone properties and threshold functions.

Full-or-part-time: 18h 30m
Theory classes: 3h
Laboratory classes: 3h
Self study : 12h 30m

Linear algebra methods in combinatorics

Description:
The Combinatorial Nullstellensatz. The polynomial method. Fisher's theorem. Applications

Full-or-part-time: 18h 30m
Theory classes: 3h
Laboratory classes: 3h
Self study : 12h 30m

GRADING SYSTEM

Continous evaluation will be based on the weekly solution of exercises. There will be also a final examination. The grading will be based on the continous evaluation during the course (60%) and the final exam (40%).

BIBLIOGRAPHY

Basic:

- Alon, Noga; Spencer, Joel H.; Erdös, Paul. The Probabilistic method. 3rd ed. New York: Wiley, 2008. ISBN 0471535885.
- Bollobás, Béla; Andrew Thomason (eds.). Combinatorics, geometry, and probability : a tribute to Paul Erdos. Cambridge: Cambridge University Press, 1997. ISBN 0521584728.
- Lint, Jacobus Hendricus van; Wilson, R. M. A Course in combinatorics. 2nd ed. Cambridge: Cambridge University Press, 2001. ISBN 0521803403.
- Flajolet, P.; Sedgewick, R. Analytic combinatorics [on line]. Cambridge: Cambridge University Press, 2009 [Consultation: 07/07/2023]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=412737. ISBN 9780521898065.
- Graham, Ronald L.; Rotschild, B.; Spencer, J. Ramsey theory. 2nd ed. New York: John Wiley & Sons, 1990. ISBN 0471500461.
- Anderson, Ian. Combinatorics of finite sets. Mineola: Dover, 2002. ISBN 0486422577.
- Lovász, László. Combinatorial problems and exercices. 2nd ed. Amsterdam: North-Holland, 1993. ISBN 044481504X.
- Oxley, J. G. Matroid theory [on line]. 2nd ed. Oxford: Oxford University Press, 2011 [Consultation: 13/12/2022]. Available on: https://academic-oup-com.recursos.biblioteca.upc.edu/book/34846?searchresult=1. ISBN 9780199603398.
- Jukna, Stasys. Extremal combinatorics. 2011. Springer, 2011. ISBN 9783642173639.

Course guide
34950 - CALG - Commutative Algebra

Last modified: 29/05/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: FRANCESC D'ASSIS PLANAS VILANOVA

Others:

PRIOR SKILLS

Linear algebra, algebraic structures, topology.

REQUIREMENTS

The two first years of a degree in mathematics.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Teaching Classes, resolution of problems

LEARNING OBJECTIVES OF THE SUBJECT

Basic course in Commutative Algebra.
An introduction to the theory of rings, ideals and modules.
Some basics on local algebra.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Rings and ideals

Description:
Basics on ring theory and ideals.
Rings of fractions. Primary decomposition. Chain conditions. Noetherian and Artinian rings.

Full-or-part-time: 28h 20m
Theory classes: 15h
Self study : 13h 20m

Modules

Description:
General properties of modules.
Modules of fractions. Chain conditions. Homomorphisms and tensor product.

Full-or-part-time: 24h
Theory classes: 12h
Self study : 12h

Algebraic varieties

Description:
The spectrum of a ring. Zariski topology.

Full-or-part-time: 24h
Theory classes: 12h
Self study : 12h

Introduction to homological algebra

Description:
Categories and functors. Complexes of modules. Derived functors.

Full-or-part-time: 24h
Theory classes: 12h
Self study : 12h

Local algebra

Description:
Regular sequences. Depth.
Homological characterizations.
Regular rings, Gorenstein rings, Cohen-Macaulay rings

Full-or-part-time: 18h 40m
Theory classes: 9h
Self study : 9h 40m

GRADING SYSTEM

The qualification will be based on:
60% Resolution of assigned exercises and/or projects
40% Final Exam

BIBLIOGRAPHY

Basic:

- Atiyah, Michael Francis; MacDonald, I. G. Introduction to commutative algebra. Reading: Addison-Wesley, 1969. ISBN 0201407515.
- Reid, Miles. Undergraduate commutative algebra. Cambridge: Cambridge University Press, 1995. ISBN 0521452554.
- Eisenbud, David. Commutative algebra : with a view toward algebraic geometry. Corrected 2nd. printing. New York: Springer-Verlag, 1996. ISBN 0387942696.
- Kunz, Ernst. Introduction to commutative algebra and algebraic geometry [on line]. Birkhäuser, 2013 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-1-4614-5987-3. ISBN 9781461459866.
- Matsumura, Hideyuki. Commutative ring theory. Cambridge: Cambridge University Press, ISBN 0521259169.
- Rotman, J.J. An Introduction to homological algebra [on line]. Academic Press, 1979 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/b98977. ISBN 0125992505.
- Bruns, Winfried; Herzog, Jürguen. Cohen-Macaulay rings. Cambridge University Press, 1993. ISBN 0521410681.

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

Course guide
34959 - CM - Computational Mechanics

Last modified: 29/05/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.
751 - DECA - Department of Civil and Environmental Engineering.
748 - FIS - Department of Physics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: JOSE JAVIER MUÑOZ ROMERO

Others:

PRIOR SKILLS

Basic knowledge of numerical methods
Basic knowledge of partial differential equations

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Four elements will be combined:
-Theory classes, where the main concepts will be presented.
-Practical classes with Matlab code in the computer room, with emphasis on the computational aspects.
-Lists of short assignments.
-Course projects in groups to be presented orally at the end of the course.

Students will work on the assignments and course projects individually or in groups.

LEARNING OBJECTIVES OF THE SUBJECT

The main objective is to provide a general perspective of the broad field of computational mechanics, covering both the modelling and the computational aspects. A broad range of problems is addressed: solids, fluids and fluid-solid interaction; linear and nonlinear models; static and dynamic problems. Some emphasis is put on applications in biomechanical problems. By the end of the course, the students should:
- Be able to choose the appropriate type of model for a specific simulation
- Be familiar with the mathematical objects (tensors) and differential operators used in computational mechanics
- Be aware of the different level of complexity of various problems (e.g. linear vs. nonlinear, static vs. dynamic).

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

CONTINUUM MECHANICS

Description:
Motivation. Definition of continuous media. Equation of motion: Eulerian and Lagrangian descriptions. Time derivatives. Strains: deformation gradient, Green and Euler-Almansi tensors; elongation and shear; small strains. Stresses: body and surface forces; Cauchy stress tensor. Balance equations: Reynolds transport theorem; mass balance; momentum balance. Constitutive equations. Applications.

Full-or-part-time: 31h 15m
Theory classes: 8h
Practical classes: 2h
Self study : 21h 15m

COMPUTATIONAL ELASTICITY

Description:
Basic concepts and motivation. Elastic constitutive equation. Displacement formulation: Navier equations. Two-dimensional elasticity: plane stresses, plane strains and axisymmetry. Weak form of the elastic problem. Finite element discretisation. Computational aspects. Applications in engineering and biomechanics.

Full-or-part-time: 31h 15m
Theory classes: 8h
Practical classes: 2h
Self study : 21h 15m

COMPUTATIONAL DYNAMICS

Description:
Weak form. Dynamic equation. Space discretisation (finite elements) and time discretisation. Solution methods: generalised eigen value problem and direct time integration. Euler, centred differences, HHT and Newmark methods. Stability, consistency and accuracy of numerical techniques in elastodynamics. Applications and relation with reduced order techniques (Proper Orthogonal Decomposition).

Full-or-part-time: 31h 15m
Theory classes: 8h
Practical classes: 2h
Self study : 21h 15m

COMPUTATIONAL PLASTICITY, FRACTURE AND VISCOELASTICITY

Description:
Basic concepts and motivation. Non-linear problems. Geometrical and material non-linearity. One-dimensional plasticity: elastic and plastic strains;
elastoplastic constitutive equation; hardening. Multi-dimensional plasticity: stress and strain
invariants; yield surface; plastic flow. Numerical time-integration of the constitutive
equation: elastic prediction and plastic correction; iterative methods for the plastic
correction. Applications. Phase field formulation and Finite Element discretisation.
Viscoelastic materials. Maxwell and Kelvin-Voigt one-dimensional models. Numerical solution. Extension to multiple dimensions.

Full-or-part-time: 31h 15m
Theory classes: 8h
Practical classes: 2h
Self study : 21h 15m

COMPUTATIONAL FLUID DYNAMICS

Description:
Basic concepts and motivation. Rate-of-deformation and spin tensors. Constitutive equation for
Newtonian fluids. Euler equations for inviscid flow. Navier-Stokes equations for viscous flow in strong form and in weak form.
Reynolds number. Stokes flow and potential flow. Finite element discretisation and numerical solution. Applications.

Full-or-part-time: 31h 15m
Theory classes: 8h
Practical classes: 2h
Self study : 21h 15m

COMPUTATIONAL METHODS FOR WAVE PROBLEMS

Description:
Basic concepts and motivation.
Acoustics: the wave equation. The scalar Helmholtz equation. Vibroacoustics: acoustic fluid-elastic solid interaction.
Computational aspects. Applications.
Electromagnetism: the Maxwell equations. Electrodynamics. The vectorial Helmholtz equation.
Computational aspects. Applications.

Full-or-part-time: 31h 15m
Theory classes: 8h
Practical classes: 2h
Self study : 21h 15m

GRADING SYSTEM

Final exam (40%), assignment problems (30%), and course project (30%, evaluated with an oral presentation and a written report).

BIBLIOGRAPHY

Basic:

- Clough, Ray W.; Penzien, J. Dynamics of structures. 2nd ed. New York: McGraw-Hill, 1993. ISBN 0071132414.
- Donea, Jean M.; Huerta, A. Finite element methods for flow problems. Chichester: John Wiley & Sons, 2003. ISBN 0471496669.
- Ihlenburg, F. Finite element analysis of acoustic scattering [on line]. New York: Springer-Verlag, 1998 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/b98828. ISBN 0387983198.
- Mase, G. Thomas; Mase, George E. Continuum mechanics for engineers. 3rd ed. Boca Raton: CRC, 2010. ISBN 9781420085389.

Complementary:

- Bathe, Klaus-Jürgen. Finite element procedures. New Jersey: Prentice-Hall, 1996. ISBN 0133014584.
- Bonet, Javier; Wood, R. D. Nonlinear continuum mechanics for finite element analysis. 2nd ed. Cambridge: Cambridge University Press, 2008. ISBN 9780521838702.
- Holzapfel, Gerhard A. Nonlinear solid mechanics : a continuum approach for engineering. Chichester: John Wiley & Sons, cop. 2000. ISBN 9780471823193.
- Marsden, Jerrold E.; Hugues, Thomas J. R. Mathematical foundations of elasticity. New York: Dover, 1994. ISBN 0486678652.
- Simo, J. C.; Hughes, T. J. R. Computational inelasticity [on line]. New York: Springer-Verlag, 1998 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/b98904. ISBN 0387975209.
- Taber, Larry A. Nonlinear theory of elasticity. Applications in Biomechanics. 2008. Singapore: World Scientific Publishing, 2004. ISBN 9812387358.
- Zienkiewicz O. C.;Taylor, R. L. The Finite element method [on line]. 6th ed. Oxford: Butterworth Heinemann, 2005 [Consultation: 10/07/2023]. Available on: http://www.sciencedirect.com/science/book/9780750664318. ISBN 9780750664318.

Course guide
34966 - VD - Differentiable Manifolds

Last modified: 29/05/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.
981 - CRM - Mathematical Research Centre.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: EVA MIRANDA GALCERÁN

Others:

PRIOR SKILLS

Basic courses on algebra, calculus, topology and differential equations, and calculus on manifolds. Students from the FME are supposed to have taken "Varietats Diferenciables" (optional 4th year course).

This is not a basic course and the students are assumed to have attended previous courses on differential geometry and smooth manifolds. Students feeling that they may not fulfill the requisites are invited to discuss their case with the lecturers. It is totally possible for prospective students with less knowledge in these topics to follow this course provided they are willing to make up for the gap with individual work during the course and/or by reading some recommended bibliography prior to the beginning of the course.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Theory classes and tutorial sessions will be used to present and develop the contents of the course. Along the course the students will be given problems to solve as homework.

LEARNING OBJECTIVES OF THE SUBJECT

The subject focuses on some of the fundamental topics of differential geometry and its applications to different areas including mathematical physics and Dynamical systems.

By the end of the course, students should be able to:
- understand all the ideas developed along the course.
- apply the studied concepts to other areas of pure mathematics, physics and engineering.
- integrate in a research group on these kinds of topics and their applications.
- search and understand the scientific literature on the subject.
- write and present an essay on mathematics.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Complements in Differential Geometry

Description:
Brief survey of manifold theory and differential geometry including differential forms.
We also plan to talk about differentiable distributions and study its integration via the theorem of Frobenius. This will lead us to introducing several examples of foliations.

Full-or-part-time: 14h 52m
Theory classes: 6h
Self study : 8h 52m

Introduction to Differential Topology

Description:
We present a brief introduction to the theory of Differential Topology which includes basic notions in transversality, singularity theory and Morse theory.

Full-or-part-time: 14h 40m
Theory classes: 8h
Self study : 6h 40m

Introduction to Lie theory

Description:
A Lie group is a group endowed with a smooth manifold structure which is compatible with the group operation. In this chapter we provide an
introduction to the main aspects of the theory of Lie groups and Lie algebras taking matrix Lie groups as starting point.

Full-or-part-time: 16h 20m
Theory classes: 8h
Self study : 8h 20m

Lie group actions on smooth manifolds

Description:
We study Lie group actions on smooth manifolds and relate both geometries via the notions of isotropy group and orbit.

Full-or-part-time: 18h
Theory classes: 4h
Theory classes: 4h
Self study : 5h
Self study : 5h

Basic notions on De Rham Cohomology

Description:
We define De Rham cohomology and compare it to other cohomologies. (Depending on the preliminary knowledge of the students, this chapter may be considered as an APPENDIX)

Full-or-part-time: 8h
Theory classes: 3h
Self study : 5h

Introduction to Symplectic and Poisson Geometry

Description:
We provide a comprehensive introduction to symplectic and Poisson manifolds with special focus on examples. Starting with symplectic manifolds, we will explain Moser's trick and some applications to normal form theorems such as the Darboux theorem and the classification of symplectic surfaces. We introduce the notion of Hamiltonian vector field, symplectic vector field and Hamiltonian System. Special attention will be given to examples provided by the realm of integrable systems. In particular the action-angle theorem of Arnold-Liouville will be presented and the notion of moment map and Hamiltonian group action. We end the chapter introducing other geometries: that of contact geometry and Poisson geometry discussing the most recent progress on open conjectures in the topic.

Full-or-part-time: 31h 40m
Theory classes: 15h
Self study : 16h 40m

GRADING SYSTEM

There will be exam(s) which will contribute to the final grade in a 40% and an essay that will contribute to the final grade in another 40%. Students would choose, together with the lecturers, a topic that complements or advances the material taught during the course, according to their mathematical interests. The remaining 20% is reserved to problem solving which can be evaluated by simply solving some assignments of the regular list of problems.

EXAMINATION RULES.

The final grade awarded to the student would we computed as follows:

40% exam(s) + 40% essay+ 20% problem solving

The grade "exam(s)" includes the one of final exam but may also include other examination material such as ATENEA questionnaires or take-home exercises. The choices and number
of exams will depend on several factors including the ratio presential versus online teaching.

BIBLIOGRAPHY

Basic:

- Bröcker, T.; Jänich, K. Introduction to differential topology. Cambridge University Press, 1982. ISBN 9780521284707.
- Cannas da Silva, Anna. Lectures on symplectic geometry [on line]. Springer-Verlag, 2008 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-540-45330-7. ISBN 354045330X.
- Duistermaat, J. J.; Kolk, Johan A.C. Lie groups. Berlin: Springer-Verlag, 2000. ISBN 3540152938.
- Fegan, H.D. Introduction to compact lie groups. World Scientific, 1991. ISBN 9810236867.
- Guillemin, V.; Sternberg, Shlomo. Symplectic techniques in physics. Repr. with corrections. Cambridge [etc.]: Cambridge University Press, 1990. ISBN 9780521389907.
- Guillemin, Victor; Pollack, Alan. Differential topology. Reprint of the 1974 original. AMS Chelsea Publishing, 2014. ISBN 9780821851937.
- Knauf, A. Mathematical physics : classical mechanics. Springer, 2018. ISBN 9783662557747.
- Lee, John M. Introduction to smooth manifolds [on line]. New York: Springer-Verlag, 2003 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-1-4419-9982-5. ISBN 0387954481.
- Milnor, John W. Topology from the differentiable viewpoint. Rev. ed. Princenton: Princeton University Press, 1997. ISBN 9780691048338.
- Munkres, J.R. Elementary differential topology. ISBN 9780691090931.
- Tu, Loring W. An Introduction to manifolds [on line]. 2nd ed. New York: Springer, 2011 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-1-4419-7400-6. ISBN 9781441973993.
- Warner, Frank W. Foundations of differentiable manifolds and Lie groups. New York [etc.]: Springer, cop. 1971. ISBN 0387908943.

Complementary:

- Bott, Raoul; Tu, Loring W. Differential forms in algebraic topology. New York: Springer-Verlag, 1982. ISBN 0387906134.
- Audin, Michèle. Torus actions on symplectic manifolds. 2nd ed. Birkhäuser, 2004. ISBN 3764321768.
- Warner, Frank W. Foundations of differentiable manifolds and lie groups. New York, NY [etc.]: Springer-Verlag, cop. 1983. ISBN 0387908943.
- Olver, Peter J. Applications of Lie groups to differential equations. New York: Springer-Verlag, 1986. ISBN 0387940073.
- Nakahara, Mikio. Geometry, topology, and physics. 2nd ed. New York [etc.]: Taylor & Francis, cop. 2003. ISBN 0750306068.
- Audin, Michèle; Damian, Mihai. Morse theory and Floer homology. New York: Springer, [2014]. ISBN 9781447154952.
- Gallier, Jean; Quaintance, Jocelyn. Notes on differential geometry and Lie groups [on line]. University of Pennsylvania, 2016 [Consultation: 10/07/2023]. Available on: http://www.cis.upenn.edu/~jean/gbooks/manif.html.

RESOURCES

Other resources:

Notes on the Geometry and Dynamics of singular symplectic manifolds (notes on the FSMP course by Eva Miranda)

Course on youtube by Professor Eva Miranda on Lie group actions
https://www.youtube.com/channel/UC8Fzyf58s0EiZ-gdYgz2ghw?view_as=subscriber />Course on youtube by Professor Eva Miranda on Symplectic and Poisson Geometry
https://www.youtube.com/channel/UC8Fzyf58s0EiZ-gdYgz2ghw?view_as=subscriber

Course guide
34956 - DG - Discrete and Algorithmic Geometry

Last modified: 24/06/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: HUEMER, CLEMENS

Others:

PRIOR SKILLS

- Elementary combinatorics.
- Elementary graph theory.
- Elementary algorithmics.
- Elementary data structures.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Theory classes will be used to present and develop the contents of the course. Most of the topics will be presented by the instructors, but there can be some sessions devoted to students presentations.

There will be lists of problems, which will not contain solutions. Problems will be designed to help students deepen and mature their command of the concepts and techniques presented in class. Some problems will be solved in class, some will be left as homework. In the problem sessions, the goal will be to propose and analyze alternative strategies to solve each problem, and to show how the results presented in class are applied. Most of the problems solved in class will be presented by the students.

LEARNING OBJECTIVES OF THE SUBJECT

Discrete, combinatorial and computational geometry are facets of a common body of knowledge that integrates fundamental elements from mathematics -mainly from algebra, topology and classical branches of geometry- with elements and problems from theoretical computer science and its applications.

The area focuses on the combinatorial and structural study of discrete geometric objects, as well as the design of algorithms to construct or analyze them. Among the objects studied, we can mention discrete sets of points, curves and manifolds, polytopes, convex bodies, packings, space decompositions, graphs, and geometric matroids.

By the end of the course, students should:
- Be able to recognize and formally express discrete geometric problems.
- Be able to discretize geometric problems, when possible.
- Be able to apply combinatorial techniques, as well as data structures and algorithms to discrete geometric problems.
- Be able to search the bibliography, and to understand the scientific literature on the subject.
- Be aware of the wide range of fields and problems to which discrete geometry results apply.
- Be aware of the most commonly used software in the field.

STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

Total learning time: 187.5 h

CONTENTS

Discrete Projective Geometry

Description:
Homogeneous coordinates. Incidences between points and hyperplanes. Infinitely far points. Linear, affine, and projective transformations

Specific objectives:
Work with finite and infinitely far points, and with parallel and non-parallel vectors and planes, without needing case-by-case distinction. Appreciate and apply the power and limitations of linear, affine and projective transformations.

Full-or-part-time: 19h 30m
Theory classes: 4h
Practical classes: 2h
Self study : 13h 30m

Convex polytopes

Description:
H/V polytopes; beneath-beyond convex hull computation; face lattice; constructions; extension complexity

Specific objectives:
Learn about the combinatorics and geometry of general convex polytopes and special families thereof. Inroduction to current areas of research

Full-or-part-time: 37h 15m
Theory classes: 8h
Practical classes: 4h
Self study : 25h 15m

Spectrahedra

Description:
Linear matrix inequalities. Projections of spectrahedra. SDP relaxations and vector programs. Convexity and faces of spectrahedra

Specific objectives:
After the convex piecewise-linear structure of convex polytopes, the next step up are convex piecewise-quadratic objects, namely spectrahedra. We will learn about different formulations to describe them and limitations of these, discuss their facial structure, and encounter some applications and open problems.

Full-or-part-time: 37h 15m
Theory classes: 8h
Practical classes: 4h
Self study : 25h 15m

Computational complexity and data structures

Description:
Computational complexity. Data structures. Representation of geometric objects.

Full-or-part-time: 12h 30m
Theory classes: 4h
Self study : 8h 30m

Convexity in low dimensions

Description:
Convex hull computation. Linear programming in low dimensions.

Full-or-part-time: 19h
Theory classes: 6h
Self study : 13h

Decompositions and arrangements

Description:
Subdivisions and triangulations of point sets and polygons. Visibility and motion planning. Duality. Special decompositions in dimension 2. The zone theorem. Incremental construction and randomized algorithms. Complexity. Levels and k-sets.

Full-or-part-time: 31h
Theory classes: 10h
Self study : 21h

Proximity Structures

Description:
Proximity problems. Voronoi diagram, Delaunay triangulation. Shape reconstruction.

Full-or-part-time: 31h
Theory classes: 10h
Self study : 21h

GRADING SYSTEM

The course consists in two parts, each contributes with 50 % to the final grade.
For each part: students will obtain marks by turning in their solutions to problems from the problem sets, by presenting solutions to problems or summarizing relevant research paper, or, optionally, by an exam.

BIBLIOGRAPHY

Basic:
- Berg, Mark de; Cheong, Otfried; Kreveld, Marc van; Overmars, Mark. Computational geometry : algorithms and applications [on line]. 3rd ed. revised. Berlin: Springer, 2008 [Consultation: 10/07/2023]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=6311912. ISBN 9783642096815.
- Boissonnat, J. D.; Yvinec, M. Algorithmic geometry. Cambridge: Cambridge University Press, 1997. ISBN 0521565294.
- Edelsbrunner, Herbert. Algorithms in combinatorial geometry. Berlin: Springer-Verlag, 1987. ISBN 354013722X.
- Matousek, Jirí. Lectures on discrete geometry. New York: Springer, 2002. ISBN 0387953736.
- Pach, János; Agarwal, Pankaj K. Combinatorial geometry. New York: John Wiley & Sons, cop. 1995. ISBN 0471588903.
- Ziegler, Günter M. Lectures on polytopes. New York: Springer-Verlag, 1995. ISBN 038794365X.

Complementary:
- Bokowski, Jürgen. Computational oriented matroids : equivalence classes of matrices within a natural framework. Cambridge: Cambridge University Press, 2006. ISBN 9780521849302.
- Conway, John Horton; Sloane, N. J. A. Sphere packings, lattices and groups. 3rd ed. Berlin: Springer, 1999. ISBN 0387985859.
- Schurmann, Achill. Computational geometry of positive definite quadratic forms : polyhedral reduction theories, algorithms, and applications. Providence: American Mathematical Society, 2009. ISBN 9780821847350.
- Weeks, Jeffrey R. The Shape of space. 2nd. ed. New York: M. Dekker, 2002. ISBN 0824707095.
- Richter-Gebert, Jürgen. Perspectives on projective geometry : a guided tour through real and complex geometry [on line]. Berlin: Springer, 2011 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-642-17286-1. ISBN 9783642172854.
- Beck, Matthias; Robins, Sinai. Computing the continuous discretely : integer-point enumeration in polyhedra [on line]. New York: Springer, cop. 2007 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-46112-0. ISBN 9780387291390.
- Edelsbrunner, Herbert; Harer, John. Computational Topology : an introduction. American Mathematical Society, cop. 2010. ISBN 9780821849255.
- Rabadán, Raúl; Blumberg, Andrew J. Topological data analysis for genomics and evolution. Cambridge University Press, 2020. ISBN 9781107159549.

RESOURCES

Audiovisual material:
- Mathfilm festival 2008 [Enregistrament vídeo]: a collection of mathematical videos. Berlin : Springer, 2008. https://discovery.upc.edu/discovery/fulldisplay?docid=alma991003546329706711&context=L&vid=34CSUC_UPC:VU1&lang=ca- Videomath Festival at International Congress of Mathematicians, Berlin, Germany 1998 [Enregistrament vídeo] / edited and produced Hans Christian Hege, Konrad Polthier. [Berlin] : Springer, 1998. https://discovery.upc.edu/discovery/fulldisplay?docid=alma991001865709706711&context=L&vid=34CSUC_UPC:VU1&lang=ca- Not knot [Enregistrament vídeo] / directed by Charlie Gunn and Delle Maxwell ; [written by David Epstein ... [et al.]]. Minnesota : Geometry Center, University of Minnesota, 1991. https://discovery.upc.edu/discovery/fulldisplay?docid=alma991001445799706711&context=L&vid=34CSUC_UPC:VU1&lang=ca- Flatland [Enregistrament vídeo] : a journey of many dimensions / written by Seth Caplan, Dano Johnson, Jeffrey Travis ; directed by Jeffrey Travis, Dano Johnson. [S.l.] : Flat World Productions, cop. 2007. https://discovery.upc.edu/discovery/fulldisplay?docid=alma991003683649706711&context=L&vid=34CSUC_UPC:VU1&lang=ca

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

Course guide
34957 - GT - Graph Theory

Last modified: 23/06/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: Perarnau Llobet, Guillem

Others:

PRIOR SKILLS

Elementary Calculus and Linear Algebra; basic notions and skills in combinatorics and probability.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Combination of theoretical lectures and exercise classes, with student presenting their solutions to the proposed problems. The active participation of students is a requirement for the course assessment.

LEARNING OBJECTIVES OF THE SUBJECT

Basics of Graph Theory.
Random graphs.
Applications of random graphs: the probabilistic method.
Spectral techniques to the study of graphs.
Applications of spectral techniques: expansion, clustering, embedding and random walks.
Extremal graph theory
Applications of extremal techniques: graph limits.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

An introduction to Graph Theory

Description:
Introduction to random graphs, main properties and classical theorems.

Specific objectives:
Basic terminology and notation
Paths and cycles
Distance and Diameter
Connectivity
Trees
Matchings
Colorings

Related competencies :
MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.
MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
06 URI. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.
05 TEQ. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
03 TLG. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
04 COE. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
07 AAT. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

Full-or-part-time: 12h
Theory classes: 12h

Random graphs

Description:
Introduction to classical model of random graphs and its main combinatorial properties.

Specific objectives:
Erdos-Rényi model of random graphs.
Properties of almost all graphs.
First and second moment methods.
The probabilistic method.
Threshold functions.
Method of Moments
Chernoff's inequality
Component phase transitions

Related competencies :
MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.
MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
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07 AAT. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

Full-or-part-time: 12h
Theory classes: 12h

Spectral Graph Theory

Description:
Introduction to spectral graph theory, graph expanders and applications to clustering, embedding and random walks on graphs.

Specific objectives:
Adjacency and Laplacian matrices. Courant-Fischer Theorem. Interlacing. Cospectral graphs. Graph invariants: independence number, chromatic number, diameter,... Spectral gap and second largest eigenvalue. Isoperimetry and Cheeger's inequality. Matrix tree theorem. Clustering graph algorithm. Graph embedding and drawing algorithms. Random walks in graphs.

Related competencies :
MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.
MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
06 URI. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.
05 TEQ. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
03 TLG. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
04 COE. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
07 AAT. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

Full-or-part-time: 16h
Theory classes: 16h

Extremal Graph Theory

Description:
Introduction to Extremal Graph Theory, the Szemeredi Regularity Lemma, and applications to Graph limits

Specific objectives:
Mantel Theorem, Turán Theorem, Erdos-Stone-Simonovits Theorem, Szemeredi Regularity Lemma, Counting Lemma, Triangle Removal Lemma, Graph Limits

Full-or-part-time: 20h
Theory classes: 20h

GRADING SYSTEM

The assessment of the course is as follows:
- weekly work on the proposed problems and their presentation during the lectures, 30% of the mark
- a final comprehensive exam on the course topics, 70% of the mark

EXAMINATION RULES.

The active participation in the course is a requirement for the final assessment.

BIBLIOGRAPHY

Basic:

- Diestel, Reinhard. Graph theory. 3rd ed. Berlin: Springer, 2005. ISBN 3540261826.
- Hell, Pavol; Nesetril, Jaroslav. Graphs and homomorphisms. Oxford: Oxford University Press, 2004. ISBN 0198528175.
- Frieze, Alan; Karonski, Michal. Introduction to random graphs [on line]. Cambridge, [etc.]: Cambridge University Press, cop. 2016 [Consultation: 10/07/2023]. Available on: https://www-cambridge-org.recursos.biblioteca.upc.edu/core/books/introduction-to-random-graphs/0F67A19795B731B0C97EAB5BB5748CF2. ISBN 9781107118508.
- Brouwer, A. E; Haemers, W. H. Spectra of graphs [on line]. New York [etc.]: Springer, cop. 2012 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-1-4614-1939-6. ISBN 9781461419389.
- Alon, Noga; Spencer, Joel. The Probabilistic method. 2016. Wiley, 2015. ISBN 9781119061953.

Complementary:

- Lovász, László. Large networks and graph limits. Providence, R.I.: American Mathematical Society, cop. 2012. ISBN 9780821890851.
- Chung, Fan R. K. Spectral graph theory. Providence: American Mathematical Society, 1997. ISBN 0821803158.
- Biggs, Norman L. Algebraic graph theory. 2nd ed. New York: Cambridge University Press, 1993. ISBN 0521458978.

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

Course guide
34962 - HS - Hamiltonian Systems

Last modified: 29/05/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: PAU MARTIN DE LA TORRE

Others:

PRIOR SKILLS

Knowledge of calculus, algebra and ordinary differential equations.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Standard exposition in front of the blackboard, resolution of exercices, completion of a project and attendance to the JISD summer school http://www.ma1.upc.edu/recerca/jisd

LEARNING OBJECTIVES OF THE SUBJECT

To comprehend the basic foundations of the theory of Hamiltonian systems, and to understand its applications to Celestial Mechanics and other fields.

STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

Total learning time: 187.5 h

CONTENTS

Hamiltonian formalism

Description:
Hamiltonian dynamical systems: symplectic maps, symplectic manifolds. Linear Hamiltonian systems and their application to the study of stability of equilibrium points. Canonical transformations.

Full-or-part-time: 28h
Theory classes: 10h
Self study : 18h

Celestial mechanics

Description:
The two body problem, first integrals. Resolution. The three body problem, different coordinates. The restricted three body problem. Central configurations. Periodic orbits, invariant manifolds.

Full-or-part-time: 34h
Theory classes: 12h
Self study : 22h

Geometric theory and invariant objects of Hamiltonian systems

Description:
Continuous and discrete dynamical systems, Poincaré map. Flow box Theorem. Noether Theorem. Periodic orbits. Continuation of periodic orbits. Lyapunov Center Theorem.

Full-or-part-time: 24h
Theory classes: 8h
Self study : 16h

Integrable systems

Description:
Complete integrability and Liouville-Arnold theorem. Action-Angle coordinates. Quasi-periodic flows on a torus, resonances.

Full-or-part-time: 10h
Theory classes: 4h
Self study : 6h

Quasi-integrable Hamiltonian systems

Description:
Examples of quasi-integrable systems. Small divisors and Diophantine inequalities. Averaging Theory. Lie Method. KAM Theory (Kolmogorov-Arnold Moser). Effective stability and Nekhoroshev theorem. Melnikov Potential. Arnold diffusion.

Full-or-part-time: 26h
Theory classes: 8h
Self study : 18h

Lagrangian systems and variational methods

Description:
Lagrangian systems. Legendre transformation. Principle of minimal action. Twist maps. Existence of periodic orbits. Aubry-Mather Theory.

Full-or-part-time: 12h
Theory classes: 4h
Self study : 8h

Hamiltonian Partial Differential Equations

Description:
Linear Hamiltonian Partial Differential Equations. Examples. Periodic, quasi-periodic and almost-periodic solutions. Nonlinear Hamiltonian Partial Differential Equations. Lyapunov stability/instability of invariant objects. Transfer of energy.

Full-or-part-time: 4h
Theory classes: 2h
Self study : 2h

- Interactions between Dynamical Systems and Partial Differential Equations

Description:
Summer School and Research workshop on topics between Dynamical Systems and Partial Differential Equations

Full-or-part-time: 49h 30m
Theory classes: 12h
Self study : 37h 30m

ACTIVITIES

JISD summer school

Description:
Attendance to the JISD summer school

Specific objectives:
To learn from oustanding researchers a view of the state of the art in several research topics, interacting with students of the rest of Spain and of the World.

GRADING SYSTEM

The students have to do some problems (60%) and a research work (25%). There will be also a final exam covering on the theoretical part of the subject (15%). Moreover, they will attend the JISD.

BIBLIOGRAPHY

Basic:
- Meyer, Kenneth R.; Hall, Glen R.; Offin, Dan. Introduction to Hamiltonian dynamical systems and the n-body problem [on line]. 2nd ed. New York: Springer-Verlag, 2009 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-09724-4. ISBN 9780387097237.
- Arnold, V. I.; Kozlov, Valerii V.; Neishtadt, Anatoly I. Mathematical aspects of classical and celestial mechanics [on line]. 3rd ed. Berlin: Springer-Verlag, 2006 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-540-48926-9. ISBN 3540282467.
- Treschev, Dmitry; Zubelevich, Oleg. Introduction to the perturbation theory of Hamiltonian systems [on line]. Berlin: Springer Verlag, 2010 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-642-03028-4. ISBN 9783642030277.
- Celletti, Alessandra. Stability and chaos in celestial mechanics [on line]. Springer-Praxis, 2010 [Consultation: 10/07/2023]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=993277. ISBN 9783540851455.
- Wintner, Aurel. The Analytical foundations of celestial mechanics. Dover Publications, ISBN 978-0486780603.
- Katok, Anatole; Hasselblatt, Boris. Introduction to the modern theory of dynamical systems. Cambridge [etc.]: Cambridge University Press, 1997. ISBN 9780521575577.
- Berti, Massimiliano. Nonlinear oscillations of Hamiltonian PDEs [on line]. Boston, MA: Birkhäuser Boston, Inc, 2007 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-8176-4681-3. ISBN 9780817646806.
- Marsden, Jerrold E; Ratiu, Tudor S. Introduction to mechanics and symmetry : a basic exposition of classical mechanical systems. 2a ed. New York [etc.]: Springer, 1999. ISBN 9780387986432.
- Kanuf, Andreas. Mathematical physics : classical mechanics. 1. Springer-Verlag, 2018. ISBN 9783662557723.

RESOURCES

Hyperlink:
- Grup de sistemes dinàmicshttps://recerca.upc.edu/sd. Pàgina web del Grup de Sistemes Dinàmics de la UPC on es descriuen diversos projectes i els investigadors que hi treballen així com diverses activitats relacionades

Course guide
200900 - ML - Machine Learning

Last modified: 18/06/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 748 - FIS - Department of Physics.
749 - MAT - Department of Mathematics.
715 - EIO - Department of Statistics and Operations Research.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).
MASTER'S DEGREE IN STATISTICS AND OPERATIONS RESEARCH (Syllabus 2013). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: PEDRO FRANCISCO DELICADO USEROS

Others:

PRIOR SKILLS

The student should have knowledge of fundamental mathematical and statistical topics, such as linear algebra, calculus, probability distributions, optimization algorithms and basic multivariate statistical methods.

REQUIREMENTS

The student should have knowledge of basic machine learning concepts. These concepts can be acquired simultaneously, for example being enrolled in the "Statistical Learning" subject offered in the MESIO master.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

TEACHING METHODOLOGY

On-Site Learning: Learning will be organized into sessions that combine theory and practice. All sessions will be held in a standard classroom. Students should bring their own laptops to class. Lectures will consist of 75% expository instruction and 25% guided practical work. During the expository portion, theoretical aspects will be presented and discussed alongside practical examples using slides that will be provided to students in advance. The practical portion of the sessions will primarily use R and/or Python environments. Students should have intermediate knowledge of these environments and basic programming skills.

Off-site learning consists of studying and solving problems and real-world cases that students should submit throughout the course. Some of these exercises will require students to complete programming tasks in R or Python and prepare short reports using R Markdown (or a similar tool). In some cases, students will be required to give an oral presentation on their work in class.

LEARNING OBJECTIVES OF THE SUBJECT

Upon completion of the course, students should have acquired advanced competencies in general topics and techniques for unsupervised and supervised machine learning. In unsupervised learning, the course covers density estimation, clustering, and dimensionality reduction. Supervised learning topics include Bayesian methods, kernel-based learning, support vector machines, feedforward neural networks, recurrent neural networks, and reinforcement learning.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Introduction to the course

Description:
Overview of the course objectives, structure, and main topics. Introduction to the importance of Machine Learning and its applications in various fields.

Full-or-part-time: 1h
Theory classes: 1h

Introduction to unsupervised learning

Description:
Introduction to key concepts in unsupervised learning. Focus on density estimation, clustering techniques, and dimensionality reduction.

Topics:
a. Introduction to the basic concepts of unsupervised learning.
b. Density estimation.
c. Clustering techniques.

Full-or-part-time: 8h
Theory classes: 8h

Dimensionality reduction

Description:
Advanced dimensionality reduction techniques for high-dimensional and infinite-dimensional datasets. Applications in visualization and exploratory analysis.

Topics:
a. Classical Dimensionality Reduction.
- Principal Component Analysis (PCA).
- Multidimensional Scaling (MDS).
b. Nonlinear Dimensionality Reduction.
- Principal curves.
- Local Multidimensional Scaling.
- ISOMAP.
- t-Stochastic Neighbor Embedding.

Full-or-part-time: 9h
Theory classes: 9h

Bayesian Methods

Description:
(Theory) Introduction to Bayesian thinking for machine learning. Concepts of Bayesian inference, including conjugate probability distributions, Bayesian linear regression, and Bayesian networks. Illustrative examples of probabilistic modeling and model comparison.
(Practice) Hands-on Bayesian inference using known software libraries for visualizing prior and posterior distributions and for Bayesian regression. Implementing the Bayesian regression solution and comparing it with the maximum likelihood estimator. Applications to real-world data.

Topics:
a. Bayesian Thinking and Inference.
b. Bayesian linear regression.
c. Bayesian Networks.

Full-or-part-time: 6h
Theory classes: 6h

Learning in Functional Spaces

Description:
(Theory) Exploration of reproducing kernel Hilbert spaces. The representer theorem and practical examples of kernel-based learning.
(Practice) Implementing Kernel Ridge Regression from scratch or using known libraries and exploring the effect of different kernels (linear, polynomial, Gaussian).

Topics:
a. Reproducing kernel Hilbert spaces (RKHS).
b. The representer theorem.
c. Kernel ridge regression as an example of kernel-based learning and comparison with Bayesian linear regression.

Full-or-part-time: 4h
Theory classes: 4h

Support Vector Machines

Description:
Exploration of support vector machines (SVM) as a flagship kernel method. Focus on classification, regression, and novelty detection.

Topics:
a. Introduction to SVMs: concept of separating hyperplanes and maximum margin.
b. Kernel trick: transforming data into higher-dimensional spaces.
c. Soft margin classification.

Full-or-part-time: 4h
Theory classes: 4h

Feedforward Neural Networks

Description:
Feedforward Neural Networks
Introduction to feedforward neural networks, including multi-layer Perceptrons and convolutional neural networks. Key aspects such as backpropagation, regularization, and examples are covered.

Topics:
a. Multi-layer Perceptron (MLP).
b. Backpropagation and regularization.
c. Convolutional Neural Networks (CNN).

Full-or-part-time: 6h
Theory classes: 6h

Recurrent Models

Description:
Recurrent Models
Exploration of sequential data modeling using recurrent neural networks (RNN), including advanced architectures such as LSTMs, GRUs, and transformers.

Topics:
a. Recurrent Neural Networks (RNN).
b. LSTM and GRU layers.
c. Interpretation of the networks.
d. Transformers as an alternative to RNN.

Full-or-part-time: 8h
Theory classes: 8h

Reinforcement Learning

Description:
Introduction to reinforcement learning (RL) and its foundational concepts, focusing on decision-making models and their mathematical underpinnings.

Topics:
a. Introduction to RL: basic principles and agent-environment interactions.
b. Dynamic Programming. Bellman equations: recursive relationships in value estimation.
c. Monte Carlo Methods.
d. Temporal-Difference Learning: Q-learning and SARSA algorithms.

Full-or-part-time: 6h
Theory classes: 6h

GRADING SYSTEM

The grading method will be based on two marks:

1) Practical work completed throughout the course: 40%.
2) Final exam: 60%.

The practical work consists of studying and solving problems and real-world cases, which students should submit throughout the course. Some of these exercises will require students to complete programming tasks in R or Python and prepare short reports using R Markdown or a similar tool. In some cases, students will be required to give an oral presentation on their work in class. The practical work can be done in groups, and the format will be specified onsite. However, the exam must be completed individually.

EXAMINATION RULES.

The precise format for the exam will be specified with sufficient advance. It may include restrictions on the allowed knowledge sources, such as written notes, books, internet connection, etc.

BIBLIOGRAPHY

Basic:
- Bishop, Christopher M. Pattern recognition and machine learning. New York: Springer, cop. 2006. ISBN 9780387310732.
- Goodfellow, Ian; Bengio, Yoshua; Courville, Aaron. Deep learning [on line]. 2016 [Consultation: 19/06/2025]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=6287197. ISBN 9780262035613.
- Hastie, Trevor; Tibshirani, Robert; Friedman, Jerome. The Elements of statistical learning : data mining, inference, and prediction [on line]. 2nd ed. New York, NY: Springer New York, 2009 [Consultation: 19/06/2025]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-84858-7. ISBN 9780387848587.
- Kung, S. Y. Kernel methods and machine learning [on line]. Cambridge University Press, 2014 [Consultation: 19/06/2025]. Available on: https://www-cambridge-org.recursos.biblioteca.upc.edu/core/books/kernel-methods-and-machine-learning/4B52092A98E1553A26EB5271D832D29E. ISBN 9781107024960.
- Murphy, Kevin P. Probabilistic machine learning: advanced topics. Cambridge: The MIT Press, 2023. ISBN 9780262048439.
- Schölkopf, Bernhard; Smola, Alexander J. Learning with Kernels : support vector machines, regularization, optimization, and beyond. Cambridge ; London: The MIT Press, 2002. ISBN 9780262194754.
- Sutton, Richard S.; Barto, Andrew G. Reinforcement learning : an introduction [on line]. Second edition. Cambridge, Massachusetts: The MIT Press, 2020 [Consultation: 19/06/2025]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=6260249. ISBN 9780262039246.
- Vapnik, Vladimir Naumovich. The Nature of statistical learning theory. 2nd ed. New York ; Barcelona [etc.]: Springer, 2000. ISBN 9780387987804.

Complementary:

- Smola, Alexander J. Advances in large margin classifiers [on line]. Cambridge, Mass: MIT Press, 2000 [Consultation: 19/06/2025]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=5966303. ISBN 9780262194488.

Course guide
34958 - MMPDE - Mathematical Modelling with Partial Differential Equations

Last modified: 23/06/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.
751 - DECA - Department of Civil and Environmental Engineering.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: MATTEO GIACOMINI

Others:

PRIOR SKILLS

Good knowledge of calculus techniques, including integral theorems. Elementary solution of ODEs and PDEs.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

The course mainly consists of theoretical lectures, but it also includes problem solving and computer sessions, with numerical codes provided to illustrate the behaviour of the models.

LEARNING OBJECTIVES OF THE SUBJECT

Many phenomena taking place in physical and engineering systems can be described by means of systems of partial differential equations (PDEs). This is, e.g., the case of heat transfer phenomena, contaminant transport, fluid dynamics, mechanics of complex media (inhomogeneous materials, metamaterials, …), and wave propagation, just to name a few. The module provides a general overview on the use of PDEs to construct mathematical models of physical phenomena and engineering systems.

The module aims to provide a first introduction to the mathematical tools to model physical phenomena, rigorously setting the governing equations describing the engineering systems under analysis. By the end of the module, students are expected to be able to:
- describe basic physical phenomena using PDEs;
- provide intuitive interpretation of the mathematical operators appearing in a PDE;
- predict the physical behaviour of an engineering system in view of its mathematical description using PDEs.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Introduction

Description:
Motivation and examples. Review of fundamental mathematical tools for the module. Primal, dual, and mixed formulations.

Full-or-part-time: 6h
Theory classes: 6h

Potential theory

Description:
Potential energy. Classic gravitation. Electrostatics. Minimal surfaces.

Full-or-part-time: 6h
Theory classes: 6h

Potential flows

Description:
Ideal flows. Complex analysis theory. Conformal maps.

Full-or-part-time: 12h
Theory classes: 12h

Convection-diffusion-reaction phenomena

Description:
Fourier and Fick laws. Heat and diffusion equations. Random walks. Convection-diffusion-reaction equations. Turing patterns.

Full-or-part-time: 12h
Theory classes: 12h

Continuum mechanics

Description:
Conservation laws. Constitutive laws. Elasticity. Viscoelasticity.

Full-or-part-time: 6h
Theory classes: 6h

Fluid dynamics

Description:
Viscous incompressible flows. Highly viscous flows. Inviscid compressible flows.

Full-or-part-time: 12h
Theory classes: 12h

Porous media flow

Description:
Heterogeneous materials. Homogenisation theory. Darcy’s law.

Full-or-part-time: 6h
Theory classes: 6h

GRADING SYSTEM

The grade of the course is obtained from a continuous assessment during the module. This consists of several activities of incremental training, carried out during the module, both in and out of the classroom.

The final grade will be computed as follows:
- 35% exam 1;
- 35% exam 2;
- 30% continuous evaluation (assignments and exercises).

EXAMINATION RULES.

The assignments must be submitted via ATENeA by the announced deadline. Late submissions or assignments submitted using other means will not be accepted and will be graded 0. The assignments must be performed individually: students are encouraged to discuss about the assignments but the submitted work must be the result of one own effort. Plagiarism in the assignments will be punished with a 0 in the classwork grade.

The written exams must be performed individually and will be closed-book. Plagiarism will be punished with a 0 in the module grade.

BIBLIOGRAPHY

Basic:

- Salsa, Sandro. Partial differential equations in action : from modelling to theory [on line]. Milan [etc.]: Springer, cop. 2008 [Consultation: 07/07/2023]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=3062992. ISBN 9788847007512.
- Howison, Sam. Practical applied mathematics : modelling, analysis, approximation. New York: Cambridge University Press, 2005. ISBN 0521603692.
- Ockendon, J.R. [et al.]. Applied partial differential equations. Revised ed. Oxford: Oxford University Press, 2003. ISBN 0198527713.
- Witelsky, T.; Bowen, M. Methods of mathematical modelling. Cham (Switzerland): Springer, 2015. ISBN 9783319230412.
- Chorin, A. J.; Marsden, J.E. A Mathematical introduction to fluid mechanics. 3rd ed. Springer, ISBN 978-0387979182.

Complementary:

- Friedman, A.; Litman, W. Industrial mathematics : a course in solving real-world problems. Philadelphia: SIAM, 1994. ISBN 0898713242.
- Fowler, A.C. Mathematical models in the applied sciences. Cambridge: Cambridge University Press, 1997. ISBN 0521467039.
- Dacorogna, Bernard. Introduction to the calculus of variations [on line]. 2015 [Consultation: 07/07/2023]. Available on: https://www-worldscientific-com.recursos.biblioteca.upc.edu/worldscibooks/10.1142/p967#t=toc. ISBN 9781783265534.
- LeVeque, Randall J. Finite difference methods for ordinary and partial differential equations : steady state and time-dependent problems [on line]. SIAM, 2007 [Consultation: 07/07/2023]. Available on: https://faculty.washington.edu/rjl/fdmbook/. ISBN 9780898716290.

RESOURCES

Other resources:

Teaching material including slides, class notes, lists of exercises are available on ATENeA.

Course guide
34960 - MMB - Mathematical Models in Biology

Last modified: 03/06/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: Lazaro Ochoa, Jose Tomas

Others:

PRIOR SKILLS

* Proficiency in undergraduate mathematics: calculus, algebra, probability and statistics.
* Ability to perform basic operations in linear algebra: eigenvalues and eigenvectors, computation of determinants, rank of matrices...
* Ability to analyze and solve linear differential equations and discuss the stability of simple vector fields.
* Interest towards biological applications of mathematics and/or previous working experience.

REQUIREMENTS

* Basic knowledge of undergraduate mathematics: calculus, ordinary differential equations, linear algebra, probability and statistics.
* First course in ordinary differential equations: linear differential equations, qualitative and stability theory and numerical simulation.
* Basic knowledge of computer programming for scientific purposes.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

The course will be structured in five blocks each consisting of a brief introduction through theoretical lectures, the development of a short project in groups and wrap-up sessions with oral presentations, discussion and complementary lectures. The central part intended to develop the short project will held at the computer lab.

LEARNING OBJECTIVES OF THE SUBJECT

This course is an introduction to the most common mathematical models in biology: in populations dynamics, ecology, neurophysiology, sequence analysis and phylogenetics. At the end of the course the student should be able to:

* Understand and discuss basic models of dynamical systems of biological origin, in terms of the parameters.
* Model simple phenomena, analyze them (numerically and/or analytically) and understand the effect of parameters.
* Understand the diversity of mechanisms and the different levels of modelization of physiological activity.
* Obtain and analyze genomic sequences of real biological species and databases containing them.
* Use computer software for gene prediction, alignment and phylogenetic reconstruction.
* Understand different gene prediction, alignment and phylogenetic reconstruction methods.
* Compare the predictions given by the models with real data.
* Communicate results in interdisciplinary teams.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Models of Population Dynamics

Description:
1.- Short introduction to epidemiology.
2.- Dynamics of a single species, including a short review on bifurcation theory.
3.- Dynamics of interacting species (competition, cooperation, predator-prey, Lotka-Volterra models)
4.- Short introduction to evolutionary game theory.

Full-or-part-time: 37h 30m
Theory classes: 10h
Laboratory classes: 2h
Self study : 25h 30m

Mathematical Models in Neuroscience

Description:
1. Membrane biophysics.
2. Excitability and action potentials: the Hodgkin-Huxley model, the Morris-Lecar model, integrate & fire models.
3. Bursting oscillations.
4. Synaptic transmission and dynamics.
5. Neuronal networks. Firing rate models. Wilson-Cowan equations.
6. Applications to cognitive tasks (working memory, decision making and visual perception).
7. Recurrent Neural Networks.
8. Statistical Models.

Full-or-part-time: 75h
Theory classes: 12h
Laboratory classes: 12h
Self study : 51h

Mathematical models in phylogenetics

Description:
1. Brief introduction to genomics and phylogenetics (genome, gen structure, alignments, evolution of species...). Retrieving genomic sequences and alignments.
2. Markov models of molecular evolution (Jukes-Cantor, Kimura, Felsenstein hierarchy...), evolutionary distances, phylogenetic trees.
3. Phylogenetic tree reconstruction: distance and character-based methods.
4. Genomics: Markov chains and Hidden Markov models for gene prediction. Tropical arithmetics and Viterbi algorithm. Forward and Expectation-Maximization algorithms. Multiple sequence alignment.

Full-or-part-time: 75h
Theory classes: 12h
Laboratory classes: 12h
Self study : 51h

GRADING SYSTEM

50%: Each of the five blocks will give a part (10%) of the qualification, based on the perfomance on the short-projects.
20%: Overall evaluation of the participation, interest and proficiency evinced along the course.
30%: Final exam aiming at validating the acquisition of the most basic concepts of each block.

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BIBLIOGRAPHY

Basic:

- Allman, Elizabeth S.; Rhodes, John A. Mathematical models in biology : an introduction. Cambridge: Cambridge University Press, 2004. ISBN 9780521819800.
- Ermentrout, Bard G.; Terman, David H. Mathematical foundations of neuroscience [on line]. New York: Springer, 2010 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-87708-2. ISBN 9780387877082.
- Istas, Jacques. Mathematical modeling for the life sciences [on line]. Berlin: Springer, 2005 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/3-540-27877-X. ISBN 354025305X.
- Izhikevich, Eugene M. Dynamical systems in neuroscience : the geometry of excitability and bursting. Cambridge: MIT Press, 2007. ISBN 0262090430.
- Keeling, Matthew J.; Rohani, Pejman. Modeling infectious diseases in humans and animals. Princeton: Princeton University Press, cop. 2008. ISBN 9780691116174.
- Murray, J.D. Mathematical biology, vol. 1 [on line]. 3rd ed. Berlin: Springer, 2002 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/b98868. ISBN 9780387952239.

Complementary:

- Pachter, Lior; Sturmfels, Bernd. Algebraic statistics for computational biology. Cambridge: Cambridge University Press, 2005. ISBN 0521857007.
- Durbin, Richard [et al.]. Biological sequence analysis : probabilistic models of proteins and nucleic acids [on line]. Cambridge: Cambridge University Press, 1998 [Consultation: 10/07/2023]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=320915. ISBN 0521629713.
- Feng, Jianfeng. Computational neuroscience : a comprehensive approach [on line]. Boca Raton: Chapman & Hall/CRC, 2004 [Consultation: 10/07/2023]. Available on: http://nba.uth.tmc.edu/homepage/cnjclub/2007summer/renart_chapter.pdf. ISBN 1584883626.
- Keener, James P.; Sneyd, James. Mathematical physiology. Vol 1 [on line]. 2nd ed. New York: Springer Verlag, 2009 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-75847-3. ISBN 9780387758473.
- Miller, Paul. An Introductory course in computational neuroscience. Cambridge, Masachusetts: The MIT Press, [2018]. ISBN 9780262038256.
- Stein, William A. [et al.]. Sage mathematics software (Version 4.4.2) [on line]. 2010 [Consultation: 10/07/2023]. Available on: http://www.sagemath.org/.
- Bacaër, N.; Bravo de la Parra, R.; Ripoll, J. Breve historia de los modelos matemáticos en dinámica de poblaciones [on line]. [Consultation: 10/07/2023]. Available on: https://rafaelbravodelaparra.web.uah.es/Breve%20historia%20de%20los%20modelos%20matem%C3%A1ticos%20en%20din%C3%A1mica%20de%20poblaciones-N%20Bacaer.pdf. ISBN 9791034365883.
- Steel, Mike. Phylogeny: Discrete and random processes in evolution. CBMS-NSF Regional conference series in Applied Mathematics, SIAM, 2016. ISBN 9781611974478.
- Felsenstein, Joseph. Inferring phylogenies. Sunderland, Massachusetts: Sinauer Associates, cop. 2004. ISBN 9780878931774.

Course guide
34951 - NCA - Non-Commutative Algebra

Last modified: 29/05/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: ENRIC VENTURA CAPELL

Others:

PRIOR SKILLS

The concept of group and subgroup, and the concept of homomorphism. Basic algebraic properties, binary operations and their properties. Equivalence relations and related set-theoretic properties.

REQUIREMENTS

The basic algebra courses from the degree in mathematics.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Classes follow the traditional structure of lecture by the professor, together with the assignment of problems and exercises for the students to solve and present, either in written or in oral form.

LEARNING OBJECTIVES OF THE SUBJECT

The main goal is to introduce the student into the basic ideas and techniques of non-commutative algebra, to the extend of being able to enroll himself/herself into some initial research project in the area, if there is interest to do so.

Non-commutative algebra plays a significant role in the research panorama in modern mathematics and students of any degree in mathematics have been introduced to it. The main goal of the present topic is to go a bit deeper into this area of mathematics, offering a general but consistent introduction into the topic.

We'll center our attention towards the so-called "Geometric Group Theory", a relatively young and very active research area. This election is done because it allows to go, within a full semester, from the basics of the theory to the description, with a good level of details and context, of some open problems that are currently being object of active research today.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Generalities about infinite groups

Description:
The free group: basic definitions.
Presentations: generators and relations.
Short exact sequences, direct and semidirect products.
Free products, amalagams, HNN extensions.

Full-or-part-time: 45h
Theory classes: 15h
Self study : 30h

Cayley graphs and growth of groups

Description:
Cayley graph and growth of a group
Quasi-isometries, geometric properties
Growth of groups: polynomial, intermediate, exponential, uniformly exponential
Gromow theorem

Full-or-part-time: 45h
Theory classes: 15h
Self study : 30h

Subgroups of free groups

Description:
- Nielsen-Schreier theorem
- Stallings graphs
- Main properties of the lattice of subgroups of a free group
- Finite index subgroups
- Pull-back and intersection of subgroups.

Full-or-part-time: 45h
Theory classes: 15h
Self study : 30h

Algorithmic problems in groups

Description:
The three classical algorithmic problems in group theory: word, conjugacy and isomorphism problems.
Resolution in simple cases: abelian, free, free-like constructions, residually finite, etc.
Tietze transformations, an attack to the isomorphism problem
Some unsolvability results: Novikov, Miller, Mihailova, etc.

Full-or-part-time: 45h
Theory classes: 15h
Self study : 30h

GRADING SYSTEM

Students will have to present in written and/or oral form some exercises assigned along the development of the course. At the end of the course, there will be a written exam with two parts: (a) the development of a theoretical topic chosen by the teachers among a closed public list of topics about the contents of the course (70%); and (b) a short list of problems/exercises about the topics of the course (30%).

BIBLIOGRAPHY

Basic:
- Rotman, Joseph. An Introduction to the theory of groups. 4th ed. New York: Springer, 1995. ISBN 0387942858.
- Lyndon, Roger C.; Schupp, Paul E. Combinatorial group theory. 2nd ed. Berlin: Springer, 2001. ISBN 9783540411581.
- Bogopolskij, Oleg Vladimirovic. Introduction to group theory. Zürich: European Mathematical Society, 2008. ISBN 9783037190418.
- Loh, Clara. Geometric group theory : an introduction [on line]. Universitext. Springer, 2017 [Consultation: 10/07/2023]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=6312482. ISBN 9783319722535.

Complementary:
- Epstein, David B.A. [et al.]. Word processing in groups. Boston: Jones and Bartlett, 1992. ISBN 9780867202441.
- Ghys, E.; La Harpe, P. de. Sur les groupes hyperboliques d'après Mikhael Gromov. Boston: Birkhäuser, 1990. ISBN 9780817635084.

RESOURCES

Other resources:
Several interesting papers and notes by Chuck Miller:

https://researchers.ms.unimelb.edu.au/ cfm/papers

Course guide
34953 - NT - Number Theory

Last modified: 23/06/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: JOAN CARLES LARIO LOYO

Others:

PRIOR SKILLS

Basic knowledge of algebraic structures: groups, rings and fields.

REQUIREMENTS

Basic material covered in any standard courses on arithmetic, group theory and Galois theory.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Most of the lectures will take place on the blackboard (replaced by online lessons if necessary), explaining carefully the contents of the course and providing as much explicit examples, exercises and applications as possible. The students will be encouraged to consult suitable references and to discuss between them and with the professor in order to achieve a good understanding of the material.

LEARNING OBJECTIVES OF THE SUBJECT

1) Algebraic number theory.
2) Number theory in function fields.
2) Cyclotomic theory.

STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

Total learning time: 187.5 h

CONTENTS

Algebraic Number Theory

Description:
Chapter 1: p-adic Arithmetic

1.1. Introduction to p-adic Numbers

Motivation: Why study p-adics?
Construction of â□¤â□□ and â□□â□□ via p-adic expansions.
Examples and Software.

1.2. Arithmetic in â□□â□□

Basic operations (addition, multiplication, inverses).
Algebraic structure: â□¤â□□ as a DIP
Hensel's Lemma
Multiplicative structure: Units in â□¤â□□ and p-adic exponents.

Chapter 2: Valuations and Local Fields

2.1. Absolute Values and Completions

p-adic valuations and the ultrametric inequality.
Completing â□□ to get â□□â□□.
Ostrowski's Theorem.

2.2. Topology of â□□â□□

Metric space properties (totally disconnected, compactness of â□¤â□□).
Comparison with real numbers (â□□ vs. â□□â□□).

2.3. Local Fields

Definition of local fields (finite extensions of â□□â□□ or ð□□½â□□((t))).
Structure of the valuation ring and residue field.

Chapter 3: Extensions of â□□â□□

3.1. Finite Extensions

Extending valuations (ramification index, inertia degree).
Norm and trace in p-adic fields.
Unramified and totally ramified extensions.
Krasner's Lemma.
Galois groups of p-adic extensions.

3.2. Algebraic Closures and â□□â□□

The algebraic closcar of â□□â□□ and its completion â□□â□□.

Chapter 4: Applications

4.1. Hasse-Minkowski Theorem

Quadratic forms over â□□â□□.
Local-global principle for â□□ (counterexamples and applications).

4.2. p-adic Analysis

p-adic integration (Haar measure, Volkenborn integral).
Zeta functions and p-adic L-functions (brief intro).

4.3. p-adic Galois Representations

Motivation from number theory (Fermat's Last Theorem).
Tate modules and â□□â□□-linear representations.

Full-or-part-time: 10h
Theory classes: 10h

Number Theory in function fields

Description:
Functions fields over finite fields
Carlitz polynomials
Carlitz extensions

Full-or-part-time: 10h
Theory classes: 10h

Cyclotomic Theory

Description:
Cyclotomic integers
Cyclotomic units
Unique factorization
Class numbers
Galois action
Kronecker-Weber theorem
Regular polygons
Fermat equation
Quadratic reciprocity

Carlitz modules
Galois action
Carlitz-Hayes theorem
Cyclotomic and Carlitz analogies
Quadratic reciprocity
Drinfeld modules

Full-or-part-time: 40h
Theory classes: 40h

GRADING SYSTEM

There will be a final exam.

EXAMINATION RULES.

Solved exercises and works must be delivered according to schedule. Final exam, if necessary.

BIBLIOGRAPHY

Basic:
- Serre, Jean Pierre. A Course in arithmetic. New York, 1973. ISBN 0387900403.
- Gouvêa, Fernando Q. P-adic Numbers: an introduction. Berlin, 1993.
- Neukirch, J. Algebraic number theory. Berlin, 1999. ISBN 3540653996.
- Koblitz, Neal. P-adic numbers, p-adic analysis, and zeta-functions. New York, 1996. ISBN 0387960171.
- Cassels, J.W.S. Local fields. Cambridge, 1986. ISBN 0521304849.
- Robert, A. A Course in p-adic analysis. New York, 2000. ISBN 0387986693.

RESOURCES

Computer material:
- SAGE. Mathematical Software
- Matlab. Mathematical software

Course guide
34964 - NMDS - Numerical Methods for Dynamical Systems

Last modified: 29/05/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: MARIA MERCEDES OLLE TORNER

Others:

PRIOR SKILLS

Good knowledge of a programming language.

REQUIREMENTS

Knowledge of theory of systems of differential equations, algebra, calculus and numerical analysis.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Theoretical sessions (presence of the students is necessary) and weekly practical tutorized assignments.

LEARNING OBJECTIVES OF THE SUBJECT

-To reach an advanced formation in using numerical methods applied to dynamical systems
- Carry out numerical simulations of particular examples
- To relate different aspects of the dynamics in order to have a global picture of the
behavior of a given problem
- To learn different tools to analyse and deal with a problem
- Ability in programming algorithms dessigned to solve particular problems in dynamical systems

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Numerical (preliminary) tools for practical purposes: integrators for ODE and graphical interfaces. Examples.

Full-or-part-time: 4h
Theory classes: 2h
Practical classes: 2h

Dynamical systems: introduction, definitions. Continuous and discrete dynamical systems. Orbit generation. Numerical computation of Poincare maps. Examples.

Full-or-part-time: 6h
Theory classes: 3h
Practical classes: 3h

Computation and stability of fixed points. Vector fields and maps. Implementation and examples.

Full-or-part-time: 10h
Theory classes: 5h
Practical classes: 5h

Computation of tori: representation, computation and continuation. Implementation and examples.

Full-or-part-time: 15h
Theory classes: 7h 30m
Practical classes: 7h 30m

Analysis of bifurcations. Some examples.

Full-or-part-time: 15h
Theory classes: 7h 30m
Practical classes: 7h 30m

GRADING SYSTEM

65% of the qualification will be obtained from the practical assignments done and 35% from short exams.

EXAMINATION RULES.

No rules, in principle.

BIBLIOGRAPHY

Basic:

- Lichtenberg, Allan J; Lieberman, M. A. Regular and stochastic motion. New York: Springer-Verlag, 1983. ISBN 0387907076.
- Press, William H. Numerical recipes in C : the art of scientific computing. 2nd. Cambridge: Cambridge University Press, 1992. ISBN 0521431085.
- Arrowsmith, D. K; Place, C. M. An introduction to dynamical systems. Cambridge: Cambridge University Press, 1990. ISBN 0521303621.
- Particular articles related to the topics of the course and some notes from suitable web pages.
- Stuart, A.M.; Humphries, A. R. Dynamical systems and numerical analysis. Cambridge University Press, 1996. ISBN 0521496721.
- Strogatz, S.H.. Nonlinear dynamics and chaos : with applications to physics, biology, chemistry, and engineering [on line]. 2nd ed. CRC Press, Taylor and Francis Group, 2018 [Consultation: 10/07/2023]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=1181622. ISBN 9780813349107.

Course guide
34965 - NMPDE - Numerical Methods for Partial Differential Equations

Last modified: 29/05/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 751 - DECA - Department of Civil and Environmental Engineering.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: SONIA FERNANDEZ MENDEZ

Others:

PRIOR SKILLS

Basics on numerical methods, differential equations and calculus.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Lectures, practical work at computer room, exercises and home works.

LEARNING OBJECTIVES OF THE SUBJECT

This course is an introduction to numerical methods for the solution of partial differential equations, with application to applied sciences, engineering and biosciences.

The course recalls the theoretical basis of the Finite Element Method (FEM) for the solution of elliptic and parabolic equations, an introduction to stabilization techniques for convection-dominated problems and the FEM for compressible flow problems, and for wave problems.

The course will include frontal lectures and exercises, as well as computer sessions aimed at introducing the bases of the programming of the numerical methods.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Fundamentals of Finite Element Methods (FEM)

Description:
Basic concepts of the Finite Element Method (FEM) for elliptic and parabolic equations: strong and weak form, discretization, implementation, functional analysis tools, error bounds and convergence.
Application to the numerical modelling of flow in porous medium, and potential flow.
Introduction to a posteriori error estimation and adaptivity.
Time integration for transient problems.
Solution of the convection-diffusion equation. Stabilized formulations for convection dominated problems.
Numerical solution of linear elasticity problems.

Full-or-part-time: 32h
Theory classes: 16h
Laboratory classes: 16h

FEM for incompressible flow problems

Description:
Weak form and discretization of the Stokes equations. Stable FEM discretizations for incompressible flow problems: LBB condition.
Introduction to the numerical solution of the incompressible Navier-Stokes equations.

Full-or-part-time: 14h
Theory classes: 7h
Practical classes: 7h

FEM for wave problems

Description:
FEM solution of Helmholtz equation. Non-reflecting boundary conditions. Application to acoustics.
Introduction to DG for first order conservation laws.

Full-or-part-time: 14h
Theory classes: 7h
Laboratory classes: 7h

GRADING SYSTEM

Exams (50%) and continuous assesment (exercises, projects and/or oral presentations) (50%).

BIBLIOGRAPHY

Basic:
- Wait, R.; Mitchell, A. R. Finite element analysis and applications. Chichester: John Wiley, 1985. ISBN 0471906778.
- Zienkiewicz, O.C.; Taylor, R. L. The finite element method [on line]. 6th ed. Oxford: Butterworth Heinemann, 2005 [Consultation: 07/07/2023]. Available on: https://www-sciencedirect-com.recursos.biblioteca.upc.edu/book/9781856176347/the-finite-element-method-for-solid-and-structural-mechanics?via=ihub=.
- Donea, Jean M; Huerta, A. Finite element methods for flow problems [on line]. Chichester: John Wiley & Sons, 2003 [Consultation: 19/05/2020]. Available on: https://onlinelibrary.wiley.com/doi/book/10.1002/0470013826. ISBN 0471496669.
- Ainsworth, M.; Oden, J. T. A posteriori error estimation in finite element analysis. New York: John Wiley & sons, 2000. ISBN 047129411X.
- Quarteroni, Alfio. Numerical models for differential problems [on line]. Springer, 2009 [Consultation: 07/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-88-470-1071-0. ISBN 1280384433.

Complementary:
- Hoffman, Joe D. Numerical methods for engineers and scientists. 2nd ed. rev. and exp. New York: Marcel Dekker, 2001. ISBN 0824704436.
- Johnson, Claes. Numerical solution of partial differential equations by the finite element. Mineola, New York: Dover Publications, 2009. ISBN 9780486469003.
- Strang, G.; Fix, G. J. An Analysis of the finite element method. Englewood Cliffs, NJ: Prentice-Hall, 1973. ISBN 0130329460.
- Trefethen, Lloyd N.; Bau, David. Numerical linear algebra. Philadelphia: SIAM, 1997. ISBN 9780898713619.

Course guide
34961 - QQMDS - Quantitative and Qualitative Methods in Dynamical Systems

Last modified: 29/05/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: PAU MARTIN DE LA TORRE

Others:

PRIOR SKILLS

Good knowledge of calculus, algebra and differential equations. It is strongly recommended a good understanding of the basic theory of ordinary differential equations as well as a basic knowledge of dynamical systems from a local point of view.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

We do not distinguish theoretical and practical classes. Some results about modern theory in Dynamical systems are presented in class. The main idea is to give basic knowledge and useful tools in the study of a dynamical system from both quantitative and qualitative points of view. We will stress the relation between different kind of systems and we will mainly focus in the use of perturvatives techniques to study a dynamical system globally.

LEARNING OBJECTIVES OF THE SUBJECT

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Invariant objects in Dynamical Systems

Description:
Continuous and discrete Dynamical Systems.
Poincaré map.
Local behaviour of hyperbolic invariant objects. Conjugation.
Invariant manifolds.

Full-or-part-time: 10h
Theory classes: 10h

Normal forms

Description:
Poincaré-Dulac normal forms. Convergence: Poincaré and Siegel domains.

Full-or-part-time: 10h
Theory classes: 10h

Perturbation theory in Dynamical Systems

Description:
Clasic perturbation theory. Averaging theory. Perturbed homoclinic orbits in the plane. Melnikov method. Singular pertubation theory.

Full-or-part-time: 15h
Theory classes: 15h

Bifurcations

Description:
Local bifurcations for planar vector fields and real maps. Saddle node and Hopf bifurcations.

Full-or-part-time: 10h
Theory classes: 10h

Homoclinic points and chaotic Dynamics

Description:
Smale horseshoe. Homoclinic points and bifurcations. Hyperbolic sets and transversal homoclinic points. Dynamical systems with chaotic dynamics. Newhouse phenomenum.

Full-or-part-time: 10h
Theory classes: 10h

Non-smooth systems

Description:
Introduction to non-smooth diferential equations. Definition and motivating examples.
Filipov's convention.

Full-or-part-time: 5h
Theory classes: 5h

GRADING SYSTEM

The students have to do some problems (60%) and a research work (25%). There will be also a final exam covering on the theoretical part of the subject (15%). On the other hand they will attend the winter courses "Recent trends in non-linear science" and produce a document about them.

EXAMINATION RULES.

There will be a final exam covering the theoretical part of the course.

BIBLIOGRAPHY

Basic:
- Arrowsmith, D. K; Place, C. M. An Introduction to dynamical systems. Cambridge [England] ; New York: Cambridge University Press, 1990. ISBN 0-521-30362-1.
- Guckenheimer, John; Holmes, Philip. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. New York, NY [etc.]: Springer-Verlag, 1983. ISBN 0-387-90819-6.
- Katok, Anatole; Hasselblatt, Boris. Introduction to the modern theory of dynamical systems. Cambridge [etc.]: Cambridge University Press, 1995. ISBN 0521341876.
- Hasselblatt, Boris; Katok, A. B. A First course in dynamics : with a panorama of recent developments. Cambridge [etc.]: Cambridge University Press, 2003. ISBN 0521583047.
- Hirsch, Morris W.; Smale, Stephen; Devaney, Robert L. Differential equations, dynamical systems, and an introduction to chaos. 3rd Edition. Oxford: Elsevier, 2013. ISBN 9780123820105.

Course guide
200901 - SAGDM - Seminar on Algebra, Geometry and Discrete Mathematics

Last modified: 23/06/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 3.0 **Languages:** English

LECTURER

Coordinating lecturer: Enric Ventura

Others:

PRIOR SKILLS

The student must know the basics on graph theory. Additionally, it is necessary to have some knowledge on probability theory, group theory and arithmetics.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

TEACHING METHODOLOGY

This seminar will be based on the presentation (from the professors of the course and the students) of material in the context of the theory of expanders. This material will be taken from specialized books, research papers and surveys.

LEARNING OBJECTIVES OF THE SUBJECT

The main objective of the seminar is to show an area of â□□â□□mathematics that intersects both algebra and geometry, discrete mathematics and other related areas, such as computer science, low-dimensional geometry and probability theory, among others.

The main objective is to get the student to gain a basic knowledge of expander theory, as well as the various applications in various branches of contemporary mathematics. In this direction, the student will also be encouraged to learn to conduct technical talks in public and in the preparation of technical scientific documents.

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STUDY LOAD

Type	Hours	Percentage
Self study	51,0	68.00
Hours large group	24,0	32.00

Total learning time: 75 h

CONTENTS

Spectral graph Theory and expanders

Description:
- Spectral graph theory.
- Spectral bounds: spectral gap, Mixing Lemma, Alon-Boppana. Ramanujan graphs.
- Existence of graph expanders: Margulis construction. Probabilistic contructions.
- Zig-zag product and construction of expanders.

Full-or-part-time: 20h
Theory classes: 6h 40m
Self study : 13h 20m

Graph expanders and group theory

Description:
- Cayley graphs. Properties
- Random walks in Cayley graphs.

Full-or-part-time: 9h 20m
Theory classes: 6h
Self study : 3h 20m

Graf expanders and Number Theory

Description:
- Sum-product phenomena. Balog-Szemerédi-Gowers Theorem.
- Classical algebraic groups and quasi-random groups.
- Expansion in SL2(Fq): Helfgott Theorem and Bourgain-Gamburd.

Full-or-part-time: 12h
Theory classes: 6h
Self study : 6h

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Other applications of graph expanders

Description:
- Applications in knot theory: the Barzdin-Kolmogorov theorem.
- Applications in theoretical computer science: design of concentrators and coding theory.
- Applications in theoretical computing: design of algorithms.
- Analogues in Riemannian geometry.

Full-or-part-time: 12h
Theory classes: 6h
Self study : 6h

GRADING SYSTEM

The grading of this seminar will be based on three points: (CA) Continuous evaluation, (MP) Material preparation and (PT) presentation.

(CA): will be based on the understanding of the material, as well as the meetings that will be held between the student and the responsible to prepare the student's presentation (or presentations). It will also include the fact of being active during the seminar sessions.
(MP): preparation of both the presentation, its good preparation and the summary sheet.
(PT): presentation. This will include questions from the teacher and students.

Overall grading of the seminar: 30% (CA)+20% (MP)+50% (PT)

BIBLIOGRAPHY

Basic:
- Kowalski, Emmanuel. An Introduction to expander graphs. Sóciété Mathématique de France, 2019. ISBN 9782856298985.
- Davidoff,Giuliana; Sarnak, Peter; Valette, Alain. Elementary number theory, group theory and Ramanujan graphs [on line]. London Mathematical Society, 2003 [Consultation: 10/07/2023]. Available on: https://www-cambridge-org.recursos.biblioteca.upc.edu/core/books/elementary-number-theory-group-theory-and-ramanujan-graphs/7932F64548F1B38B95AA2593E0B986B2. ISBN 9780521531436.
- Hoory, Shlomo; Linial, Nathan; Wigderson, Avi. "Expander graphs and their applications". Bulletin of the American Mathematical Society [on line]. [Consultation: 10/07/2023]. Available on: https://www.ams.org/journals/bull/2006-43-04/S0273-0979-06-01126-8/S0273-0979-06-01126-8.pdf.- Krebs, Mike; Shaheen, Anthony. Expander families and Cayley graphs : a beginner's guide [on line]. Oxford University Press, 2011 [Consultation: 10/07/2023]. Available on: https://web-p-ebscohost-com.recursos.biblioteca.upc.edu/ehost/ebookviewer/ebook?sid=2edb9be3-0d08-4c98-8afd-062a20793602%40redis&vid=0&format=EB.
- Ireland, K.; Rosen, M.. A Classical introduction to modern number theory. New York: Springer Verlag, 1990. ISBN 038797329X.

Complementary:
- Lubotzky, Alex. Discrete groups, expanding graphs and invariant measures. Birkhäuser, 1994. ISBN 376435075X.

Course guide
200902 - SADEM - Seminar on Analysis, Differential Equations and Modelling

Last modified: 23/06/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 748 - FIS - Department of Physics.
751 - DECA - Department of Civil and Environmental Engineering.
749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 3.0 **Languages:** English

LECTURER

Coordinating lecturer: Jose Muñoz

Others:

PRIOR SKILLS

It is important to have some background in one of the following areas: Dynamical Systems, Partial Differential Equations, Numerical methods and discretisation methods for PDEs.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

TEACHING METHODOLOGY

Students will present a topic related to Mathematical Analysis for Oscillatory Systems autonomously on a seminar format. This material will be taken from specialized books and research papers.
There will be some meetings between the instructor and the students before the oral presentation.
Students must prepare an abstract/report of the seminar to help the rest of the students attending the seminar to understand the topic.
Students must attend at least 90% of the lectures and be active in all the presentations.

LEARNING OBJECTIVES OF THE SUBJECT

The main goal is to provide the students with a basic knowledge on mathematical methods for the analysis for oscillatory systems, as well as, several applications in different branches of applied mathematics. The topic intersects with dynamical systems, partial differential equations as well as numerical methods, amongst others.

Additionally, the students will learn how to conduct technical talks in public and prepare technical scientific documents.

STUDY LOAD

Type	Hours	Percentage
Hours large group	24,0	32.00
Self study	51,0	68.00

Total learning time: 75 h

CONTENTS

Mathematical tools for periodic orbits and stability analysis

Description:
Poincaré Section. Perturbation analysis. Characteristic equations

Full-or-part-time: 15h
Theory classes: 4h
Self study : 11h

Biological oscillators

Description:
Reaction-Diffusion equations and Turing patterns. Biological oscillators in neural systems. Electrical + ion concentration fields. Hodgkin-Huxley equations.

Full-or-part-time: 30h
Theory classes: 10h
Self study : 20h

Oscillations in mechanics

Description:
Chemo-mechanical oscillators in elasticity. Delay systems in mechanics and biology. Wave Propagation in solids and fluids: acoustics, surface waves, etc. Periodic orbits in celestial mechanics.

Full-or-part-time: 30h
Theory classes: 10h
Self study : 20h

GRADING SYSTEM

The grading of this seminar will be based on three aspects: (C) Continuous evaluation, (M) Material preparation and (P) Presentation.

(C): the grade will be based on the understanding of the material, as well as the meetings that will be held between the student and the instructor to prepare the student's presentation (or presentations). The goal of the seminar is to ensure that the audience understands the lectures and its technicalities.
(M): the grade will be based on the quality of both the presentation slides and the abstract/report.
(P): the grade will be based on the clarity of the presentation. This will include questions from the teacher and students.

Overall grading of the seminar: 30% (C) + 20% (M) + 50% (P)

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BIBLIOGRAPHY

Basic:
- Erneux, Thomas. Applied delay differential equations [on line]. New York, NY: Springer New York, 2009 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-74372-1. ISBN 9780387743721.
- Murray, J. D. Mathematical biology, vol. 1 [on line]. 3rd ed. New York [etc.]: Springer, 2002-2003 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/b98868. ISBN 9780387224374.
- Keener, James; Sneyd, James. Mathematical physiology [on line]. New York, NY: Springer New York, 2009 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-75847-3 (v.1) - https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-79388-7 (v.2). ISBN 9780387758473.
- Guckenheimer, John; Holmes, Philip. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. New York, NY [etc.]: Springer-Verlag, 1983. ISBN 0387908196.
- Strogatz, Steven H. Nonlinear dynamics and chaos : with applications to physics, biology, chemistry and engineering [on line]. 2nd ed. CRC Press, Taylor, 2018 [Consultation: 10/07/2023]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=1181622. ISBN 9780813349107.