



GUIA DOCENT

24/25

Curs Srinivasa Ramanujan

Master's degree in  
Advanced  
Mathematics and  
Mathematical  
Engineering  
(MAMME)



Srinivasa Ramanujan  
22/12/1887 – 26/04/1920

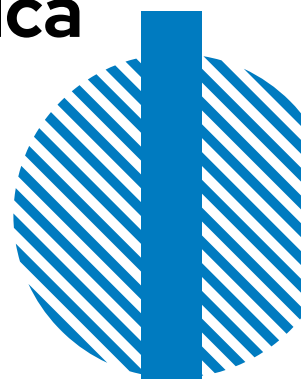
 UNIVERSITAT POLITÈCNICA DE CATALUNYA  
BARCELONATECH  Curs 2024-2025  
Facultat de Matemàtiques i Estadística

Facultat de Matemàtiques i Estadística



UNIVERSITAT POLITÈCNICA DE CATALUNYA  
BARCELONATECH

Facultat de Matemàtiques i Estadística



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# Master's degree in Advanced Mathematics and Mathematical Engineering (MAMME)

El **master's degree in Advanced Mathematics and Mathematical Engineering (MAMME)** (**màster universitari en Matemàtica Avançada i Enginyeria Matemàtica**) ([web del màster](#)) és un programa de màster ofertat per la Facultat de Matemàtiques i Estadística (FME) de 60 ECTS, dividits en 45 ECTS de cursos, més 15 ECTS de tesi de màster.

L'oferta de cursos permet als nostres estudiants dissenyar el seu currículum amb dues possibles orientacions: un currículum en matemàtica pura (orientat a recerca en matemàtica fonamental) o un currículum en matemàtica aplicada (preparant els estudiants per a recerca en matemàtica aplicada, y per a treballar en equips interdisciplinaris en col·laboració amb enginyers, físics, biòlegs, economistes, etc).

El MAMME ofereix també la possibilitat de cursar fins a 22.5 ECTS a altres màsters en matemàtiques o estadística, o a altres màsters de la UPC, permetent dissenyar un currículum interdisciplinari basat en la selecció de cursos en màsters en enginyeria o ciències aplicades.

Veure les propostes de focalització a <http://mamme.masters.upc.edu/en>.

## DADES GENERALS

### Durada i inici

Un curs acadèmic, 60 crèdits ECTS. Inici: setembre i febrer

### Horaris i modalitat

Tarda. Presencial

### Preus i beques

Preu aproximat del màster **sense altres despeses addicionals** (no inclou taxes acadèmiques de caràcter no docent ni expedició del títol):

1.660 € (6.331 € per a no residents a la UE).

[Més informació sobre preus i pagament de la matrícula](#)

[Més informació de beques i ajuts](#)

### Idiomes

Anglès

[Informació sobre l'ús de llengües a l'aula i els drets lingüístics de l'estudiantat.](#)

### Lloc d'impartició

[Facultat de Matemàtiques i Estadística](#) (Barcelona)

### Títol oficial

[Inscrit en el registre del Ministeri d'Educació, Cultura i Esport](#)

## ACCÉS

### Requisits generals

[Requisits acadèmics d'accés a un màster](#)

### Places

30 places setembre i 3 places al febrer

### Preinscripció

Preinscripció tancada (consulta els nous períodes de preinscripció al [calendari acadèmic](#)).

[Com es formalitza la preinscripció?](#)

## Admissió i matrícula

[Com es formalitza la matrícula?](#)

## Legalització de documents

Els documents expedits per estats no membres de la Unió Europea ni signataris de l'Acord sobre l'espai econòmic europeu han d'estar [legalitzats per via diplomàtica](#) o amb la postil·la corresponent.

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## ACORDS DE DOBLE TITULACIÓ

### Amb altres universitats internacionals

- *Master's degree in Advanced Mathematics and Mathematical Engineering (FME) + Master of Science in Applied Mathematics* (Illinois Institute of Technology (ITT), Chicago, USA). (Flux solament de l'FME a Illinois)
- *Master's degree in Advanced Mathematics and Mathematical Engineering (FME) + Master of Science in Industrial and Applied Mathematics (MSIAM)* (Institute d'Ingénierie et de Management, Université Grenoble Alpes (UGA), Grenoble, França)

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## SORTIDES PROFESSIONALS

### Sortides professionals

Algunes de les sortides professionals dels titulats i titulades d'aquest màster són la recerca acadèmica (fent un doctorat en matemàtiques, ciència o enginyeria, per exemple), la modelització matemàtica en la indústria, les finances, l'estadística i la recerca aplicada (centres de recerca biomèdica, visió per ordinador, etc.).

## Competències

### Competències transversals

Les competències transversals descriuen allò que un titulat o titulada ha de saber o ha de ser capaç de fer en acabar el procés d'aprenentatge, amb independència de la titulació. **Les competències transversals establertes a la UPC** són emprenedoria i innovació, sostenibilitat i compromís social, coneixement d'una tercera llengua (preferentment l'anglès), treball en equip i ús solvent dels recursos d'informació.

### Competències específiques

1. (Recerca). Llegir i comprendre articles avançats de recerca en matemàtiques. Utilitzar tècniques de recerca en matemàtiques per produir i transmetre nous resultats.
2. (Modelització). Formular, analitzar i validar models matemàtics de problemes pràctics utilitzant les eines matemàtiques més adequades.
3. (Càlcul). Obtenir solucions (exactes o aproximades) a aquests models amb els recursos disponibles, incloent-hi mitjans computacionals.
4. (Avaluació crítica). Discutir la validesa, l'abast i la importància d'aquestes solucions; presentar resultats i defensar conclusions.
5. (Docència). Ensenyar matemàtiques a nivell universitari.

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## ORGANITZACIÓ ACADÈMICA: NORMATIVES, CALENDARIS

### Centre docent UPC

[Facultat de Matemàtiques i Estadística \(FME\)](#)

### Responsable acadèmic del programa

[Jordi Saludes Closa](#)

### Calendari acadèmic

[Calendari acadèmic dels estudis universitaris de la UPC](#)

### Normatives acadèmiques

[Normativa acadèmica dels estudis de màster de la UPC](#)

**PLA D'ESTUDIS**

<b>Assignatures</b>	<b>crèdits ECTS</b>	<b>Tipus</b>
<b>PRIMER QUADRIMESTRE</b>		
Àlgebra Commutativa	7.5	Optativa
Àlgebra No Commutativa	7.5	Optativa
Geometria Discreta i Algorítmica	7.5	Optativa
Mètodes Numèrics per a Equacions en Derivades Parcial	7.5	Optativa
Mètodes Numèrics per a Sistemes Dinàmics	7.5	Optativa
Mètodes Quantitatius i Qualitatius en Sistemes Dinàmics	7.5	Optativa
Modelització Matemàtica amb Equacions en Derivades Parcial	7.5	Optativa
Models Matemàtics en Biologia	7.5	Optativa
Teoria de Grafs	7.5	Optativa
Teoria de Nombres	7.5	Optativa
<b>SEGON QUADRIMESTRE</b>		
Aprenentatge Automàtic	7.5	Optativa
Codis i Criptografia	7.5	Optativa
Combinatòria	7.5	Optativa
Curs Avançat d'Equacions en Derivades Parcial	7.5	Optativa
Geometria Algebraica	7.5	Optativa
Mecànica Computacional	7.5	Optativa
Seminari d'Àlgebra, Geometria i Matemàtica Discreta	3	Optativa
Seminari d'Anàlisi, Equacions Diferencials i Modelització	3	Optativa
Sistemes Hamiltonians	7.5	Optativa
Varietats Diferenciables	7.5	Optativa
Treball de Fi de Màster	15	Projecte

# Master's degree in Advanced Mathematics and Mathematical Engineering (MAMME)

The **master's degree in Advanced Mathematics and Mathematical Engineering (MAMME)** ([master's degree website](#)) is a master's programme in mathematics offered at the School of Mathematics and Statistics (FME).

The courses offered in MAMME allow our students to design their curriculum with two different orientations: a pure mathematics curriculum (oriented to research in fundamental mathematics) or an applied mathematics curriculum (preparing them for applied mathematics research and for interdisciplinary teamwork, in collaboration with engineers, physicists, biologists, economists, etc.).

The curriculum comprises a total of 60 ECTS credits, divided into 45 credits for courses and 15 for the master's thesis. It is intended to be completed in one academic year. In addition, MAMME offers the possibility of enrolling for up to 22.5 ECTS credits in other master's degrees in mathematics or statistics, or in other UPC master's programmes, opening the path for an interdisciplinary curriculum based on selected courses in master's degrees in engineering and applied sciences. See the MAMME focus proposals at <http://mamme.masters.upc.edu/en>.

## GENERAL DETAILS

### Duration and start date

One academic year, 60 ECTS credits. Starting September and February

### Timetable and delivery

Afternoons. Face-to-face

### Fees and grants

Approximate fees for the master's degree, **excluding other costs** (does not include non-teaching academic fees and issuing of the degree certificate):

€1,660 (€6,331 for non-EU residents).

[More information about fees and payment options](#)

[More information about grants and loans](#)

### Language of instruction

English

Information on [language use in the classroom and students' language rights](#).

### Location

[School of Mathematics and Statistics](#) (Barcelona)

### Official degree

[Recorded in the Ministry of Education's degree register](#)

## ADMISSION

### General requirements

[Academic requirements for admission to master's degrees](#)

### Specific requirements

This master's degree is aimed at students with good abstract reasoning, an interest in problem solving, strong work

habits and a liking for mathematics.

A scientific background is required, with basic mathematical foundations. For this reason, a bachelor's degree in mathematics, statistics, physics, engineering, economics or science is recommended. This list is non-exhaustive, and all applications will be reviewed on an individual basis.

## Admission criteria

The following elements will be taken into consideration during the evaluation process: the academic record, the CV, a statement of purpose and, if deemed necessary, a personal interview and recommendation letters.

## Places

33

## Pre-enrolment

Pre-enrolment closed (consult the new pre-enrolment periods in the [academic calendar](#)).

[How to pre-enrol](#)

## Enrolment

[How to enrol](#)

## Legalisation of foreign documents

All documents issued in non-EU countries must be [legalised and bear the corresponding apostille](#).

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## DOUBLE-DEGREE AGREEMENTS

### Double-degree pathways with universities around the world

- Master's degree in Advanced Mathematics and Mathematical Engineering (FME) + Master of Science in Applied Mathematics (Illinois Institute of Technology (ITT), Chicago, USA). (Only FME students to Illinois, not vice versa.)
- Master's degree in Advanced Mathematics and Mathematical Engineering (FME) + Master of Science in Industrial and Applied Mathematics (MSIAM) (Institute d'Ingénierie et de Management, Université Grenoble Alpes (UGA), Grenoble, France)

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## PROFESSIONAL OPPORTUNITIES

### Professional opportunities

Some of the career prospects of master's degree graduates are academic research (by pursuing a PhD in mathematics, science or engineering, for instance), mathematical modelling in industry, finance, statistics and applied research (biomedical research centres, computer vision, etc.).

## Competencies

### Generic competencies

Generic competencies are the skills that graduates acquire regardless of the specific course or field of study. The generic competencies established by the UPC are capacity for innovation and entrepreneurship, sustainability and social commitment, knowledge of a foreign language (preferably English), teamwork and proper use of information resources.

### Specific competencies

On completing this master's degree, students will be able to:

1. (Research). Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. (Modelling). Formulate, analyse and validate mathematical models of practical problems by using the

- appropriate mathematical tools.
3. (Calculus). Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
  4. (Critical assessment). Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.
  5. (Teaching). Teach mathematics at university level.

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## ORGANISATION: ACADEMIC CALENDAR AND REGULATIONS

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### UPC school

[School of Mathematics and Statistics \(FME\)](#)

### Academic coordinator

[Jordi Saludes Closa](#)

### Academic calendar

[General academic calendar for bachelor's, master's and doctoral degrees courses](#)

### Academic regulations

[Academic regulations for master's degree courses at the UPC](#)

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## CURRICULUM

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Subjects	ECTS credits	Type
<b>FIRST SEMESTER</b>		
Commutative Algebra	7.5	Optional
Discrete and Algorithmic Geometry	7.5	Optional
Graph Theory	7.5	Optional
Mathematical Modelling with Partial Differential Equations	7.5	Optional
Mathematical Models in Biology	7.5	Optional
Non-Commutative Algebra	7.5	Optional
Number Theory	7.5	Optional
Numerical Methods for Dynamical Systems	7.5	Optional
Numerical Methods for Partial Differential Equations	7.5	Optional
Quantitative and Qualitative Methods in Dynamical Systems	7.5	Optional
<b>SECOND SEMESTER</b>		
Advanced Course in Partial Differential Equations	7.5	Optional
Algebraic Geometry	7.5	Optional
Codes and Cryptography	7.5	Optional
Combinatorics	7.5	Optional
Computational Mechanics	7.5	Optional
Differentiable Manifolds	7.5	Optional
Hamiltonian Systems	7.5	Optional
Machine Learning	7.5	Optional
Seminar on Algebra, Geometry and Discrete Mathematics	3	Optional



<b>Subjects</b>	<b>ECTS credits</b>	<b>Type</b>
Seminar on Analysis, Differential Equations and Modelling	3	Optional
Master's Thesis	15	Project

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October 2024. [UPC](#). Universitat Politècnica de Catalunya · BarcelonaTech

# Master's degree in Advanced Mathematics and Mathematical Engineering (MAMME)

El **master's degree in Advanced Mathematics and Mathematical Engineering (MAMME)** (**máster universitario en Matemática Avanzada e Ingeniería Matemática**) ([web del máster](#)) es un programa de máster ofrecido por la Facultad de Matemáticas y Estadística (FME) de 60 ECTS, divididos en 45 ECTS de cursos, más 15 ECTS de tesis de máster.

La oferta de cursos permite a nuestros estudiantes diseñar su currículum con dos posibles orientaciones diferentes: un currículum en matemática pura (orientado a investigación en matemática fundamental) o un currículum en matemática aplicada (preparando los estudiantes para investigación en matemática aplicada, y para trabajar en equipos interdisciplinares en colaboración con ingenieros, físicos, biólogos, economistas, etc).

Además, MAMME ofrece la posibilidad de cursar hasta 22.5 ECTS en otros másteres en matemáticas o estadística, o en otros másteres de la UPC, permitiendo diseñar un currículum interdisciplinar basado en la selección de cursos en másteres en ingeniería o ciencias aplicadas.

Vea las propuestas de focalización en <http://mamme.masters.upc.edu/en>.

## DATOS GENERALES

### Duración e inicio

Un curso académico, 60 créditos ECTS. Inicio septiembre y febrero

### Horarios y modalidad

Tarde. Presencial

### Precios y becas

Precio aproximado del máster **sin otros gastos adicionales** (no incluye tasas académicas de carácter no docente ni expedición del título):

1.660 € (6.331 € para no residentes en la UE).

[Más información sobre precios y pago de la matrícula](#)

[Más información de becas y ayudas](#)

### Idiomas

Inglés

Información sobre el [uso de lenguas en el aula y los derechos lingüísticos de los estudiantes](#).

### Lugar de impartición

[Facultad de Matemáticas y Estadística](#) (Barcelona)

### Título oficial

[Inscrito en el registro del Ministerio de Educación, Cultura y Deporte](#)

## ACCESO

### Requisitos generales

[Requisitos académicos de acceso a un máster](#)

### Plazas

33

### Preinscripción

Preinscripción cerrada (consulta los nuevos periodos de preinscripción en el [calendario académico](#)).  
[¿Cómo se formaliza la preinscripción?](#)

## Matrícula

[¿Cómo se formaliza la matrícula?](#)

## Legalización de documentos

Los documentos expedidos por estados no miembros de la Unión Europea ni firmantes del Acuerdo sobre el espacio económico europeo tienen que estar [legalizados por vía diplomática o con correspondiente apostilla](#).

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## ACUERDOS DE DOBLE TITULACIÓN

### Con otras universidades internacionales

- *Master's degree in Advanced Mathematics and Mathematical Engineering (FME) + Master of Science in Applied Mathematics* (Illinois Institute of Technology (ITT), Chicago, USA). (Flujo sólo de la FME en Illinois)
- *Master's degree in Advanced Mathematics and Mathematical Engineering (FME) + Master of Science in Industrial and Applied Mathematics (MSIAM)* (Institute d'Ingénierie et de Management, Université Grenoble Alpes (UGA), Grenoble, Francia)

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## SALIDAS PROFESIONALES

### Salidas profesionales

Algunas de las salidas profesionales de los titulados de este máster son la investigación académica (haciendo un doctorado en matemáticas, ciencia o ingeniería, por ejemplo), la modelización matemática en la industria, las finanzas, la estadística y la investigación aplicada (centros de investigación biomédica, visión por ordenador, etc.).

## Competencias

### Competencias transversales

Las competencias transversales describen aquello que un titulado o titulada es capaz de saber o hacer al concluir su proceso de aprendizaje, con independencia de la titulación. **Las competencias transversales establecidas en la UPC** son la capacidad de espíritu empresarial e innovación, sostenibilidad y compromiso social, conocimiento de una tercera lengua (preferentemente el inglés), trabajo en equipo y uso solvente de los recursos de información.

### Competencias específicas

1. (Investigación). Leer y comprender artículos avanzados de investigación en matemáticas. Utilizar técnicas de investigación en matemáticas para producir y transmitir nuevos resultados.
2. (Modelización). Formular, analizar y validar modelos matemáticos de problemas prácticos utilizando las herramientas matemáticas más adecuadas.
3. (Cálculo). Obtener soluciones (exactas o aproximadas) a estos modelos con los recursos disponibles, incluyendo medios computacionales.
4. (Evaluación crítica). Discutir la validez, el alcance y la importancia de estas soluciones; presentar resultados y defender conclusiones.
5. (Docencia). Enseñar matemáticas a nivel universitario.

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## ORGANIZACIÓN ACADÉMICA: NORMATIVAS, CALENDARIOS

### Centro docente UPC

[Facultad de Matemáticas y Estadística \(FME\)](#)

### Responsable académico del programa

[Jordi Saludes Closa](#)

### Calendario académico

[Calendario académico de los estudios universitarios de la UPC](#)

### Normativas académicas

[Normativa académica de los estudios de máster de la UPC](#)

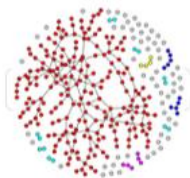
**PLAN DE ESTUDIOS**

<b>Asignaturas</b>	<b>créditos ECTS</b>	<b>Tipo</b>
<b>PRIMER CUATRIMESTRE</b>		
Álgebra Conmutativa	7.5	Optativa
Álgebra No Conmutativa	7.5	Optativa
Geometría Discreta y Algorítmica	7.5	Optativa
Métodos Cuantitativos y Cualitativos en Sistemas Dinámicos	7.5	Optativa
Métodos Numéricos para Ecuaciones en Derivadas Parciales	7.5	Optativa
Métodos Numéricos para Sistemas Dinámicos	7.5	Optativa
Modelización Matemática con Ecuaciones en Derivadas Parciales	7.5	Optativa
Modelos Matemáticos en Biología	7.5	Optativa
Teoría de Grafos	7.5	Optativa
Teoría de Números	7.5	Optativa
<b>SEGUNDO CUATRIMESTRE</b>		
Aprendizaje Automático	7.5	Optativa
Códigos y Criptografía	7.5	Optativa
Combinatoria	7.5	Optativa
Curso Avanzado de Ecuaciones en Derivadas Parciales	7.5	Optativa
Geometría Algebraica	7.5	Optativa
Mecánica Computacional	7.5	Optativa
Seminario de Álgebra, Geometría y Matemática Discreta	3	Optativa
Seminario de Análisis, Ecuaciones Diferenciales y Modelización	3	Optativa
Sistemas Hamiltonianos	7.5	Optativa
Variedades Diferenciables	7.5	Optativa
Trabajo de Fin de Máster	15	Proyecto

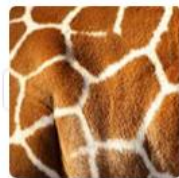
# FOCUS PROPOSALS

The following is a **NON-EXHAUSTIVE** list of focus proposals that may be considered for the selection of courses in MAMME, including both MAMME courses and courses in other master programs. They are **just examples** for selection of courses, and they will not be mentioned in the master's degree certificate. Students are encouraged to design their own curriculum with total freedom.

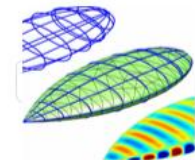
In any case, recall that a minimum of 22.5 ECTS in MAMME courses is mandatory. Registration in non-MAMME courses requires the approval of the director of MAMME and of the director of the other master.



Discrete Mathematics



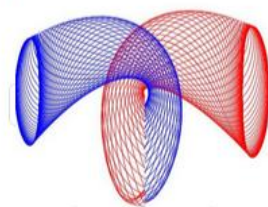
Partial Differential Equations and Analysis



Mathematical and Computational Modelling



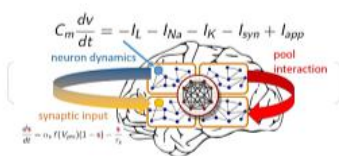
Optimization and Operation Research



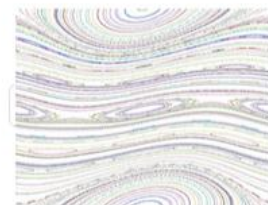
Dynamical Systems and Applications to Celestial Mechanics



Algebra, Geometry and Number Theory



Modelling and analysis in Biomedical Science



Geometry and its applications

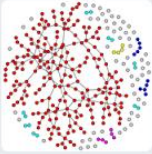
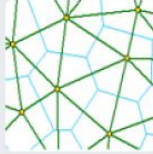



Mathematical Physics

# Focus on Discrete Mathematics

Discrete Mathematics has had a strong development from the second half of the XXth century fostered by the development of computers and communication technologies. The main topics include algorithms, coding theory, combinatorics, cryptography, discrete and computational geometry, finite geometry, game theory, graph theory, logic, operation research and random structures. Besides the wealth of problems which have become central in the development of contemporary mathematics, discrete mathematics holds a strong connection with applications in Bioinformatics, Computer Graphics, Information Theory, Networks or Theoretical Computer Science, as well as with other areas of mathematics like Algebra, Analysis, Number Theory or Topology.

The UPC gathers one of the strongest research groups in Spain in the area with a broad international projection providing a sound training. Most of the former students of the master have found job opportunities in industry and in academics by pursuing a PhD in UPC or in prestigious universities in Europe, the USA or Canada.

		
Random graphs, the basic model for random structures	Triangulations, a basic tool for computational geometry	Cryptography, one of the key applications of discrete mathematics

Students interested in focusing on Discrete Mathematics are invited to select 45 ECTS from this list:


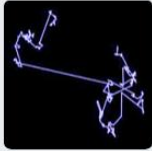
<a href="#">Combinatorics</a>	7.5 ECTS	English	MAMME
<a href="#">Graph Theory</a>	7.5 ECTS	English	MAMME
<a href="#">Codes and Cryptography</a>	7.5 ECTS	English	MAMME
<a href="#">Discrete and Algorithmic Geometry</a>	7.5 ECTS	English	MAMME
Optimización Entera y Combinatoria	5 ECTS	Spanish	Máster Univ. en Estadística e Investigación Operativa, UPC-UB
Algorithmic Methods for Mathematical Models	6 ECTS	English	Master in Innovation and Research in Informatics, UPC
Computational Complexity	6 ECTS	English	Master in Innovation and Research in Informatics, UPC
Combinatorial Set Theory	6 ECTS	English	Master in Pure and Applied Logic, UB-UPC

Registration to non-MAMME courses requires the approval of the director of the corresponding master program.

Recall that a minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

# Focus on partial Differential Equations and Analysis

Partial Differential Equations (PDEs) play a central role in physics, chemistry, biology, industry, mathematical finance, and image processing. Their analysis often requires deep mathematical techniques, which makes PDEs to at the heart of both historical and recent developments in analysis, geometry, and probability. Because of this and their applications, PDEs is a very active area of mathematics, the one with the largest number of publications.

		
Pattern formation with reaction-diffusion systems of PDEs	Free boundaries and PDEs: the Stefan problem for melting ice	Lévy flights and PDEs in finance, biological invasions...

Students interested in focusing on PDEs and Analysis are invited to select 45 ECTS from this list and the suggestions below:

<a href="#">Partial Differential Equations</a>	6 ECTS	English	Master in Advanced Mathematics, UB
<a href="#">Functional Analysis and PDEs</a>	7.5 ECTS	English	MAMME
<a href="#">Mathematical Modeling with PDEs</a>	7.5 ECTS	English	MAMME
<a href="#">Numerical Methods for PDEs</a>	7.5 ECTS	English	MAMME
Stochastic Calculus	6 ECTS	English	Master in Advanced Mathematics, UB
Complex Analysis	6 ECTS	English	Master in Advanced Mathematics, UB

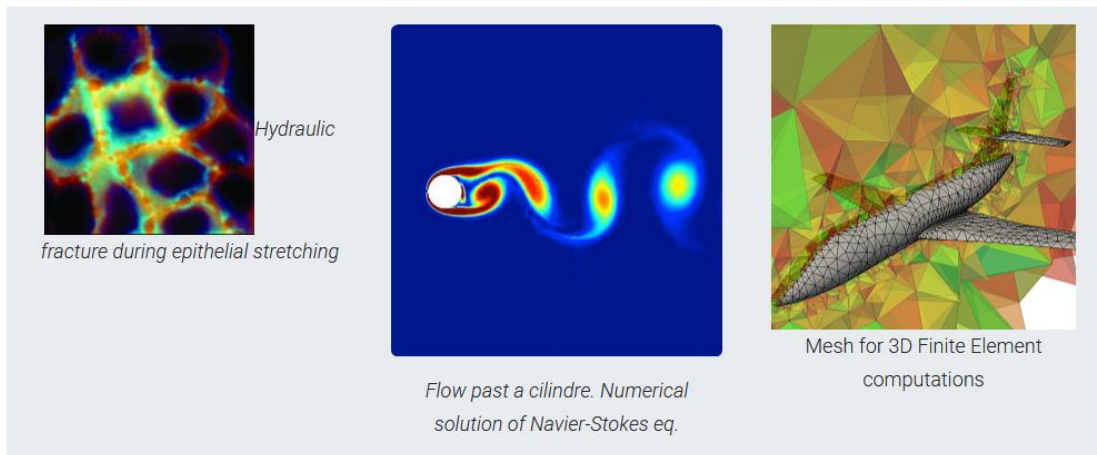
A minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

Other appropriate courses (depending on the student interests) with connections to PDEs are: Quantitative and Qualitative Methods in Dynamical Systems (Q1 MAMME), Hamiltonian Systems (Q2 MAMME), and courses within the Barcelona universities masters offer in Mathematical Finance, Mathematical Biology, Image Processing, Functional Analysis, or Differential Geometry.

The UPC master course "Functional Analysis and PDEs" (currently "Advanced Course in Partial Differential Equations") and the UB master course "Partial Differential Equations" cover different topics of the theory of PDEs. Their contents do not overlap (or intersect), but both courses complement each other. At the same time, their contents do not overlap with a possible first undergraduate course in PDEs that the student may have taken.

# Focus on Mathematical and Computational Modelling with PDEs

Mathematical and computational modelling with Partial Differential Equations (PDEs) is nowadays an essential tool for analysing, understanding and predicting phenomena in physics, biology, engineering, economics, social sciences and related fields. The applications cover a wide spectrum ranging from the modelling of the aerodynamical behaviour of an airfoil, to the simulation of the impact of a tsunami in a coastal area, or the study of fracture in epithelial cell sheets.



Students interested in focusing on modeling with PDEs are invited to select 45 ECTS from this list:

Mathematical Modelling with PDEs	7.5 ECTS	English	MAMME
Numerical Methods for PDEs	7.5 ECTS	English	MAMME
Computational Mechanics	7.5 ECTS	English	MAMME
Advanced Course in PDEs	7.5 ECTS	English	MAMME
Advanced Fluid Mechanics**	5 ECTS	English	Master in Numerical Methods in Engineering, UPC
Finite Elements in Fluids**	5 ECTS	English	Master in Numerical Methods in Engineering, UPC
Advanced Discretization Methods**	5 ECTS	English	Master in Numerical Methods in Engineering, UPC
Numerical Modelling*	9 ECTS	English	Master en Enginyeria de Camins, Canals i Ports, UPC

Recall that a minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

(\*) "Numerical Modeling" is recommended to students that do not have a solid background in numerical methods and programming. Registration to this course requires the approval of the director of the corresponding master.



# Focus on Optimization and Operation Research for Efficient Decision Making

Efficient decision making based on quantitative results is essential for success in business and management. Operations Research (also known as “Management Sciences” or “Analytics”) is a discipline that deals with the application of advanced analytical methods to help make better decisions. Project planning, network optimization, facility location, routing, supply chain management, scheduling, among others, are real problems tackled by Operation Research. Industrial sectors that benefit from Operation Research range from airlines (scheduling, tariff policy), to hospitals (scheduling), to electric utilities (production, trading) and logistics (route scheduling).



Travelling salesman problem solution



Traffic simulation system

Students interested in focusing on Optimization and Operation Research should select 45 ECTS from this list:

Continuous Optimization	5 ECTS	Spanish	MESIO UPC-UB
Optimization in Energy Systems and Markets	5 ECTS	Spanish	MESIO UPC-UB
Stochastic Optimization	5 ECTS	English	MESIO UPC-UB
Large Scale Optimization	5 ECTS	English	MESIO UPC-UB
Integer and Combinatorial Optimization*	5 ECTS	Spanish	MESIO UPC-UB
Statistical Data Protection*	5 ECTS	English	MESIO UPC-UB
Graph Theory*	7.5 ECTS	English	MAMME
Combinatorics*	7.5 ECTS	English	MAMME
Mathematical Models in Biology	7.5 ECTS	English	MAMME
Numerical Methods for Dynamical Systems	7.5 ECTS	English	MAMME
Numerical Methods for Partial Differential Equations	7.5 ECTS	English	MAMME

Recall that a minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

(\*) These courses are proposed to students willing to get a deeper focus on discrete and combinatorial optimization.

# Focus on Dynamical Systems and Applications to Celestial Mechanics

Dynamical Systems provide a powerful mathematical background to explore a great variety of models involving natural and social sciences, physics, chemistry, ecology, economics, neuroscience, astrodynamics among other fields. As a consequence Dynamical Systems theory has become an important and attractive branch of mathematics to students in many disciplines.

The courses proposed below aim at acquiring a basic and transversal knowledge of both the theory of Dynamical Systems as well as computational tools. Along the courses several applications are considered (see the course on 'Mathematical methods in Biology') but special emphasis is focused on Celestial Mechanics.

Other complementary courses from the Master at the Universitat de Barcelona are also given.

Qualitative and quantitative methods in dynamical systems	7.5 ECTS	English	MAMME
Numerical methods for dynamical systems	7.5 ECTS	English	MAMME
Hamiltonian systems	7.5 ECTS	English	MAMME
Mathematical models in biology	7.5 ECTS	English	MAMME
Advanced course in partial differential equations	7.5 ECTS	English	MAMME
Dynamical systems	6 ECTS	English	Master's degree in advanced and professional mathematics (UB)
Simulation methods	6 ECTS	English	Master's degree in advanced and professional mathematics (UB)

# Focus on Algebra, Geometry and Number Theory

Mathematics departments at UPC gather several research groups specialized in [Number Theory](#), [Algebraic Geometry](#), [Differential Geometry](#) and commutative and [non-commutative Algebra](#). All of them collaborate closely with other researchers of the Universitat de Barcelona and the Universitat Autònoma de Barcelona, and with research groups of some of the most prestigious universities around the world. Many young researchers began their scientific careers by coursing the Master and/or doing the PhD in one of the UPC groups: [Ariadna](#), [Biel](#), [Carlos](#), [Enrique](#), [Francesc](#), [Francesc](#), [Marc](#), [María](#), [Martí](#), [Pere-Daniel](#), [Santi](#), [Víctor](#), [Xevi](#), among others.

A knowledge of some basics in Algebra, Geometry and Number Theory is also very useful for people aimed to work in applications of Mathematics to Cryptography, Coding Theory, Discrete Mathematics, Control Theory, Mathematical Physics, Algorithmics, Biosciences, etc.

Students interested in focusing on Algebra, Geometry and Number Theory are invited to select 45 ECTS from this list:

<a href="#">Commutative Algebra*</a>	MAMME
<a href="#">Non-Commutative Algebra*</a>	MAMME
<a href="#">Differentiable Manifolds*</a>	MAMME
<a href="#">Number Theory*</a>	MAMME
<a href="#">Algebraic Geometry*</a>	MAMME
Geometry and Topology of Varieties	Master in Advanced Mathematics, UB
Algebraic Curves**	Master in Advanced Mathematics, UB
Computational Algebra	Master in Advanced Mathematics, UB
Geometrical Methods in Number Theory	Master in Advanced Mathematics, UB
Local Algebra	Master in Advanced Mathematics, UB

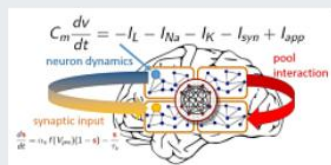
A minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

(\*) Please check at the [Study Program web page](#) if this course is offered in the academic year you are interested in.

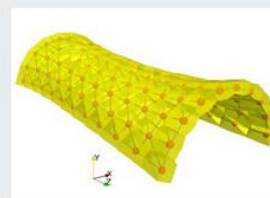
(\*\*) This course is not recommended to students who have followed the course "Geometria Algebraica" of *Grau en Matemàtiques* at FME.

# Focus on Modelling and Analysis in Biomedical Sciences

Research in biomedical sciences increasingly involves mathematical modelling as a support to validate theories, to test computational replicas, to manage biomedical data and to deal with new challenges that are hard to explore either clinically or experimentally. All these goals require scientists with the solid basis provided in standard mathematical undergraduate programs, but also equipped with advanced mathematical and computational tools, as well as a practical spirit, to serve at the interface of biology, medicine, mathematics and computation. Fortunately, while the MAMME program gives the opportunity of acquiring a basic knowledge of mathematical models in biology and advanced mathematical/computational tools, other master's programs at UPC, allow offering a complete training to prepare our students in this stimulating interdisciplinary area. Students interested in joining this area through the MAMME will receive advice from the master's coordination to tailor their curriculum according to different scopes in mathematical modelling of biomedical sciences. We aim at giving a broad training in the mathematical modelling of medically significant biological problems and, additionally, endow their careers with an initial subfocus in some specific problems. The list of courses below represent the wide offer at the UPC to tailor specific profiles (the student has to select 45 ECTS from it), which can be also complemented with problem-oriented master's theses (for example, study of phylogenetic trees, mathematical and computational neuroscience, electro-mechanical models in cardiac physiology, mathematical epidemiology,...), eventually co-advised with partners in biomedical labs. Researchers involved in the area offer their advice to adapt the curriculum to each student's background and interests.



Brain dynamics: modelling and analysis at different levels, with different tools (differential equations, graphs, statistics,...)



Simulation of curved cellular monolayers with computational mechanics

Mathematical models in biology	7.5 ECTS	English	MAMME
Numerical Methods for dynamical systems	7.5 ECTS	English	MAMME
Mathematical Modeling with PDEs	7.5 ECTS	English	MAMME
Numerical Methods for PDEs	7.5 ECTS	English	MAMME
Computational Mechanics	7.5 ECTS	English	MAMME
Qualitative and quantitative methods in dynamical systems	7.5 ECTS	English	MAMME
Graph theory	7.5 ECTS	English	MAMME
Inferencia estadística avanzada	5 ECTS	Spanish	MESIO UPC-UB
Fundamentos de bioinformática	5 ECTS	Spanish	MESIO UPC-UB
Multivariate data analysis	5 ECTS	Spanish-English	MESIO UPC-UB
Probability and stochastic processes	5 ECTS	English	MESIO UPC-UB
Time series	5 ECTS	Spanish-English	MESIO UPC-UB
Numerical Modeling*	9 ECTS	English	Màster en Enginyeria de Camins, Canals i Ports, UPC
Técnicas básicas en neurociencia**	2.5 ECTS		Màster oficial en neurociencia, UB-UPF-UDL-URV
Biología Celular y Molecular de la Neurona**	3 ECTS		Màster oficial en neurociencia, UB-UPF-UDL-URV
Diseño y análisis de datos en neurociencia cognitive**	2.5 ECTS		Màster oficial en neurociencia, UB-UPF-UDL-URV
Neurociencia computacional**	2.5 ECTS		Màster oficial en neurociencia, UB-UPF-UDL-URV

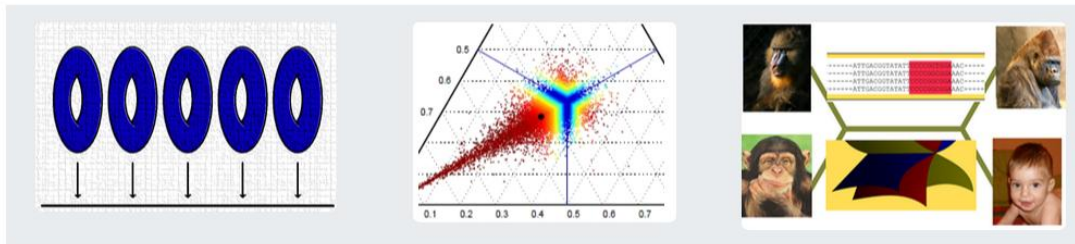
We remind that a minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

(\*) "Numerical Modeling" is recommended to students that do not have a solid background in numerical methods and programming. Registration to this course requires the approval of the director of the corresponding master.

(\*\*) These courses are proposed to students willing to get a deeper focus neuroscience. Registration to these courses requires the approval of the director of the corresponding master.

# Focus on Geometry and its Applications

Geometry is a multifaceted research field which is at the crossroad of other topics such as Mathematical Physics and Applied Mathematics. The different branches of Geometry include Algebraic Geometry and its applications to Phylogenetics and Robotics, Algebraic Topology and its applications to Computational Topology, Differential Geometry and its applications to Mathematical Physics and Control Theory.



Students interested in focusing on Geometry and its applications are invited to select 45 ECTS from this list:

Students interested in focusing on Geometry and its applications are invited to select 45 ECTS from this list:

Commutative Algebra (7,5 ECTS)	MAMME
Non-Commutative Algebra (7,5 ECTS)	MAMME
Differentiable Manifolds (7,5 ECTS)	MAMME
Algebraic Geometry (7,5 ECTS)	MAMME
Geometry and Topology of Varieties (6 ECTS)	Master in Advanced Mathematics, UB
Algebraic Curves (6 ECTS)	Master in Advanced Mathematics, UB
Computational Algebra (6 ECTS)	Master in Advanced Mathematics, UB
Local Algebra (6 ECTS)	Master in Advanced Mathematics, UB

A minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

# Focus on Mathematical Physics

The Focus on *Mathematical Physics* concentrates on the interplay between mathematics and physics, exploring the deep and intricate connections that bridge these two fundamental disciplines. This focus illustrates how mathematical theories can provide precise and predictive frameworks for understanding physical phenomena, while physical insights, in turn, inspire the development of new mathematical concepts and techniques.



Students interested in focusing on Physics and Mathematics are invited to select 45 ECTS from this list:

Differentiable Manifolds	7.5 ECTS	MAMME
Hamiltonian Systems	7.5 ECTS	MAMME
Quantitative and Qualitative Methods in Dynamical Systems	7.5 ECTS	MAMME
Graph theory	7.5 ECTS	MAMME
Computational Mechanics	7.5 ECTS	MAMME
Mathematical Modelling with PDEs	7.5 ECTS	MAMME
Numerical methods for dynamical systems	7.5 ECTS	MAMME
Codes and Cryptography	7.5 ECTS	MAMME
Numerical Methods for PDEs	7.5 ECTS	MAMME
Critical Phenomena and complexity	5 ECTS	MEPH <a href="#">e</a>
Quantum Matter	5 ECTS	MEPH <a href="#">e</a>
Computational Astrophysics	4 ECTS	MEPH <a href="#">e</a>
Complexity in Biological systems	4 ECTS	MEPH <a href="#">e</a>
Stochastic Methods for Optimization & Simulation	4 ECTS	MEPH <a href="#">e</a>
Materials Science of Drugs	4 ECTS	MEPH <a href="#">e</a>
Atomic and molecular physics	4 ECTS	MEPH <a href="#">e</a>

A minimum of 3 MAMME courses (22.5 ECTS) is mandatory.

# Double degrees

Attached there is the list of double degrees in association with MAMME

[MAMME-IIT \(IIT, USA\)](#)

[MAMME-INP Grenoble \(Grenoble, France\)](#)



# Master's thesis

All students are required to write and defend a master thesis, usually during the second term of the academic year.

It may be carried out at a research group in UPC (see the [research groups associated to the PhD program in Applied Mathematics at UPC](#)), at a research group from another university, at a research center or at a company.

A list of proposals for master thesis can be found at the FME intranet at [Borsa de Projectes](#). Even though the list includes many proposals, other researchers not included there will also be willing to supervise your work. Thus, if you are interested in a particular area, you can contact the coordinator or the [course](#) closer to it, or the master's director or a member of the [academic committee](#) for guidance.

## Regulations, calendar and templates

The calendar and the academic regulations for the master's thesis can be found at the [FME web page](#).

Templates for the document of the master thesis can be downloaded here:

- [LaTeX template](#)
- [Cover page](#)
- [Master Thesis evaluation](#)

## Master thesis defence

The master thesis is defended publically and evaluated by 3 members of UPC. The usual session works as follow:

- 1) The defence of the master thesis done by the candidate (around 25 minutes). The defence must be done preferably in English.
- 2) A round of questions (around 20 minutes) from the committee.
- 3) A round of questions (if any) from people in the audience with a master degree.

## Forthcoming defenses

A list of forthcoming presentations of master's thesis can be found at this [link](#).

# Index

34963 - Advanced Course in Partial Differential Equations  
34952 - Algebraic Geometry  
34954 - Codes and Cryptography  
34955 - Combinatorics  
34950 - Commutative Algebra  
34959 - Computational Mechanics  
34966 - Differentiable Manifolds  
34956 - Discrete and Algorithmic Geometry  
34957 - Graph Theory  
34962 - Hamiltonian Systems  
200900 - Machine Learning  
34958 - Mathematical Modelling with Partial Differential Equations  
34960 - Mathematical Models in Biology  
34951 - Non-Commutative Algebra  
34953 - Number Theory  
34964 - Numerical Methods for Dynamical Systems  
34965 - Numerical Methods for Partial Differential Equations  
34961 - Quantitative and Qualitative Methods in Dynamical Systems  
200901 - Seminar on Algebra, Geometry and Discrete Mathematics  
200902 - Seminar on Analysis, Differential Equations and Modelling



# Course guide

## 34963 - ACPDE - Advanced Course in Partial Differential Equations

Last modified: 11/04/2024

<b>Unit in charge:</b>	School of Mathematics and Statistics	
<b>Teaching unit:</b>	749 - MAT - Department of Mathematics.	
<b>Degree:</b>	MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).	
<b>Academic year:</b> 2024	<b>ECTS Credits:</b> 7.5	<b>Languages:</b> English

### LECTURER

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<b>Coordinating lecturer:</b>	OLLI SAARI
<b>Others:</b>	Segon quadrimestre: OLLI SAARI - A

### PRIOR SKILLS

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Basic knowledge of Partial Differential Equations (undergraduate level).  
Basic knowledge of Mathematical Analysis (undergraduate level).  
Basic knowledge of Functional Analysis (undergraduate level).

### REQUIREMENTS

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Undergraduate courses in Partial Differential Equations and in Mathematical Analysis.  
It is helpful an undergraduate course in Functional Analysis.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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#### Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

#### Transversal:

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.



## TEACHING METHODOLOGY

Classes will combine theoretical aspects and proofs with resolution of concrete problems and exercises. Further reading from the bibliography will be given often.

## LEARNING OBJECTIVES OF THE SUBJECT

This course is intended to be an introduction to modern methods for solving elliptic partial differential equations. However, some insights to classical solutions to parabolic and hyperbolic equations will also be given. The objectives of the course are:

- understand the classical methods to solve the transport, wave, heat, Laplace, and Poisson equations,
- understand the role of Sobolev norms and compact embeddings to solve PDE and find spectral decompositions,
- learn the modern methods to solve elliptic PDE.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

**Total learning time:** 187.5 h

## CONTENTS

### Classical methods in PDEs

**Description:**

[This topic will only be treated in the exercises sessions.] Classical solutions to the transport, wave, heat, Laplace, and Poisson equations. Maximum principles, Green's functions, separation of variables, energy methods, probabilistic interpretation.

**Full-or-part-time:** 46h

Theory classes: 15h

Self study : 31h

### Hilbert space techniques

**Description:**

Orthogonal projections, Riesz-Fréchet representation theorem, Lax-Milgram theorem.

**Full-or-part-time:** 25h

Theory classes: 8h

Self study : 17h

### Sobolev spaces

**Description:**

Mollifiers, Fréchet-Kolmogorov theorem, distributions, Sobolev norms, Poincaré inequality, compact embeddings, approximation by smooth functions, traces.

**Full-or-part-time:** 29h

Theory classes: 9h

Self study : 20h



### Weak formulation and the weak maximum principle

**Description:**

Weak solutions via Hilbert space techniques and interpretation, comparison principles in the weak formulation.

**Full-or-part-time:** 25h

Theory classes: 8h

Self study : 17h

### Regularity theory

**Description:**

Boundedness of weak solutions, Sobolev-Gagliardo-Nirenberg inequality, regularity in Sobolev spaces, the translation method, bootstrap technique.

**Full-or-part-time:** 25h 30m

Theory classes: 8h

Self study : 17h 30m

### Eigenvalues

**Description:**

Spectral decompositions, applications to (time dependent) evolution equations, Rayleigh quotient, description of the first eigenvalue for the Dirichlet problem on a bounded domain.

**Full-or-part-time:** 17h

Theory classes: 6h

Self study : 11h

### Nonlinear problems

**Description:**

Calculus of variations, monotone iteration method, obstacle problems.

**Full-or-part-time:** 20h

Theory classes: 6h

Self study : 14h

## GRADING SYSTEM

The evaluation of the course is based on:

- the resolution of problems proposed in class (40%),
- a midterm exam (20%),
- a final comprehensive exam (40%).

The active participation during the course will be a requirement for the evaluation of the final exam.



## BIBLIOGRAPHY

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### Basic:

- Chipot, M. Elliptic equations : an introductory course [on line]. Basel: Birkhäuser Basel, 2009 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-7643-9982-5>. ISBN 9783764399825.
- Evans, Lawrence C. Partial differential equations. Providence, Rhode Island: American Mathematical Society, 1998. ISBN 0821807722.
- Salsa, Sandro. Partial differential equations in action : from modelling to theory [on line]. Milan: Springer, 2008 [Consultation: 10/07/2023]. Available on: <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=3062992>. ISBN 9788847007512.
- Brézis, H. Análisis funcional : teoría y aplicaciones. Madrid: Alianza, 1984. ISBN 8420680885.

### Complementary:

- Folland, G. B. Introduction to partial differential equations. 2nd ed. Princeton (N.J.): Princeton University Press, 1995. ISBN 0691043612.
- Struwe, Michael. Variational methods : applications to nonlinear partial differential equations and hamiltonian systems [on line]. 2nd rev. and substantially expanded ed. Berlin: Springer, 1996 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-540-74013-1>. ISBN 3540520228.
- Gilbarg, David; Trudinger, Neil S. Elliptic partial differential equations of second order. 2nd ed., rev. third printing. Berlin: Springer-Verlag, 1998. ISBN 354013025X.
- Necas, Jindrich. Introduction to the theory of nonlinear elliptic equations. Chichester: John Wiley & Sons, 1986. ISBN 0471908940.



# Course guide

## 34952 - AG - Algebraic Geometry

Last modified: 11/04/2024

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2024    **ECTS Credits:** 7.5    **Languages:** English

### LECTURER

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**Coordinating lecturer:** MARIA ALBERICH CARRAMIÑANA

**Others:** Segon quadrimestre:  
MARIA ALBERICH CARRAMIÑANA - A  
ROGER GÓMEZ LÓPEZ - A

### PRIOR SKILLS

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Aquaintance with mathematical computations, both by hand and with a computer, and mathematical reasoning, including proofs.

### REQUIREMENTS

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Basic abstract Algebra, Topology and Differential Geometry.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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**Specific:**

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

**Transversal:**

4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

### TEACHING METHODOLOGY

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Approximately 50% of class time will be dedicated to interactive master classes, in which the lecturer will discuss course topics and propose small challenges and questions to solve. The other half of the class time will be structured as a problem-solving session, in which students will solve problems from a proposed list on the blackboard, based on the course syllabus, and their solutions will be discussed by the class.



## LEARNING OBJECTIVES OF THE SUBJECT

The main objective of the course is to introduce students to local algebraic geometry, with a focus on plane curve singularities. It aims to provide insight into the singularity theory of plane curves and the geometric theory of valuations of the ring of convergent series of two variables over the complex numbers. The course will demonstrate that singular points of algebraic curves in the complex plane is a meeting point for various areas of mathematics.

The course will heavily rely on examples, emphasizing the geometric significance of the subject. The specific topics for the final projects will be determined based on the students' interests.

## STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

**Total learning time:** 187.5 h

## CONTENTS

### Chapter 1: Algebraic and analytic varieties

**Description:**

Algebraic preliminaries. Affine algebraic varieties, Nullstellensatz. Analytic varieties, Teorema de Rückert.

**Full-or-part-time:** 11h 30m

Theory classes: 5h

Self study : 6h 30m

### Chapter 2: Parametrizing branches of plane curves

**Description:**

Newton-Puiseux algorithm, Weierstrass preparation and division theorems, Hensel's lemma. Intersection multiplicity: Halphen's formula. Testing analytic reducibility.

**Full-or-part-time:** 12h 30m

Theory classes: 6h

Self study : 6h 30m

### Chapter 3: Infinitely near points and resolutions of singularities

**Description:**

Proximity, Enriques diagrams, dual graph. Resolutions of singularities. Rings in successive neighbourhoods. The order of singularity.

**Full-or-part-time:** 23h

Theory classes: 10h

Self study : 13h





#### Chapter 4: Topological classification of singularities

**Description:**

Topological classification of plane curves: equisingularity, semigroup of values, complete equisingularity invariants. Milnor number. Topological conic structure of a singularity. Topology of the singularity link. The Milnor fibration.

**Full-or-part-time:** 23h

Theory classes: 10h

Self study : 13h

#### Chapter 5: Constructions on the resolution tree

**Description:**

Homology of a blow-up. The exceptional divisor of a curve. Functions on the resolution tree. Complete ideals. Multiplier ideals and jumping numbers. Topological zeta function.

**Full-or-part-time:** 18h

Theory classes: 8h

Self study : 10h

#### Chapter 6: Analytic classification of plane curves

**Description:**

Jacobian ideal and its semimodule of values. Tjurina algebra and Tjurina number. Unfoldings of equations. Deformations of curves: versal and miniversal deformations. Zariski's moduli space. Teissier's monomial curve.

**Full-or-part-time:** 18h

Theory classes: 8h

Self study : 10h

#### Chapter 7: Valuations and complete ideals

**Description:**

Classification of valuations. Zariski decomposition of complete ideals.

**Full-or-part-time:** 18h

Theory classes: 8h

Self study : 10h

#### Chapter 8: Final projects

**Description:**

The final essays of the course on the topics chosen by the students will be presented by the students themselves and commented by the course lecturers.

**Full-or-part-time:** 25h

Theory classes: 5h

Self study : 20h

## GRADING SYSTEM

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Students who solve a sufficient number of problems on the blackboard during the problem-solving class will pass the course. If they wish to improve their grade from a passing grade to a higher score, they will be assigned a final project. The final project will involve studying, writing an essay and delivering a lecture on an additional topic towards the end of the course.

Students who have not actively participated enough in the problem-solving class, or still wish to improve their grade even after completing the problem class and final project, will be required to take a final exam lasting approximately 4 hours.

## EXAMINATION RULES.

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The problem list for participation in the problem-solving class will be published at the beginning of each course unit. Students are expected to prepare these problems in advance at home.

The topics for optional final projects aimed at increasing grades will be proposed around Easter. Students will be responsible for preparing the lecture and the essay of the final project independently at home.

Students who choose to take the final exam will be required to do so without any notes, books, or other materials whatsoever.

## BIBLIOGRAPHY

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### Basic:

- Casas Alvero, Eduardo. Singularities of plane curves. Cambridge: Cambridge University Press, 2000. ISBN 0521789591.
- Fischer, Gerd. Plane algebraic curves. American Mathematical Society, cop. 2001. ISBN 0821821229.
- Ghys, E. A Singular mathematical promenade. Lyon: CNRS-ENS Éditions, 2017. ISBN 9782847889390.
- Wall, C. T. C. Singular points of plane curves. Cambridge, UK ; New York: Cambridge University Press, cop. 2004. ISBN 0521547741.
- Zariski, Oscar; Linchtin, Ben; Teissier, Bernard. The Moduli problem for plane branches. American Mathematical Society, 2006. ISBN 9780821829837.

### Complementary:

- Brieskorn, Egbert; Knörrer, Horst. Plane algebraic curves : translated by John Stillwell [on line]. Springerlink, 2012 [Consultation: 15/06/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-0348-0493-6>. ISBN 9783034804929.
- Casas-Alvero, Eduardo. Algebraic curves, the brill and noether way. Springer, ISBN 9783030290153.
- Chenciner, Alain. Courbes algébriques planes [on line]. Berlin: Springer, 2008 [Consultation: 07/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-540-33708-9>. ISBN 9783540337072.
- Greuel, Gert-Martin; Lossen, Christoph; Shustin, Eugenii. Singular algebraic curves : with an appendix by Oleg Viro. Springer, ISBN 9783030033491.
- Kollar, János. Lectures on resolution of singularities. Princeton University Press, ISBN 9780691129235.

# Course guide

## 34954 - CC - Codes and Cryptography

Last modified: 11/04/2024

<b>Unit in charge:</b>	School of Mathematics and Statistics	
<b>Teaching unit:</b>	749 - MAT - Department of Mathematics.	
<b>Degree:</b>	MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).	
<b>Academic year:</b> 2024	<b>ECTS Credits:</b> 7.5	<b>Languages:</b> English

### LECTURER

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<b>Coordinating lecturer:</b>	SIMEON MICHAEL BALL MARKS
<b>Others:</b>	Segon quadrimestre: SIMEON MICHAEL BALL MARKS - A JAVIER HERRANZ SOTOCA - A

### PRIOR SKILLS

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Basic probability, basic number theory and linear algebra

### REQUIREMENTS

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Undergraduate mathematics

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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#### Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

#### Transversal:

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

### TEACHING METHODOLOGY

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The course is divided in two parts: codes and cryptography. Each part consists of 26 h of ordinary classes, including theory and problem sessions.



## LEARNING OBJECTIVES OF THE SUBJECT

This course aims to give a solid understanding of the uses of mathematics in Information technologies and modern communications. The course focuses on the reliable and efficient transmission and storage of the information. Both the mathematical foundations and the description of the most important cryptographic protocols and coding systems are given in the course.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

**Total learning time:** 187.5 h

## CONTENTS

### Introduction

**Description:**

The problem of communication. Information theory, Coding theory and Cryptographic theory

**Full-or-part-time:** 6h 15m

Theory classes: 2h

Self study : 4h 15m

### Information and Entropy

**Description:**

Uncertainty or information. Entropy. Mutual information

**Full-or-part-time:** 18h 45m

Theory classes: 6h

Self study : 12h 45m

### Source codes without memory

**Description:**

Codes. Average length. Huffman codes. Extensions of a source. Theory of a noiseless communication. Notes of compression.

**Full-or-part-time:** 12h 30m

Theory classes: 4h

Self study : 8h 30m

### Channel coding

**Description:**

Discrete channels without memory. Symmetric channels. Shannon's theorem.

**Full-or-part-time:** 18h 45m

Theory classes: 6h

Self study : 12h 45m



### Block codes

**Description:**

Hamming's distance. Detection and correction of errors. Bounds. Linear codes.

**Full-or-part-time:** 18h 45m

Theory classes: 6h

Self study : 12h 45m

### Cyclic codes

**Description:**

Cyclic codes. Generator and control matrices. Factorization of  $x^n-1$ . Roots of a cyclic code. BCH codes. Primitive Reed-Solomon codes. Meggit's decoder.

**Full-or-part-time:** 18h 45m

Theory classes: 6h

Self study : 12h 45m

### Introduction to modern cryptography

**Description:**

The setting: secure storage and symmetric key encryption. Turing machines and complexity classes. Security definitions. Adversarial models. Reductionist security proofs.

**Full-or-part-time:** 15h 37m

Theory classes: 5h

Self study : 10h 37m

### Symmetric key cryptography

**Description:**

Symmetric key encryption. Pseudorandom generators. Block ciphers. Message authentication codes.

**Full-or-part-time:** 15h 38m

Theory classes: 5h

Self study : 10h 38m

### Public key encryption

**Description:**

Definitions and security notions. One way functions. Probabilistic encryption. Main constructions. Homomorphic encryption. Chosen ciphertext security.

**Full-or-part-time:** 15h 37m

Theory classes: 5h

Self study : 10h 37m



### Digital signatures

**Description:**

Security definitions. RSA and Schnorr signatures.

**Full-or-part-time:** 15h 38m

Theory classes: 5h

Self study : 10h 38m

### Proofs of knowledge and other cryptographic protocols

**Description:**

Ring signatures. Distributed signatures. Identity and attribute based protocols.

**Full-or-part-time:** 15h 37m

Theory classes: 5h

Self study : 10h 37m

### Multiparty computation

**Description:**

Secret sharing schemes. Unconditionally and computationally secure multiparty computation.

**Full-or-part-time:** 15h 38m

Theory classes: 5h

Self study : 10h 38m

## GRADING SYSTEM

Exam of coding part (50%) and exam of crypto part (50%). If the average is less than 5 out of 10, there is a chance to pass the subject in a final exam.

## EXAMINATION RULES.

All the subjects are important. To pass the course it is required to fulfill all the items.

## BIBLIOGRAPHY

**Basic:**

- Delfs, Hans; Knebl, Helmut. Introduction to cryptography : principles and applications. 2nd ed. Berlin: Springer, 2007. ISBN 9783540492436.
- Katz, Jonathan; Lindell, Yehuda. Introduction to modern cryptography : principles and protocols. Boca Raton: Chapman & Hall, 2008. ISBN 9781584885511.
- Ball, Simeon. A Course in algebraic error-correcting codes [on line]. Birkhauser, 2020 [Consultation: 07/07/2023]. Available on: <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=6194949>. ISBN 9783030411534.
- Jones, Gareth A.; Jones, J. Mary. Information and coding theory. Springer, 2000. ISBN 9781447103615.

**Complementary:**

- Huffman, W. Cary; Pless, Vera. Fundamentals of error-correcting codes. Cambridge: Cambridge University Press, 2003. ISBN 0521782805.
- Justesen, Jorn; Hoholdt, Tom. A Course in error-correcting codes. Zürich: European Mathematical Society, 2004. ISBN 3037190019.



- Welsh, Dominic. Codes and cryptography. Oxford: Oxford university Press, 1988. ISBN 0198532881.
- Xambó Descamps, Sebastián. Block error-correcting codes : a computational primer. Berlin: Springer, 2003. ISBN 3540003959.
- Goldreich, Oded. Foundations of cryptography : basic tools. New York: Cambridge University Press, 2001. ISBN 0521791723.
- Goldreich, Oded. Foundations of cryptography : basic applications. New York: Cambridge University Press, 2004. ISBN 9780521830843.



## Course guide

### 34955 - COMB - Combinatorics

Last modified: 11/04/2024

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2024    **ECTS Credits:** 7.5    **Languages:** English

#### LECTURER

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**Coordinating lecturer:** JUAN JOSÉ RUE PERNA

**Others:** Segon quadrimestre:  
RICHARD JOHANNES LANG - A  
PATRICK MORRIS - A  
JUAN JOSÉ RUE PERNA - A  
ORIOL SERRA ALBO - A  
LLUIS VENA CROS - A

#### PRIOR SKILLS

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Basic calculus and linear algebra. Notions of probability.

#### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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**Specific:**

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
  2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
  3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.
- MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.

**Transversal:**

4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

#### TEACHING METHODOLOGY

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There will be a lecture each week, followed by a problem session.





## LEARNING OBJECTIVES OF THE SUBJECT

To use algebraic, probabilistic and analytic methods for studying combinatorial structures. The main topics of study are: partially ordered sets, extremal set theory, finite geometries, matroids, Ramsey theory and enumerative combinatorics.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

**Total learning time:** 187.5 h

## CONTENTS

### Partially ordered sets

**Description:**

Sperner's theorem. LYM inequalities. Bollobás's theorem. Erdos-Ko-Rado Theorem. Dilworth's theorem. Applications of Dilworth Theorem. Lattices and distributive lattices. The 4 functions theorem and applications

**Full-or-part-time:** 24h 40m

Practical classes: 4h

Laboratory classes: 4h

Self study : 16h 40m

### Ramsey theory

**Description:**

Theorems of Ramsey and Hales-Jewett. Theorems of Schur, Van der Waerden and Rado.

**Full-or-part-time:** 31h 40m

Theory classes: 5h

Laboratory classes: 5h

Self study : 21h 40m

### Probabilistic methods in combinatorics

**Description:**

First and second moment. Lovász Local Lemma and entropy methods. Applications: Permanents, transversals, hypergraph coloring. Monotone properties and threshold functions.

**Full-or-part-time:** 18h 30m

Theory classes: 3h

Laboratory classes: 3h

Self study : 12h 30m



### Linear algebra methods in combinatorics

**Description:**

The polynomial method and applications. Fisher's theorem. Applications

**Full-or-part-time:** 18h 30m

Theory classes: 3h

Laboratory classes: 3h

Self study : 12h 30m

### GRADING SYSTEM

Continuous evaluation will be based on the weekly solution of exercises. There will be also a final examination. The grading will be based on the continuous evaluation during the course (60%) and the final exam (40%).

### BIBLIOGRAPHY

**Basic:**

- Alon, Noga; Spencer, Joel H.; Erdős, Paul. The Probabilistic method. 3rd ed. New York: Wiley, 2008. ISBN 0471535885.
- Bollobás, Béla; Andrew Thomason (eds.). Combinatorics, geometry, and probability : a tribute to Paul Erdos. Cambridge: Cambridge University Press, 1997. ISBN 0521584728.
- Lint, Jacobus Hendricus van; Wilson, R. M. A Course in combinatorics. 2nd ed. Cambridge: Cambridge University Press, 2001. ISBN 0521803403.
- Flajolet, P.; Sedgewick, R. Analytic combinatorics [on line]. Cambridge: Cambridge University Press, 2009 [Consultation: 07/07/2023]. Available on : <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=412737>. ISBN 9780521898065.
- Graham, Ronald L.; Rothschild, B.; Spencer, J. Ramsey theory. 2nd ed. New York: John Wiley & Sons, 1990. ISBN 0471500461.
- Anderson, Ian. Combinatorics of finite sets. Mineola: Dover, 2002. ISBN 0486422577.
- Lovász, László. Combinatorial problems and exercises. 2nd ed. Amsterdam: North-Holland, 1993. ISBN 044481504X.
- Oxley, J. G. Matroid theory [on line]. 2nd ed. Oxford: Oxford University Press, 2011 [Consultation: 13/12/2022]. Available on: <https://academic-oup-com.recursos.biblioteca.upc.edu/book/34846?searchresult=1>. ISBN 9780199603398.
- Jukna, Stasys. Extremal combinatorics. 2011. Springer, 2011. ISBN 9783642173639.



## Course guide

# 34950 - CALG - Commutative Algebra

**Last modified:** 15/05/2024

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2024    **ECTS Credits:** 7.5    **Languages:** English

### LECTURER

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**Coordinating lecturer:** FRANCESC D'ASSIS PLANAS VILANOVA

**Others:** Primer quadrimestre:  
FRANCESC D'ASSIS PLANAS VILANOVA - A

### PRIOR SKILLS

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Linear algebra, algebraic structures, topology.

### REQUIREMENTS

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The two first years of a degree in mathematics.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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**Specific:**

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

**Transversal:**

4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

### TEACHING METHODOLOGY

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Teaching Classes, resolution of problems



## LEARNING OBJECTIVES OF THE SUBJECT

Basic course in Commutative Algebra.  
An introduction to the theory of rings, ideals and modules.  
Some basics on local algebra.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

## CONTENTS

### Rings and ideals

**Description:**

Basics on ring theory and ideals.  
Rings of fractions. Primary decomposition. Chain conditions. Noetherian and Artinian rings.

**Full-or-part-time:** 28h 20m

Theory classes: 15h  
Self study : 13h 20m

### Modules

**Description:**

General properties of modules.  
Modules of fractions. Chain conditions. Homomorphisms and tensor product.

**Full-or-part-time:** 24h

Theory classes: 12h  
Self study : 12h

### Algebraic varieties

**Description:**

The spectrum of a ring. Zariski topology.

**Full-or-part-time:** 24h

Theory classes: 12h  
Self study : 12h

### Introduction to homological algebra

**Description:**

Categories and functors. Complexes of modules. Derived functors.

**Full-or-part-time:** 24h

Theory classes: 12h  
Self study : 12h



## Local algebra

### Description:

Regular sequences. Depth.  
Homological characterizations.  
Regular rings, Gorenstein rings, Cohen-Macaulay rings

**Full-or-part-time:** 18h 40m

Theory classes: 9h

Self study : 9h 40m

## GRADING SYSTEM

The qualification will be based on:

60% Resolution of assigned exercises and/or projects

40% Final Exam

## BIBLIOGRAPHY

### Basic:

- Atiyah, Michael Francis; MacDonald, I. G. Introduction to commutative algebra. Reading: Addison-Wesley, 1969. ISBN 0201407515.
- Reid, Miles. Undergraduate commutative algebra. Cambridge: Cambridge University Press, 1995. ISBN 0521452554.
- Eisenbud, David. Commutative algebra : with a view toward algebraic geometry. Corrected 2nd. printing. New York: Springer-Verlag, 1996. ISBN 0387942696.
- Kunz, Ernst. Introduction to commutative algebra and algebraic geometry [on line]. Birkhäuser, 2013 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-1-4614-5987-3>. ISBN 9781461459866.
- Matsumura, Hideyuki. Commutative ring theory. Cambridge: Cambridge University Press, ISBN 0521259169.
- Rotman, J.J. An Introduction to homological algebra [on line]. Academic Press, 1979 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/b98977>. ISBN 0125992505.
- Bruns, Winfried; Herzog, Jürgen. Cohen-Macaulay rings. Cambridge University Press, 1993. ISBN 0521410681.

# Course guide

## 34959 - CM - Computational Mechanics

**Last modified:** 11/04/2024

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 749 - MAT - Department of Mathematics.  
751 - DECA - Department of Civil and Environmental Engineering.  
748 - FIS - Department of Physics.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2024    **ECTS Credits:** 7.5    **Languages:** English

### LECTURER

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**Coordinating lecturer:** JOSE JAVIER MUÑOZ ROMERO

**Others:** Segon quadrimestre:  
DAVID CODONY GISBERT - A  
JOSE JAVIER MUÑOZ ROMERO - A

### PRIOR SKILLS

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Basic knowledge of numerical methods  
Basic knowledge of partial differential equations

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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**Specific:**

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

**Transversal:**

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

## TEACHING METHODOLOGY

Four elements will be combined:

- Theory classes, where the main concepts will be presented.
- Practical classes with Matlab code in the computer room, with emphasis on the computational aspects.
- Lists of short assignments.
- Course projects in groups to be presented orally at the end of the course.

Students will work on the assignments and course projects individually or in groups.

## LEARNING OBJECTIVES OF THE SUBJECT

The main objective is to provide a general perspective of the broad field of computational mechanics, covering both the modelling and the computational aspects. A broad range of problems is addressed: solids, fluids and fluid-solid interaction; linear and nonlinear models; static and dynamic problems. Some emphasis is put on applications in biomechanical problems. By the end of the course, the students should:

- Be able to choose the appropriate type of model for a specific simulation
- Be familiar with the mathematical objects (tensors) and differential operators used in computational mechanics
- Be aware of the different level of complexity of various problems (e.g. linear vs. nonlinear, static vs. dynamic).

## STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

**Total learning time:** 187.5 h

## CONTENTS

### CONTINUUM MECHANICS

#### Description:

Motivation. Definition of continuous media. Equation of motion: Eulerian and Lagrangian descriptions. Time derivatives. Strains: deformation gradient, Green and Euler-Almansi tensors; elongation and shear; small strains. Stresses: body and surface forces; Cauchy stress tensor. Balance equations: Reynolds transport theorem; mass balance; momentum balance. Constitutive equations. Applications.

**Full-or-part-time:** 31h 15m

Theory classes: 8h

Practical classes: 2h

Self study : 21h 15m

### COMPUTATIONAL ELASTICITY

#### Description:

Basic concepts and motivation. Elastic constitutive equation. Displacement formulation: Navier equations. Two-dimensional elasticity: plane stresses, plane strains and axisymmetry. Weak form of the elastic problem. Finite element discretisation. Computational aspects. Applications in engineering and biomechanics.

**Full-or-part-time:** 31h 15m

Theory classes: 8h

Practical classes: 2h

Self study : 21h 15m



### COMPUTATIONAL DYNAMICS

**Description:**

Weak form. Dynamic equation. Space discretisation (finite elements) and time discretisation. Solution methods: generalised eigen value problem and direct time integration. Euler, centred differences, HHT and Newmark methods. Stability, consistency and accuracy of numerical techniques in elastodynamics. Applications and relation with reduced order techniques (Proper Orthogonal Decomposition).

**Full-or-part-time:** 31h 15m

Theory classes: 8h

Practical classes: 2h

Self study : 21h 15m

### COMPUTATIONAL PLASTICITY, FRACTURE AND VISCOELASTICITY

**Description:**

Basic concepts and motivation. Non-linear problems. Geometrical and material non-linearity. One-dimensional plasticity: elastic and plastic strains;

elastoplastic constitutive equation; hardening. Multi-dimensional plasticity: stress and strain

invariants; yield surface; plastic flow. Numerical time-integration of the constitutive

equation: elastic prediction and plastic correction; iterative methods for the plastic

correction. Applications. Phase field formulation and Finite Element discretisation.

Viscoelastic materials. Maxwell and Kelvin-Voigt one-dimensional models. Numerical solution. Extension to multiple dimensions.

**Full-or-part-time:** 31h 15m

Theory classes: 8h

Practical classes: 2h

Self study : 21h 15m

### COMPUTATIONAL FLUID DYNAMICS

**Description:**

Basic concepts and motivation. Rate-of-deformation and spin tensors. Constitutive equation for

Newtonian fluids. Euler equations for inviscid flow. Navier-Stokes equations for viscous flow in strong form and in weak form.

Reynolds number. Stokes flow and potential flow. Finite element discretisation and numerical solution. Applications.

**Full-or-part-time:** 31h 15m

Theory classes: 8h

Practical classes: 2h

Self study : 21h 15m

### COMPUTATIONAL METHODS FOR WAVE PROBLEMS

**Description:**

Basic concepts and motivation.

Acoustics: the wave equation. The scalar Helmholtz equation. Vibroacoustics: acoustic fluid-elastic solid interaction.

Computational aspects. Applications.

Electromagnetism: the Maxwell equations. Electrodynamics. The vectorial Helmholtz equation.

Computational aspects. Applications.

**Full-or-part-time:** 31h 15m

Theory classes: 8h

Practical classes: 2h

Self study : 21h 15m





## GRADING SYSTEM

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Final exam (40%), assignment problems (30%), and course project (30%, evaluated with an oral presentation and a written report).

## BIBLIOGRAPHY

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### Basic:

- Clough, Ray W.; Penzien, J. Dynamics of structures. 2nd ed. New York: McGraw-Hill, 1993. ISBN 0071132414.
- Donea, Jean M.; Huerta, A. Finite element methods for flow problems. Chichester: John Wiley & Sons, 2003. ISBN 0471496669.
- Ihlenburg, F. Finite element analysis of acoustic scattering [on line]. New York: Springer-Verlag, 1998 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/b98828>. ISBN 0387983198.
- Mase, G. Thomas; Mase, George E. Continuum mechanics for engineers. 3rd ed. Boca Raton: CRC, 2010. ISBN 9781420085389.

### Complementary:

- Bathe, Klaus-Jürgen. Finite element procedures. New Jersey: Prentice-Hall, 1996. ISBN 0133014584.
- Bonet, Javier; Wood, R. D. Nonlinear continuum mechanics for finite element analysis. 2nd ed. Cambridge: Cambridge University Press, 2008. ISBN 9780521838702.
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# Course guide

## 34966 - VD - Differentiable Manifolds

Last modified: 11/04/2024

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 749 - MAT - Department of Mathematics.  
981 - CRM - Mathematical Research Centre.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2024    **ECTS Credits:** 7.5    **Languages:** English

### LECTURER

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**Coordinating lecturer:** EVA MIRANDA GALCERÁN

**Others:** Segon quadrimestre:  
EVA MIRANDA GALCERÁN - A  
PABLO NICOLÁS MARTÍNEZ - A

### PRIOR SKILLS

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Basic courses on algebra, calculus, topology and differential equations, and calculus on manifolds. Students from the FME are supposed to have taken "Varietats Diferenciables" (optional 4th year course).

This is not a basic course and the students are assumed to have attended previous courses on differential geometry and smooth manifolds. Students feeling that they may not fulfill the requisites are invited to discuss their case with the lecturers. It is totally possible for prospective students with less knowledge in these topics to follow this course provided they are willing to make up for the gap with individual work during the course and/or by reading some recommended bibliography prior to the beginning of the course.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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**Specific:**

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

**Transversal:**

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

### TEACHING METHODOLOGY

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Theory classes and tutorial sessions will be used to present and develop the contents of the course. Along the course the students will be given problems to solve as homework.



## LEARNING OBJECTIVES OF THE SUBJECT

The subject focuses on some of the fundamental topics of differential geometry and its applications to different areas including mathematical physics and Dynamical systems.

By the end of the course, students should be able to:

- understand all the ideas developed along the course.
- apply the studied concepts to other areas of pure mathematics, physics and engineering.
- integrate in a research group on these kinds of topics and their applications.
- search and understand the scientific literature on the subject.
- write and present an essay on mathematics.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

**Total learning time:** 187.5 h

## CONTENTS

### Complements in Differential Geometry

**Description:**

Brief survey of manifold theory and differential geometry including differential forms.

We also plan to talk about differentiable distributions and study its integration via the theorem of Frobenius. This will lead us to introducing several examples of foliations.

**Full-or-part-time:** 14h 52m

Theory classes: 6h

Self study : 8h 52m

### Introduction to Differential Topology

**Description:**

We present a brief introduction to the theory of Differential Topology which includes basic notions in transversality, singularity theory and Morse theory.

**Full-or-part-time:** 14h 40m

Theory classes: 8h

Self study : 6h 40m

### Introduction to Lie theory

**Description:**

A Lie group is a group endowed with a smooth manifold structure which is compatible with the group operation. In this chapter we provide an

introduction to the main aspects of the theory of Lie groups and Lie algebras taking matrix Lie groups as starting point.

**Full-or-part-time:** 16h 20m

Theory classes: 8h

Self study : 8h 20m



### Lie group actions on smooth manifolds

**Description:**

We study Lie group actions on smooth manifolds and relate both geometries via the notions of isotropy group and orbit.

**Full-or-part-time:** 18h

Theory classes: 4h

Theory classes: 4h

Self study : 5h

Self study : 5h

### Basic notions on De Rham Cohomology

**Description:**

We define De Rham cohomology and compare it to other cohomologies. (Depending on the preliminary knowledge of the students, this chapter may be considered as an APPENDIX)

**Full-or-part-time:** 8h

Theory classes: 3h

Self study : 5h

### Introduction to Symplectic and Poisson Geometry

**Description:**

We provide a comprehensive introduction to symplectic and Poisson manifolds with special focus on examples. Starting with symplectic manifolds, we will explain Moser's trick and some applications to normal form theorems such as the Darboux theorem and the classification of symplectic surfaces. We introduce the notion of Hamiltonian vector field, symplectic vector field and Hamiltonian System. Special attention will be given to examples provided by the realm of integrable systems. In particular the action-angle theorem of Arnold-Liouville will be presented and the notion of moment map and Hamiltonian group action. We end the chapter introducing other geometries: that of contact geometry and Poisson geometry discussing the most recent progress on open conjectures in the topic.

**Full-or-part-time:** 31h 40m

Theory classes: 15h

Self study : 16h 40m

## GRADING SYSTEM

There will be exam(s) which will contribute to the final grade in a 40% and an essay that will contribute to the final grade in another 40%. Students would choose, together with the lecturers, a topic that complements or advances the material taught during the course, according to their mathematical interests. The remaining 20% is reserved to problem solving which can be evaluated by simply solving some assignments of the regular list of problems.

## EXAMINATION RULES.

The final grade awarded to the student would we computed as follows:

40% exam(s) + 40% essay+ 20% problem solving

The grade "exam(s)" includes the one of final exam but may also include other examination material such as ATENEA questionnaires or take-home exercises. The choices and number of exams will depend on several factors including the ratio presential versus online teaching.

## BIBLIOGRAPHY

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### Basic:

- Bröcker, T.; Jänich, K. Introduction to differential topology. Cambridge University Press, 1982. ISBN 9780521284707.
- Cannas da Silva, Anna. Lectures on symplectic geometry [on line]. Springer-Verlag, 2008 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-540-45330-7>. ISBN 354045330X.
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- Tu, Loring W. An Introduction to manifolds [on line]. 2nd ed. New York: Springer, 2011 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-1-4419-7400-6>. ISBN 9781441973993.
- Warner, Frank W. Foundations of differentiable manifolds and Lie groups. New York [etc.]: Springer, cop. 1971. ISBN 0387908943.

### Complementary:

- Bott, Raoul; Tu, Loring W. Differential forms in algebraic topology. New York: Springer-Verlag, 1982. ISBN 0387906134.
- Audin, Michèle. Torus actions on symplectic manifolds. 2nd ed. Birkhäuser, 2004. ISBN 3764321768.
- Warner, Frank W. Foundations of differentiable manifolds and lie groups. New York, NY [etc.]: Springer-Verlag, cop. 1983. ISBN 0387908943.
- Olver, Peter J. Applications of Lie groups to differential equations. New York: Springer-Verlag, 1986. ISBN 0387940073.
- Nakahara, Mikio. Geometry, topology, and physics. 2nd ed. New York [etc.]: Taylor & Francis, cop. 2003. ISBN 0750306068.
- Audin, Michèle; Damian, Mihai. Morse theory and Floer homology. New York: Springer, [2014]. ISBN 9781447154952.
- Gallier, Jean; Quaintance, Jocelyn. Notes on differential geometry and Lie groups [on line]. University of Pennsylvania, 2016 [Consultation: 10/07/2023]. Available on: <http://www.cis.upenn.edu/~jean/gbooks/manif.html>.

## RESOURCES

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### Other resources:

Notes on the Geometry and Dynamics of singular symplectic manifolds (notes on the FSMP course by Eva Miranda)

Course on youtube by Professor Eva Miranda on Lie group actions

[https://www.youtube.com/channel/UC8Fzyf58s0EiZ-gdYgz2ghw?view\\_as=subscriber](https://www.youtube.com/channel/UC8Fzyf58s0EiZ-gdYgz2ghw?view_as=subscriber) />Course on youtube by Professor Eva Miranda on Symplectic and Poisson Geometry

[https://www.youtube.com/channel/UC8Fzyf58s0EiZ-gdYgz2ghw?view\\_as=subscriber](https://www.youtube.com/channel/UC8Fzyf58s0EiZ-gdYgz2ghw?view_as=subscriber)



## Course guide

# 34956 - DG - Discrete and Algorithmic Geometry

**Last modified:** 11/04/2024

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2024    **ECTS Credits:** 7.5    **Languages:** English

### LECTURER

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**Coordinating lecturer:** JULIAN PFEIFLE

**Others:** Primer quadrimestre:  
JULIAN PFEIFLE - A  
RODRIGO IGNACIO SILVEIRA ISOBA - A

### PRIOR SKILLS

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- Elementary combinatorics.
- Elementary graph theory.
- Elementary algorithmics.
- Elementary data structures.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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#### Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

#### Transversal:

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

## TEACHING METHODOLOGY

Theory classes will be used to present and develop the contents of the course. Most of the topics will be presented by the instructors, but there can be some sessions devoted to students presentations.

There will be lists of problems, which will not contain solutions. Problems will be designed to help students deepen and mature their command of the concepts and techniques presented in class. Some problems will be solved in class, some will be left as homework. In the problem sessions, the goal will be to propose and analyze alternative strategies to solve each problem, and to show how the results presented in class are applied. Most of the problems solved in class will be presented by the students.

## LEARNING OBJECTIVES OF THE SUBJECT

Discrete, combinatorial and computational geometry are facets of a common body of knowledge that integrates fundamental elements from mathematics -mainly from algebra, topology and classical branches of geometry- with elements and problems from theoretical computer science and its applications.

The area focuses on the combinatorial and structural study of discrete geometric objects, as well as the design of algorithms to construct or analyze them. Among the objects studied, we can mention discrete sets of points, curves and manifolds, polytopes, convex bodies, packings, space decompositions, graphs, and geometric matroids.

By the end of the course, students should:

- Be able to recognize and formally express discrete geometric problems.
- Be able to discretize geometric problems, when possible.
- Be able to apply combinatorial techniques, as well as data structures and algorithms to discrete geometric problems.
- Be able to search the bibliography, and to understand the scientific literature on the subject.
- Be aware of the wide range of fields and problems to which discrete geometry results apply.
- Be aware of the most commonly used software in the field.

## STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

**Total learning time:** 187.5 h

## CONTENTS

### Preliminaries

**Description:**

linear, affine, non-negative and convex hull; complexes and their combinatorics; homogeneous coordinates

**Full-or-part-time:** 12h 30m

Theory classes: 4h

Self study : 8h 30m

### Convex polytopes

**Description:**

H/V polytopes; beneath-beyond convex hull computation; face lattice; f-, h- and g-vector

**Full-or-part-time:** 19h

Theory classes: 6h

Self study : 13h



### Constructions and Examples

**Description:**

Prisms and pyramids; Products and direct sums; Polarity; Gale duality; Orbit polytopes; Permutahedra; Matroid polytopes

**Full-or-part-time:** 31h

Theory classes: 10h

Self study : 21h

### Complexes

**Description:**

Exterior and normal fan; the phylogenetic space of trees; tropical varieties

**Full-or-part-time:** 31h

Theory classes: 10h

Self study : 21h

### Computational complexity and data structures

**Description:**

Computational complexity. Data structures. Representation of geometric objects.

**Full-or-part-time:** 12h 30m

Theory classes: 4h

Self study : 8h 30m

### Convexity in low dimensions

**Description:**

Convex hull computation. Linear programming in low dimensions.

**Full-or-part-time:** 19h

Theory classes: 6h

Self study : 13h

### Decompositions and arrangements

**Description:**

Subdivisions and triangulations of point sets and polygons. Visibility and motion planning. Duality. Special decompositions in dimension 2. The zone theorem. Incremental construction and randomized algorithms. Complexity. Levels and k-sets.

**Full-or-part-time:** 31h

Theory classes: 10h

Self study : 21h



## Proximity Structures

### Description:

Proximity problems. Voronoi diagram, Delaunay triangulation. Shape reconstruction.

### Full-or-part-time: 31h

Theory classes: 10h

Self study : 21h

## GRADING SYSTEM

The course consists in two parts, each contributes with 50 % to the final grade.

For each part: students will obtain marks by turning in their solutions to problems from the problem sets, by presenting solutions to problems or summarizing relevant research paper, or, optionally, by an exam.

## BIBLIOGRAPHY

### Basic:

- Berg, Mark de; Cheong, Otfried; Kreveld, Marc van; Overmars, Mark. Computational geometry : algorithms and applications [on line]. 3rd ed. revised. Berlin: Springer, 2008 [Consultation: 10/07/2023]. Available on: <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=6311912>. ISBN 9783642096815.
- Boissonnat, J. D.; Yvinec, M. Algorithmic geometry. Cambridge: Cambridge University Press, 1997. ISBN 0521565294.
- Edelsbrunner, Herbert. Algorithms in combinatorial geometry. Berlin: Springer-Verlag, 1987. ISBN 354013722X.
- Matousek, Jirí. Lectures on discrete geometry. New York: Springer, 2002. ISBN 0387953736.
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- Ziegler, Günter M. Lectures on polytopes. New York: Springer-Verlag, 1995. ISBN 038794365X.

### Complementary:

- Bokowski, Jürgen. Computational oriented matroids : equivalence classes of matrices within a natural framework. Cambridge: Cambridge University Press, 2006. ISBN 9780521849302.
- Conway, John Horton; Sloane, N. J. A. Sphere packings, lattices and groups. 3rd ed. Berlin: Springer, 1999. ISBN 0387985859.
- Schurmann, Achill. Computational geometry of positive definite quadratic forms : polyhedral reduction theories, algorithms, and applications. Providence: American Mathematical Society, 2009. ISBN 9780821847350.
- Weeks, Jeffrey R. The Shape of space. 2nd. ed. New York: M. Dekker, 2002. ISBN 0824707095.
- Richter-Gebert, Jürgen. Perspectives on projective geometry : a guided tour through real and complex geometry [on line]. Berlin: Springer, 2011 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-642-17286-1>. ISBN 9783642172854.
- Beck, Matthias; Robins, Sinai. Computing the continuous discretely : integer-point enumeration in polyhedra [on line]. New York: Springer, cop. 2007 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-46112-0>. ISBN 9780387291390.
- Edelsbrunner, Herbert; Harer, John. Computational Topology : an introduction. American Mathematical Society, cop. 2010. ISBN 9780821849255.
- Rabadán, Raúl; Blumberg, Andrew J. Topological data analysis for genomics and evolution. Cambridge University Press, 2020. ISBN 9781107159549.



## RESOURCES

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### Audiovisual material:

- Mathfilm festival 2008 [Enregistrement vidéo]: a collection of mathematical videos. Berlin : Springer, 2008. [https://discovery.upc.edu/discovery/fulldisplay?docid=alma991003546329706711&context=L&vid=34CSUC\\_UPC:VU1&lang=ca](https://discovery.upc.edu/discovery/fulldisplay?docid=alma991003546329706711&context=L&vid=34CSUC_UPC:VU1&lang=ca)

- Videomath Festival at International Congress of Mathematicians, Berlin, Germany 1998 [Enregistrement vidéo] / edited and produced Hans Christian Hege, Konrad Polthier. [Berlin] : Springer, 1998. [https://discovery.upc.edu/discovery/fulldisplay?docid=alma991001865709706711&context=L&vid=34CSUC\\_UPC:VU1&lang=ca](https://discovery.upc.edu/discovery/fulldisplay?docid=alma991001865709706711&context=L&vid=34CSUC_UPC:VU1&lang=ca)

- Not knot [Enregistrement vidéo] / directed by Charlie Gunn and Delle Maxwell ; [written by David Epstein ... [et al.]]. Minnesota : Geometry Center, University of Minnesota, 1991. [https://discovery.upc.edu/discovery/fulldisplay?docid=alma991001445799706711&context=L&vid=34CSUC\\_UPC:VU1&lang=ca](https://discovery.upc.edu/discovery/fulldisplay?docid=alma991001445799706711&context=L&vid=34CSUC_UPC:VU1&lang=ca)

- Flatland [Enregistrement vidéo] : a journey of many dimensions / written by Seth Caplan, Dano Johnson, Jeffrey Travis ; directed by Jeffrey Travis, Dano Johnson. [S.l.] : Flat World Productions, cop. 2007. [https://discovery.upc.edu/discovery/fulldisplay?docid=alma991003683649706711&context=L&vid=34CSUC\\_UPC:VU1&lang=ca](https://discovery.upc.edu/discovery/fulldisplay?docid=alma991003683649706711&context=L&vid=34CSUC_UPC:VU1&lang=ca)



# Course guide

## 34957 - GT - Graph Theory

**Last modified:** 13/05/2024

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2024    **ECTS Credits:** 7.5    **Languages:** English

### LECTURER

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**Coordinating lecturer:** GUILLEM PERARNAU LLOBET

**Others:** Primer quadrimestre:  
GUILLEM PERARNAU LLOBET - A  
ORIOI SERRA ALBO - A

### PRIOR SKILLS

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Elementary Calculus and Linear Algebra; basic notions and skills in combinatorics and probability.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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**Specific:**

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

**Transversal:**

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

### TEACHING METHODOLOGY

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Combination of theoretical lectures and exercise classes, with student presenting their solutions to the proposed problems. The active participation of students is a requirement for the course assessment.

## LEARNING OBJECTIVES OF THE SUBJECT

Basics of Graph Theory.  
 Random graphs.  
 Applications of random graphs: the probabilistic method.  
 Spectral techniques to the study of graphs.  
 Applications of spectral techniques: expansion, clustering, embedding and random walks.  
 Extremal graph theory  
 Applications of extremal techniques: graph limits.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

**Total learning time:** 187.5 h

## CONTENTS

### Spectral Graph Theory

#### Description:

Introduction to spectral graph theory, graph expanders and applications to clustering, embedding and random walks on graphs.

#### Specific objectives:

Adjacency and Laplacian matrices. Courant-Fischer Theorem. Interlacing. Cospectral graphs. Graph invariants: independence number, chromatic number, diameter,... Spectral gap and second largest eigenvalue. Isoperimetry and Cheeger's inequality. Matrix tree theorem. Clustering graph algorithm. Graph embedding and drawing algorithms. Random walks in graphs.

#### Related competencies :

MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.

MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.

MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.

06 URI. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

05 TEQ. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.

03 TLG. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.

04 COE. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.

07 AAT. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

**Full-or-part-time:** 16h

Theory classes: 16h



### Extremal Graph Theory

**Description:**

Introduction to Extremal Graph Theory, the Szemerédi Regularity Lemma, and applications to Graph limits

**Specific objectives:**

Mantel Theorem, Turán Theorem, Erdős-Stone-Simonovits Theorem, Szemerédi Regularity Lemma, Counting Lemma, Triangle Removal Lemma, Graph Limits

**Full-or-part-time:** 20h

Theory classes: 20h

### Random graphs

**Description:**

Introduction to classical model of random graphs and its main combinatorial properties.

**Specific objectives:**

Erdős-Rényi model of random graphs.

Properties of almost all graphs.

First and second moment methods.

The probabilistic method.

Threshold functions.

Method of Moments

Chernoff's inequality

Component phase transitions

**Related competencies :**

MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.

MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.

MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.

06 URI. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

05 TEQ. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.

03 TLG. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.

04 COE. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.

07 AAT. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

**Full-or-part-time:** 12h

Theory classes: 12h



## An introduction to Graph Theory

### Description:

Introduction to random graphs, main properties and classical theorems.

### Specific objectives:

Basic terminology and notation  
Paths and cycles  
Distance and Diameter  
Connectivity  
Trees  
Matchings  
Colorings

### Related competencies :

MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.  
MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.  
MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.  
MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.  
06 URI. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.  
05 TEQ. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.  
03 TLG. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.  
04 COE. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.  
07 AAT. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

### Full-or-part-time: 12h

Theory classes: 12h

## GRADING SYSTEM

The assessment of the course is as follows:

- weekly work on the proposed problems and their presentation during the lectures, 30% of the mark
- a final comprehensive exam on the course topics, 70% of the mark

## EXAMINATION RULES.

The active participation in the course is a requirement for the final assessment.



## BIBLIOGRAPHY

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### Basic:

- Diestel, Reinhard. Graph theory. 3rd ed. Berlin: Springer, 2005. ISBN 3540261826.
- Hell, Pavol; Nešetřil, Jaroslav. Graphs and homomorphisms. Oxford: Oxford University Press, 2004. ISBN 0198528175.
- Frieze, Alan; Karonski, Michal. Introduction to random graphs [on line]. Cambridge, [etc.]: Cambridge University Press, cop. 2016 [Consultation: 10/07/2023]. Available on: <https://www-cambridge-org.recursos.biblioteca.upc.edu/core/books/introduction-to-random-graphs/0F67A19795B731B0C97EAB5BB5748CF2>. ISBN 9781107118508.
- Brouwer, A. E; Haemers, W. H. Spectra of graphs [on line]. New York [etc.]: Springer, cop. 2012 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-1-4614-1939-6>. ISBN 9781461419389.
- Alon, Noga; Spencer, Joel. The Probabilistic method. 2016. Wiley, 2015. ISBN 9781119061953.

### Complementary:

- Lovász, László. Large networks and graph limits. Providence, R.I.: American Mathematical Society, cop. 2012. ISBN 9780821890851.
- Chung, Fan R. K. Spectral graph theory. Providence: American Mathematical Society, 1997. ISBN 0821803158.
- Biggs, Norman L. Algebraic graph theory. 2nd ed. New York: Cambridge University Press, 1993. ISBN 0521458978.



# Course guide

## 34962 - HS - Hamiltonian Systems

Last modified: 11/04/2024

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2024    **ECTS Credits:** 7.5    **Languages:** English

### LECTURER

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**Coordinating lecturer:** PAU MARTIN DE LA TORRE

**Others:** Segon quadrimestre:  
ANDREW MICHAEL CLARKE - A  
PAU MARTIN DE LA TORRE - A

### PRIOR SKILLS

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Knowledge of calculus, algebra and ordinary differential equations.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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**Specific:**

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

**Transversal:**

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

### TEACHING METHODOLOGY

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Standard exposition in front of the blackboard, resolution of exercises, completion of a project and attendance to the JISD summer school <http://www.ma1.upc.edu/recerca/jisd>

### LEARNING OBJECTIVES OF THE SUBJECT

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To comprehend the basic foundations of the theory of Hamiltonian systems, and to understand its applications to Celestial Mechanics and other fields.





## STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

Total learning time: 187.5 h

## CONTENTS

### Hamiltonian formalism

**Description:**

Hamiltonian dynamical systems: symplectic maps, symplectic manifolds. Linear Hamiltonian systems and their application to the study of stability of equilibrium points. Canonical transformations.

**Full-or-part-time:** 28h

Theory classes: 10h

Self study : 18h

### Celestial mechanics

**Description:**

The two body problem, first integrals. Resolution. The three body problem, different coordinates. The restricted three body problem. Central configurations. Periodic orbits, invariant manifolds.

**Full-or-part-time:** 34h

Theory classes: 12h

Self study : 22h

### Geometric theory and invariant objects of Hamiltonian systems

**Description:**

Continuous and discrete dynamical systems, Poincaré map. Flow box Theorem. Noether Theorem. Periodic orbits. Continuation of periodic orbits. Lyapunov Center Theorem.

**Full-or-part-time:** 24h

Theory classes: 8h

Self study : 16h

### Integrable systems

**Description:**

Complete integrability and Liouville-Arnold theorem. Action-Angle coordinates. Quasi-periodic flows on a torus, resonances.

**Full-or-part-time:** 10h

Theory classes: 4h

Self study : 6h



### Quasi-integrable Hamiltonian systems

**Description:**

Examples of quasi-integrable systems. Small divisors and Diophantine inequalities. Averaging Theory. Lie Method. KAM Theory (Kolmogorov-Arnold Moser). Effective stability and Nekhoroshev theorem. Melnikov Potential. Arnold diffusion.

**Full-or-part-time:** 26h

Theory classes: 8h

Self study : 18h

### Lagrangian systems and variational methods

**Description:**

Lagrangian systems. Legendre transformation. Principle of minimal action. Twist maps. Existence of periodic orbits. Aubry-Mather Theory.

**Full-or-part-time:** 12h

Theory classes: 4h

Self study : 8h

### Hamiltonian Partial Differential Equations

**Description:**

Linear Hamiltonian Partial Differential Equations. Examples. Periodic, quasi-periodic and almost-periodic solutions. Nonlinear Hamiltonian Partial Differential Equations. Lyapunov stability/instability of invariant objects. Transfer of energy.

**Full-or-part-time:** 4h

Theory classes: 2h

Self study : 2h

### - Interactions between Dynamical Systems and Partial Differential Equations

**Description:**

Summer School and Research workshop on topics between Dynamical Systems and Partial Differential Equations

**Full-or-part-time:** 49h 30m

Theory classes: 12h

Self study : 37h 30m

## ACTIVITIES

### JISD summer school

**Description:**

Attendance to the JISD summer school

**Specific objectives:**

To learn from outstanding researchers a view of the state of the art in several research topics, interacting with students of the rest of Spain and of the World.



## GRADING SYSTEM

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The students have to do some problems (60%) and a research work (25%). There will be also a final exam covering on the theoretical part of the subject (15%). Moreover, they will attend the JISD.

## BIBLIOGRAPHY

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### Basic:

- Meyer, Kenneth R.; Hall, Glen R.; Offin, Dan. Introduction to Hamiltonian dynamical systems and the n-body problem [on line]. 2nd ed. New York: Springer-Verlag, 2009 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-09724-4>. ISBN 9780387097237.
- Arnold, V. I.; Kozlov, Valerii V.; Neishtadt, Anatoly I. Mathematical aspects of classical and celestial mechanics [on line]. 3rd ed. Berlin: Springer-Verlag, 2006 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-540-48926-9>. ISBN 3540282467.
- Treschev, Dmitry; Zubelevich, Oleg. Introduction to the perturbation theory of Hamiltonian systems [on line]. Berlin: Springer Verlag, 2010 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-642-03028-4>. ISBN 9783642030277.
- Celletti, Alessandra. Stability and chaos in celestial mechanics [on line]. Springer-Praxis, 2010 [Consultation: 10/07/2023]. Available on: <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=993277>. ISBN 9783540851455.
- Wintner, Aurel. The Analytical foundations of celestial mechanics. Dover Publications, ISBN 978-0486780603.
- Katok, Anatole; Hasselblatt, Boris. Introduction to the modern theory of dynamical systems. Cambridge [etc.]: Cambridge University Press, 1997. ISBN 9780521575577.
- Berti, Massimiliano. Nonlinear oscillations of Hamiltonian PDEs [on line]. Boston, MA: Birkhäuser Boston, Inc, 2007 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-8176-4681-3>. ISBN 9780817646806.
- Marsden, Jerrold E; Ratiu, Tudor S. Introduction to mechanics and symmetry : a basic exposition of classical mechanical systems. 2a ed. New York [etc.]: Springer, 1999. ISBN 9780387986432.
- Kanuf, Andreas. Mathematical physics : classical mechanics. 1. Springer-Verlag, 2018. ISBN 9783662557723.

## RESOURCES

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### Hyperlink:

- Grup de sistemes dinàmics <https://recerca.upc.edu/sd>. Pàgina web del Grup de Sistemes Dinàmics de la UPC on es descriuen diversos projectes i els investigadors que hi treballen així com diverses activitats relacionades



# Course guide

## 200900 - ML - Machine Learning

**Last modified:** 11/04/2024

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 723 - CS - Department of Computer Science.  
715 - EIO - Department of Statistics and Operations Research.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).  
MASTER'S DEGREE IN STATISTICS AND OPERATIONS RESEARCH (Syllabus 2013). (Optional subject).

**Academic year:** 2024    **ECTS Credits:** 7.5    **Languages:** English

### LECTURER

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**Coordinating lecturer:** LUIS ANTONIO BELANCHE MUÑOZ

**Others:** Segon quadrimestre:  
LUIS ANTONIO BELANCHE MUÑOZ - A  
PEDRO FRANCISCO DELICADO USEROS - A

### PRIOR SKILLS

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The student should have knowledge of fundamental mathematical and statistical topics, such as linear algebra, calculus, probability distributions, optimization algorithms and basic multivariate statistical methods.

### REQUIREMENTS

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The student should have knowledge of basic machine learning concepts. These concepts can be acquired simultaneously, for example being enrolled in the "Statistical Learning" subject offered in the MESIO master. It is not recommended that the student acquires this knowledge from the course itself.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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**Specific:**

MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.

MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

### TEACHING METHODOLOGY

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**On-Site Learning:** On-site learning will be organized into theoretical-practical sessions. All these sessions will be held in a standard classroom; students should bring their own laptops to the classroom. Lectures will normally combine a 75% of expository classes and a 25% of guided practical work. In the expository part of the sessions, the theoretical aspects are presented, discussed and accompanied by practical examples, using slides that will be previously supplied to the student. The fundamental work environment of the practical part of the sessions will be R, of which an intermediate knowledge is presumed (use of the environment and basic programming).

**Off-Site Learning:** Off-site learning will consist of the study and resolution of (mainly practical) problems that the student should turn in throughout the course. Some of these exercises will require completion of programming tasks in R and preparation of short reports using RMarkdown (or a similar tool).



## LEARNING OBJECTIVES OF THE SUBJECT

Upon completion of the course, the student should have acquired advanced competences on general topics and techniques for statistical machine learning (particularly for regularization-based learning algorithms) as well as for unsupervised learning, particularly data visualization. The student should be able to produce machine learning solutions for many complex problems, including those in which a reduction of dimension is necessary, those where the data comes as variables of different mixed types, or those where the number of variables greatly exceeds the number of observations, such as problems typically found in genomics.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

**Total learning time:** 187.5 h

## CONTENTS

### Introduction to unsupervised learning

**Description:**

Definition and illustrative examples of unsupervised learning

**Full-or-part-time:** 2h

Theory classes: 2h

### Nonlinear dimensionality reduction

**Description:**

- a. Principal curves.
- b. Local Multidimensional Scaling.
- c. ISOMAP.
- d. t-Stochastic Neighbor Embedding.
- e. Applications

**Full-or-part-time:** 8h

Theory classes: 4h

Laboratory classes: 4h

### Dimensionality reduction with sparsity

**Description:**

- a. Matrix decompositions, approximations, and completion.
- b. Sparse Principal Components and Canonical Correlation.
- c. Applications

**Full-or-part-time:** 8h

Theory classes: 4h

Laboratory classes: 4h



### Gentle introduction to machine learning

**Description:**

Introduction to Bayesian thinking for machine learning. Learning by solving a regularized problem. Illustrative example.

**Full-or-part-time:** 5h

Theory classes: 2h

Practical classes: 3h

### Learning in functional spaces

**Description:**

Reproducing kernel Hilbert spaces. The representer theorem. Example 1: Kernel ridge regression. Example 2: The Perceptron and the kernel Perceptron.

**Full-or-part-time:** 8h

Theory classes: 4h

Practical classes: 4h

### Kernel functions in $\mathbb{R}^d$

**Description:**

Description and demonstration of fundamental kernel functions in  $\mathbb{R}^d$ . Polynomial and Gaussian kernels. General properties of kernel functions.

**Full-or-part-time:** 4h

Theory classes: 2h

Practical classes: 2h

### The support vector machine for classification, regression and novelty detection

**Description:**

The support vector machine (SVM) is the flagship in kernel methods. Its versions for classification, regression and novelty detection are explained and demonstrated.

**Full-or-part-time:** 6h

Theory classes: 4h

Practical classes: 2h

### Kernel functions for different data types

**Description:**

Some kernel functions for different data types are presented and demonstrated, such as text, trees, graphs, categorical variables, and others.

**Full-or-part-time:** 6h

Theory classes: 4h

Practical classes: 2h



### Other kernel-based learning algorithms

**Description:**

Additional kernel-based learning methods are explained, such as kernel PCA and kernel FDA. These are illustrated in several application examples. Finally, recent research on the fusion of deep neural networks and kernel functions will be presented, if time permits.

**Full-or-part-time:** 5h

Theory classes: 3h

Practical classes: 2h

### Advanced ideas and techniques in kernel-based learning methods

**Description:**

Other advanced methods are briefly introduced, such as the RVM and GPs. Nyström acceleration and random Fourier features. Introduction to the idea of Deep Kernel Learning

**Full-or-part-time:** 2h

Theory classes: 2h

## GRADING SYSTEM

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The grading method will be based in two basic marks, as follows:

- 1) Practical work done throughout the course: 50%
- 2) Final exam: 50%

The practical work will consist in a term project as well as several exercises, all of which can be done in groups (their format will be specified onsite), but the exam is completed as an individual task.

## EXAMINATION RULES.

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The precise format for the exam will be specified with sufficient advance. It may include restrictions on the allowed knowledge sources, such as written notes, books, internet connection, etc.



## BIBLIOGRAPHY

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### Basic:

- Schölkopf, Bernhard; Smola, Alexander J. Learning with Kernels : support vector machines, regularization, optimization, and beyond. Cambridge ; London: The MIT Press, cop. 2002. ISBN 9780262194754.
- Bishop, Christopher M. Pattern recognition and machine learning. New York: Springer, cop. 2006. ISBN 9780387310732.
- Vapnik, Vladimir Naumovich. The Nature of statistical learning theory. 2nd ed. New York ; Barcelona [etc.]: Springer, cop. 2000. ISBN 9780387987804.
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<https://www-cambridge-org.recursos.biblioteca.upc.edu/core/books/kernel-methods-and-machine-learning/4B52092A98E1553A26EB5271D832D29E>. ISBN 9781107024960.
- Hastie, Trevor; Tibshirani, Robert; Wainwright, Martin. Statistical learning with sparsity : the lasso and generalizations [on line]. CRC raton, 2015 [Consultation: 28/06/2023]. Available on :  
<https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=4087701>. ISBN 78149871216.
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<https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-84858-7>. ISBN 9780387848587.

### Complementary:

- Smola, Alexander J. Advances in large margin classifiers [on line]. Cambridge, Mass: MIT Press, 2000 [Consultation: 28/06/2023]. Available on :  
<https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=5966303>. ISBN 9780262194488.



# Course guide

## 34958 - MMPDE - Mathematical Modelling with Partial Differential Equations

Last modified: 16/05/2024

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 749 - MAT - Department of Mathematics.  
751 - DECA - Department of Civil and Environmental Engineering.  
**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2024    **ECTS Credits:** 7.5    **Languages:** English

### LECTURER

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**Coordinating lecturer:** MATTEO GIACOMINI  
**Others:** Primer quadrimestre:  
JEZABEL CURBELO HERNANDEZ - A  
MATTEO GIACOMINI - A  
JOSE JAVIER MUÑOZ ROMERO - A

### PRIOR SKILLS

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Good knowledge of calculus techniques, including integral theorems. Elementary solution of ODEs and PDEs.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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#### Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

#### Transversal:

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

### TEACHING METHODOLOGY

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The course mainly consists of theoretical lectures, but it also includes problem solving and computer sessions, with numerical codes provided to illustrate the behaviour of the models.



## LEARNING OBJECTIVES OF THE SUBJECT

Many phenomena taking place in physical and engineering systems can be described by means of systems of partial differential equations (PDEs). This is, e.g., the case of heat transfer phenomena, contaminant transport, fluid dynamics, mechanics of complex media (inhomogeneous materials, metamaterials, ...), and wave propagation, just to name a few. The module provides a general overview on the use of PDEs to construct mathematical models of physical phenomena and engineering systems.

The module aims to provide a first introduction to the mathematical tools to model physical phenomena, rigorously setting the governing equations describing the engineering systems under analysis. By the end of the module, students are expected to be able to:

- describe basic physical phenomena using PDEs;
- provide intuitive interpretation of the mathematical operators appearing in a PDE;
- predict the physical behaviour of an engineering system in view of its mathematical description using PDEs.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

**Total learning time:** 187.5 h

## CONTENTS

### Introduction

**Description:**

Motivation and examples. Review of fundamental mathematical tools for the module. Primal, dual, and mixed formulations.

**Full-or-part-time:** 6h

Theory classes: 6h

### Potential theory

**Description:**

Potential energy. Classic gravitation. Electrostatics. Minimal surfaces.

**Full-or-part-time:** 6h

Theory classes: 6h

### Potential flows

**Description:**

Ideal flows. Complex analysis theory. Conformal maps.

**Full-or-part-time:** 12h

Theory classes: 12h



### Convection-diffusion-reaction phenomena

**Description:**

Fourier and Fick laws. Heat and diffusion equations. Random walks. Convection-diffusion-reaction equations. Turing patterns.

**Full-or-part-time:** 12h

Theory classes: 12h

### Continuum mechanics

**Description:**

Conservation laws. Constitutive laws. Elasticity. Viscoelasticity.

**Full-or-part-time:** 6h

Theory classes: 6h

### Fluid dynamics

**Description:**

Viscous incompressible flows. Highly viscous flows. Inviscid compressible flows.

**Full-or-part-time:** 12h

Theory classes: 12h

### Porous media flow

**Description:**

Heterogeneous materials. Homogenisation theory. Darcy's law.

**Full-or-part-time:** 6h

Theory classes: 6h

## GRADING SYSTEM

The grade of the course is obtained from a continuous assessment during the module. This consists of several activities of incremental training, carried out during the module, both in and out of the classroom.

The final grade will be computed as follows:

- 35% exam 1;
- 35% exam 2;
- 30% continuous evaluation (assignments and exercises).

## EXAMINATION RULES.

The assignments must be submitted via ATENaA by the announced deadline. Late submissions or assignments submitted using other means will not be accepted and will be graded 0. The assignments must be performed individually; students are encouraged to discuss about the assignments but the submitted work must be the result of one own effort. Plagiarism in the assignments will be punished with a 0 in the classwork grade.

The written exams must be performed individually and will be closed-book. Plagiarism will be punished with a 0 in the module grade.



## BIBLIOGRAPHY

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### Basic:

- Salsa, Sandro. Partial differential equations in action : from modelling to theory [on line]. Milan [etc.]: Springer, cop. 2008 [Consultation: 07/07/2023]. Available on: <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=3062992>. ISBN 9788847007512.
- Howison, Sam. Practical applied mathematics : modelling, analysis, approximation. New York: Cambridge University Press, 2005. ISBN 0521603692.
- Ockendon, J.R. [et al.]. Applied partial differential equations. Revised ed. Oxford: Oxford University Press, 2003. ISBN 0198527713.
- Witelsky, T.; Bowen, M. Methods of mathematical modelling. Cham (Switzerland): Springer, 2015. ISBN 9783319230412.
- Chorin, A. J.; Marsden, J.E. A Mathematical introduction to fluid mechanics. 3rd ed. Springer, ISBN 978-0387979182.

### Complementary:

- Friedman, A.; Litman, W. Industrial mathematics : a course in solving real-world problems. Philadelphia: SIAM, 1994. ISBN 0898713242.
- Fowler, A.C. Mathematical models in the applied sciences. Cambridge: Cambridge University Press, 1997. ISBN 0521467039.
- Dacorogna, Bernard. Introduction to the calculus of variations [on line]. 2015 [Consultation: 07/07/2023]. Available on: <https://www.worldscientific-com.recursos.biblioteca.upc.edu/worldscibooks/10.1142/p967#t=oc>. ISBN 9781783265534.
- LeVeque, Randall J. Finite difference methods for ordinary and partial differential equations : steady state and time-dependent problems [on line]. SIAM, 2007 [Consultation: 07/07/2023]. Available on: <https://faculty.washington.edu/rjl/fdmbook/>. ISBN 9780898716290.

## RESOURCES

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### Other resources:

Teaching material including slides, class notes, lists of exercises are available on ATENa.

## Course guide

# 34960 - MMB - Mathematical Models in Biology

Last modified: 11/04/2024

<b>Unit in charge:</b>	School of Mathematics and Statistics	
<b>Teaching unit:</b>	749 - MAT - Department of Mathematics.	
<b>Degree:</b>	MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).	
<b>Academic year:</b> 2024	<b>ECTS Credits:</b> 7.5	<b>Languages:</b> English

### LECTURER

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<b>Coordinating lecturer:</b>	MARTA CASANELLAS RIUS
<b>Others:</b>	Primer quadrimestre: MARTA CASANELLAS RIUS - A JESUS FERNANDEZ SANCHEZ - A GEMMA HUGUET CASADES - A JOSE TOMAS LAZARO OCHOA - A ADRIÁN FERNANDO PONCE ÁLVAREZ - A

### PRIOR SKILLS

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- \* Proficiency in undergraduate mathematics: calculus, algebra, probability and statistics.
- \* Ability to perform basic operations in linear algebra: eigenvalues and eigenvectors, computation of determinants, rank of matrices...
- \* Ability to analyze and solve linear differential equations and discuss the stability of simple vector fields.
- \* Interest towards biological applications of mathematics and/or previous working experience.

### REQUIREMENTS

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- \* Basic knowledge of undergraduate mathematics: calculus, ordinary differential equations, linear algebra, probability and statistics.
- \* First course in ordinary differential equations: linear differential equations, qualitative and stability theory and numerical simulation.
- \* Basic knowledge of computer programming for scientific purposes.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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#### Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

### Transversal:

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

## TEACHING METHODOLOGY

The course will be structured in five blocks each consisting of a brief introduction through theoretical lectures, the development of a short project in groups and wrap-up sessions with oral presentations, discussion and complementary lectures. The central part intended to develop the short project will held at the computer lab.

## LEARNING OBJECTIVES OF THE SUBJECT

This course is an introduction to the most common mathematical models in biology: in populations dynamics, ecology, neurophysiology, sequence analysis and phylogenetics. At the end of the course the student should be able to:

- \* Understand and discuss basic models of dynamical systems of biological origin, in terms of the parameters.
- \* Model simple phenomena, analyze them (numerically and/or analytically) and understand the effect of parameters.
- \* Understand the diversity of mechanisms and the different levels of modelization of physiological activity.
- \* Obtain and analyze genomic sequences of real biological species and databases containing them.
- \* Use computer software for gene prediction, alignment and phylogenetic reconstruction.
- \* Understand different gene prediction, alignment and phylogenetic reconstruction methods.
- \* Compare the predictions given by the models with real data.
- \* Communicate results in interdisciplinary teams.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

**Total learning time:** 187.5 h

## CONTENTS

### Models of Population Dynamics

#### Description:

1. One dimensional models in ecology. A single species dynamics. One-dimensional discrete models.
2. Modelling interactions between populations with differential equations. Stability and bifurcations.
3. Epidemiology.

**Full-or-part-time:** 12h 30m

Theory classes: 2h

Laboratory classes: 2h

Self study : 8h 30m



### Mathematical Models in Neuroscience

**Description:**

1. Membrane biophysics.
2. Excitability and action potentials: the Hodgkin-Huxley model, the Morris-Lecar model, integrate & fire models.
3. Bursting oscillations.
4. Synaptic transmission and dynamics.
5. Neuronal networks. Firing rate models. Wilson-Cowan equations.
6. Applications to cognitive tasks (working memory, decision making and visual perception).
7. Recurrent Neural Networks.
8. Statistical Models.

**Full-or-part-time:** 100h

Theory classes: 16h

Laboratory classes: 16h

Self study : 68h

### Mathematical models in phylogenetics

**Description:**

1. Brief introduction to genomics and phylogenetics (genome, gen structure, alignments, evolution of species...). Retrieving genomic sequences and alignments.
2. Markov models of molecular evolution (Jukes-Cantor, Kimura, Felsenstein hierarchy...), evolutionary distances, phylogenetic trees.
3. Phylogenetic tree reconstruction: distance and character-based methods.
4. Genomics: Markov chains and Hidden Markov models for gene prediction. Tropical arithmetics and Viterbi algorithm. Forward and Expectation-Maximization algorithms. Multiple sequence alignment.

**Full-or-part-time:** 75h

Theory classes: 12h

Laboratory classes: 12h

Self study : 51h

## GRADING SYSTEM

50%: Each of the five blocks will give a part (10%) of the qualification, based on the performance on the short-projects.

20%: Overall evaluation of the participation, interest and proficiency evinced along the course.

30%: Final exam aiming at validating the acquisition of the most basic concepts of each block.

## BIBLIOGRAPHY

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### Basic:

- Allman, Elizabeth S.; Rhodes, John A. Mathematical models in biology : an introduction. Cambridge: Cambridge University Press, 2004. ISBN 9780521819800.
- Ermentrout, Bard G.; Terman, David H. Mathematical foundations of neuroscience [on line]. New York: Springer, 2010 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-87708-2>. ISBN 9780387877082.
- Istas, Jacques. Mathematical modeling for the life sciences [on line]. Berlin: Springer, 2005 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/3-540-27877-X>. ISBN 354025305X.
- Izhikevich, Eugene M. Dynamical systems in neuroscience : the geometry of excitability and bursting. Cambridge: MIT Press, 2007. ISBN 0262090430.
- Keeling, Matthew J.; Rohani, Pejman. Modeling infectious diseases in humans and animals. Princeton: Princeton University Press, cop. 2008. ISBN 9780691116174.
- Murray, J.D. Mathematical biology, vol. 1 [on line]. 3rd ed. Berlin: Springer, 2002 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/b98868>. ISBN 9780387952239.

### Complementary:

- Pachter, Lior; Sturmfels, Bernd. Algebraic statistics for computational biology. Cambridge: Cambridge University Press, 2005. ISBN 0521857007.
- Durbin, Richard [et al.]. Biological sequence analysis : probabilistic models of proteins and nucleic acids [on line]. Cambridge: Cambridge University Press, 1998 [Consultation: 10/07/2023]. Available on: <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=320915>. ISBN 0521629713.
- Feng, Jianfeng. Computational neuroscience : a comprehensive approach [on line]. Boca Raton: Chapman & Hall/CRC, 2004 [Consultation: 10/07/2023]. Available on: [http://nba.uth.tmc.edu/homepage/cnjclub/2007summer/renart\\_chapter.pdf](http://nba.uth.tmc.edu/homepage/cnjclub/2007summer/renart_chapter.pdf). ISBN 1584883626.
- Keener, James P.; Sneyd, James. Mathematical physiology. Vol 1 [on line]. 2nd ed. New York: Springer Verlag, 2009 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-75847-3>. ISBN 9780387758473.
- Miller, Paul. An Introductory course in computational neuroscience. Cambridge, Massachusetts: The MIT Press, [2018]. ISBN 9780262038256.
- Stein, William A. [et al.]. Sage mathematics software (Version 4.4.2) [on line]. 2010 [Consultation: 10/07/2023]. Available on: <http://www.sagemath.org/>.
- Bacaër, N.; Bravo de la Parra, R.; Ripoll, J. Breve historia de los modelos matemáticos en dinámica de poblaciones [on line]. [Consultation: 10/07/2023]. Available on: <https://rafaelbravodelaparra.web.uah.es/Breve%20historia%20de%20los%20modelos%20matem%C3%A1ticos%20en%20din%C3%A1mica%20de%20poblaciones-N%20Bacaer.pdf>. ISBN 9791034365883.
- Steel, Mike. Phylogeny: Discrete and random processes in evolution. CBMS-NSF Regional conference series in Applied Mathematics, SIAM, 2016. ISBN 9781611974478.
- Felsenstein, Joseph. Inferring phylogenies. Sunderland, Massachusetts: Sinauer Associates, cop. 2004. ISBN 9780878931774.





# Course guide

## 34951 - NCA - Non-Commutative Algebra

Last modified: 11/04/2024

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2024    **ECTS Credits:** 7.5    **Languages:** English

### LECTURER

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**Coordinating lecturer:** ENRIC VENTURA CAPELL

**Others:** Primer quadrimestre:  
JOSE BURILLO PUIG - A  
JORGE DELGADO RODRÍGUEZ - A  
ENRIC VENTURA CAPELL - A

### PRIOR SKILLS

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The concept of group and subgroup, and the concept of homomorphism. Basic algebraic properties, binary operations and their properties. Equivalence relations and related set-theoretic properties.

### REQUIREMENTS

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The basic algebra courses from the degree in mathematics.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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**Specific:**

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

**Transversal:**

4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

### TEACHING METHODOLOGY

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Classes follow the traditional structure of lecture by the professor, together with the assignment of problems and exercises for the students to solve and present, either in written or in oral form.



## LEARNING OBJECTIVES OF THE SUBJECT

The main goal is to introduce the student into the basic ideas and techniques of non-commutative algebra, to the extent of being able to enroll himself/herself into some initial research project in the area, if there is interest to do so.

Non-commutative algebra plays a significant role in the research panorama in modern mathematics and students of any degree in mathematics have been introduced to it. The main goal of the present topic is to go a bit deeper into this area of mathematics, offering a general but consistent introduction into the topic.

We'll center our attention towards the so-called "Geometric Group Theory", a relatively young and very active research area. This election is done because it allows to go, within a full semester, from the basics of the theory to the description, with a good level of details and context, of some open problems that are currently being object of active research today.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

**Total learning time:** 187.5 h

## CONTENTS

### Generalities about infinite groups

**Description:**

The free group: basic definitions.  
Presentations: generators and relations.  
Short exact sequences, direct and semidirect products.  
Free products, amalgams, HNN extensions.

**Full-or-part-time:** 45h

Theory classes: 15h  
Self study : 30h

### Cayley graphs and growth of groups

**Description:**

Cayley graph and growth of a group  
Quasi-isometries, geometric properties  
Growth of groups: polynomial, intermediate, exponential, uniformly exponential  
Gromow theorem

**Full-or-part-time:** 45h

Theory classes: 15h  
Self study : 30h



### Subgroups of free groups

**Description:**

- Nielsen-Schreier theorem
- Stallings graphs
- Main properties of the lattice of subgroups of a free group
- Finite index subgroups
- Pull-back and intersection of subgroups.

**Full-or-part-time:** 45h

Theory classes: 15h

Self study : 30h

### Algorithmic problems in groups

**Description:**

The three classical algorithmic problems in group theory: word, conjugacy and isomorphism problems.

Resolution in simple cases: abelian, free, free-like constructions, residually finite, etc.

Tietze transformations, an attack to the isomorphism problem

Some unsolvability results: Novikov, Miller, Mihailova, etc.

**Full-or-part-time:** 45h

Theory classes: 15h

Self study : 30h

## GRADING SYSTEM

Students will have to present in written and/or oral form some exercises assigned along the development of the course. At the end of the course, there will be a written exam with two parts: (a) the development of a theoretical topic chosen by the teachers among a closed public list of topics about the contents of the course (70%); and (b) a short list of problems/exercises about the topics of the course (30%).

## BIBLIOGRAPHY

**Basic:**

- Rotman, Joseph. An Introduction to the theory of groups. 4th ed. New York: Springer, 1995. ISBN 0387942858.
- Lyndon, Roger C.; Schupp, Paul E. Combinatorial group theory. 2nd ed. Berlin: Springer, 2001. ISBN 9783540411581.
- Bogopolskij, Oleg Vladimirovic. Introduction to group theory. Zürich: European Mathematical Society, 2008. ISBN 9783037190418.
- Loh, Clara. Geometric group theory : an introduction [on line]. Universitext. Springer, 2017 [Consultation: 10/07/2023]. Available on : <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=6312482>. ISBN 9783319722535.

**Complementary:**

- Epstein, David B.A. [et al.]. Word processing in groups. Boston: Jones and Bartlett, 1992. ISBN 9780867202441.
- Ghys, E.; La Harpe, P. de. Sur les groupes hyperboliques d'après Mikhael Gromov. Boston: Birkhäuser, 1990. ISBN 9780817635084.

## RESOURCES

**Other resources:**

Several interesting papers and notes by Chuck Miller:



<https://researchers.ms.unimelb.edu.au/cfm/papers>



## Course guide

### 34953 - NT - Number Theory

Last modified: 11/04/2024

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2024    **ECTS Credits:** 7.5    **Languages:** English

#### LECTURER

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**Coordinating lecturer:** JOAN CARLES LARIO LOYO

**Others:** Primer quadrimestre:  
JOAN CARLES LARIO LOYO - A  
BERNAT PLANS BERENGUER - A

#### PRIOR SKILLS

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Basic knowledge of algebraic structures: groups, rings and fields.

#### REQUIREMENTS

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Basic material covered in any standard courses on arithmetic, group theory and Galois theory.

#### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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**Specific:**

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

**Transversal:**

4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

#### TEACHING METHODOLOGY

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Most of the lectures will take place on the blackboard (replaced by online lessons if necessary), explaining carefully the contents of the course and providing as much explicit examples, exercises and applications as possible. The students will be encouraged to consult suitable references and to discuss between them and with the professor in order to achieve a good understanding of the material.



## LEARNING OBJECTIVES OF THE SUBJECT

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- 1) Algebraic number theory.
- 2) Number theory in function fields.
- 2) Cyclotomic theory.

## STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

**Total learning time:** 187.5 h

## CONTENTS

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### Algebraic Number Theory

**Description:**

Introduction  
Cyclotomic extensions  
Cyclotomic polynomials

**Full-or-part-time:** 10h

Theory classes: 10h

### Number Theory in function fields

**Description:**

Functions fields over finite fields  
Carlitz polynomials  
Carlitz extensions

**Full-or-part-time:** 10h

Theory classes: 10h



## Cyclotomic Theory

### Description:

Cyclotomic integers  
Cyclotomic units  
Unique factorization  
Class numbers  
Galois action  
Kronecker-Weber theorem  
Regular polygons  
Fermat equation  
Quadratic reciprocity

Carlitz modules  
Galois action  
Carlitz-Hayes theorem  
Cyclotomic and Carlitz analogies  
Quadratic reciprocity  
Drinfeld modules

**Full-or-part-time:** 40h

Theory classes: 40h

## GRADING SYSTEM

There will be a final exam.

## EXAMINATION RULES.

Solved exercises and works must be delivered according to schedule.

## BIBLIOGRAPHY

### Basic:

- Marcus, D. A. Number fields [on line]. 2a. Springer, 2018 [Consultation: 10/07/2023]. Available on: <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=6315071>. ISBN 9783319902326.
- Washington, Lawrence C. Introduction to cyclotomic fields. 2a. Springer, 1997. ISBN 9780387947624.
- Lang, Serge. Cyclotomic fields I and II. 2a. Springer, 1990. ISBN 9780387966717.
- Rosen, Michael. Number theory in function fields [on line]. Springer, 2002 [Consultation: 10/07/2023]. Available on: <https://web-s-ebshost-com.recursos.biblioteca.upc.edu/ehost/ebookviewer/ebook?sid=c3a1ddb-02a6-4928-9cf8-61fa67b451e8%40redis&vid=0&format=EB>. ISBN 9780387953359.
- Samuel, Pierre. Algebraic theory of numbers [on line]. Dover, 2008 [Consultation: 10/07/2023]. Available on: <https://web-s-ebshost-com.recursos.biblioteca.upc.edu/ehost/ebookviewer/ebook?sid=df04de0b-9ea0-463f-837a-8c3e0eaf5c3%40redis&vid=0&format=EK>. ISBN 9780486466668.
- Weiss, Edwin. Algebraic number theory [on line]. Dover, 1998 [Consultation: 10/07/2023]. Available on: <https://web-s-ebshost-com.recursos.biblioteca.upc.edu/ehost/ebookviewer/ebook?sid=41dff72-5826-4437-902d-0e8cf20a0103%40redis&vid=0&format=EK>. ISBN 9780486154367.
- Goss, David. Basic structures of function field arithmetic. Springer, 1998. ISBN 9783540635413.



## RESOURCES

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### Computer material:

- SAGE. Mathematical Software
- Matlab. Mathematical software



## Course guide

# 34964 - NMDS - Numerical Methods for Dynamical Systems

Last modified: 11/04/2024

<b>Unit in charge:</b>	School of Mathematics and Statistics	
<b>Teaching unit:</b>	749 - MAT - Department of Mathematics.	
<b>Degree:</b>	MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).	
<b>Academic year:</b> 2024	<b>ECTS Credits:</b> 7.5	<b>Languages:</b> English

### LECTURER

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<b>Coordinating lecturer:</b>	MERCEDES OLLE TORNER
<b>Others:</b>	Primer quadrimestre: MERCEDES OLLE TORNER - A

### PRIOR SKILLS

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Good knowledge of a programming language.

### REQUIREMENTS

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Knowledge of theory of systems of differential equations, algebra, calculus and numerical analysis.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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#### Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

#### Transversal:

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

### TEACHING METHODOLOGY

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Theoretical sessions (presence of the students is necessary) and weekly practical tutorized assignments.



## LEARNING OBJECTIVES OF THE SUBJECT

- To reach an advanced formation in using numerical methods applied to dynamical systems
- Carry out numerical simulations of particular examples
- To relate different aspects of the dynamics in order to have a global picture of the behavior of a given problem
- To learn different tools to analyse and deal with a problem
- Ability in programming algorithms designed to solve particular problems in dynamical systems

## STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

**Total learning time:** 187.5 h

## CONTENTS

**Numerical (preliminary) tools for practical purposes: integrators for ODE and graphical interfaces. Examples.**

**Full-or-part-time:** 4h

Theory classes: 2h

Practical classes: 2h

**Dynamical systems: introduction, definitions. Continuous and discrete dynamical systems. Orbit generation. Numerical computation of Poincare maps. Examples.**

**Full-or-part-time:** 6h

Theory classes: 3h

Practical classes: 3h

**Computation and stability of fixed points. Vector fields and maps. Implementation and examples.**

**Full-or-part-time:** 10h

Theory classes: 5h

Practical classes: 5h

**Computation of tori: representation, computation and continuation. Implementation and examples.**

**Full-or-part-time:** 15h

Theory classes: 7h 30m

Practical classes: 7h 30m

**Analysis of bifurcations. Some examples.**

**Full-or-part-time:** 15h

Theory classes: 7h 30m

Practical classes: 7h 30m



## GRADING SYSTEM

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65% of the qualification will be obtained from the practical assignments done and 35% from short exams.

## EXAMINATION RULES.

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No rules, in principle.

## BIBLIOGRAPHY

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### Basic:

- Lichtenberg, Allan J; Lieberman, M. A. Regular and stochastic motion. New York: Springer-Verlag, 1983. ISBN 0387907076.
- Press, William H. Numerical recipes in C : the art of scientific computing. 2nd. Cambridge: Cambridge University Press, 1992. ISBN 0521431085.
- Arrowsmith, D. K; Place, C. M. An introduction to dynamical systems. Cambridge: Cambridge University Press, 1990. ISBN 0521303621.
- Particular articles related to the topics of the course and some notes from suitable web pages.
- Stuart, A.M.; Humphries, A. R. Dynamical systems and numerical analysis. Cambridge University Press, 1996. ISBN 0521496721.
- Strogatz, S.H.. Nonlinear dynamics and chaos : with applications to physics, biology, chemistry, and engineering [on line]. 2nd ed. CRC Press, Taylor and Francis Group, 2018 [Consultation: 10/07/2023]. Available on: <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=1181622>. ISBN 9780813349107.

# Course guide

## 34965 - NMPDE - Numerical Methods for Partial Differential Equations

Last modified: 11/04/2024

<b>Unit in charge:</b>	School of Mathematics and Statistics	
<b>Teaching unit:</b>	751 - DECA - Department of Civil and Environmental Engineering.	
<b>Degree:</b>	MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).	
<b>Academic year:</b> 2024	<b>ECTS Credits:</b> 7.5	<b>Languages:</b> English

### LECTURER

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<b>Coordinating lecturer:</b>	SONIA FERNANDEZ MENDEZ
<b>Others:</b>	Primer quadrimestre: SONIA FERNANDEZ MENDEZ - A ABEL GARGALLO PEIRO - A SERGI PÉREZ ESCUDERO - A

### PRIOR SKILLS

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Basics on numerical methods, differential equations and calculus.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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#### Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

#### Transversal:

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

### TEACHING METHODOLOGY

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Lectures, practical work at computer room, exercises and home works.



## LEARNING OBJECTIVES OF THE SUBJECT

This course is an introduction to numerical methods for the solution of partial differential equations, with application to applied sciences, engineering and biosciences.

The course recalls the theoretical basis of the Finite Element Method (FEM) for the solution of elliptic and parabolic equations, an introduction to stabilization techniques for convection-dominated problems and the FEM for compressible flow problems, and for wave problems.

The course will include frontal lectures and exercises, as well as computer sessions aimed at introducing the bases of the programming of the numerical methods.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

**Total learning time:** 187.5 h

## CONTENTS

### Fundamentals of Finite Element Methods (FEM)

**Description:**

Basic concepts of the Finite Element Method (FEM) for elliptic and parabolic equations: strong and weak form, discretization, implementation, functional analysis tools, error bounds and convergence.

Application to the numerical modelling of flow in porous medium, and potential flow.

Introduction to a posteriori error estimation and adaptivity.

Time integration for transient problems.

Solution of the convection-diffusion equation. Stabilized formulations for convection dominated problems.

Numerical solution of linear elasticity problems.

**Full-or-part-time:** 32h

Theory classes: 16h

Laboratory classes: 16h

### FEM for incompressible flow problems

**Description:**

Weak form and discretization of the Stokes equations. Stable FEM discretizations for incompressible flow problems: LBB condition.

Introduction to the numerical solution of the incompressible Navier-Stokes equations.

**Full-or-part-time:** 14h

Theory classes: 7h

Practical classes: 7h



## FEM for wave problems

### Description:

FEM solution of Helmholtz equation. Non-reflecting boundary conditions. Application to acoustics.  
Introduction to DG for first order conservation laws.

**Full-or-part-time:** 14h

Theory classes: 7h

Laboratory classes: 7h

## GRADING SYSTEM

Exams (50%) and continuous assesment (exercises, projects and/or oral presentations) (50%).

## BIBLIOGRAPHY

### Basic:

- Wait, R.; Mitchell, A. R. Finite element analysis and applications. Chichester: John Wiley, 1985. ISBN 0471906778.
- Zienkiewicz, O.C.; Taylor, R. L. The finite element method [on line]. 6th ed. Oxford: Butterworth Heinemann, 2005 [Consultation: 07/07/2023]. Available on: <https://www-sciencedirect-com.recursos.biblioteca.upc.edu/book/9781856176347/the-finite-element-method-for-solid-and-structural-mechanics?via=ihub=>.
- Donea, Jean M; Huerta, A. Finite element methods for flow problems [on line]. Chichester: John Wiley & Sons, 2003 [Consultation: 19/05/2020]. Available on: <https://onlinelibrary.wiley.com/doi/book/10.1002/0470013826>. ISBN 0471496669.
- Ainsworth, M.; Oden, J. T. A posteriori error estimation in finite element analysis. New York: John Wiley & sons, 2000. ISBN 047129411X.
- Quarteroni, Alfio. Numerical models for differential problems [on line]. Springer, 2009 [Consultation: 07/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-88-470-1071-0>. ISBN 1280384433.

### Complementary:

- Hoffman, Joe D. Numerical methods for engineers and scientists. 2nd ed. rev. and exp. New York: Marcel Dekker, 2001. ISBN 0824704436.
- Johnson, Claes. Numerical solution of partial differential equations by the finite element. Mineola, New York: Dover Publications, 2009. ISBN 9780486469003.
- Strang, G.; Fix, G. J. An Analysis of the finite element method. Englewood Cliffs, NJ: Prentice-Hall, 1973. ISBN 0130329460.
- Trefethen, Lloyd N.; Bau, David. Numerical linear algebra. Philadelphia: SIAM, 1997. ISBN 9780898713619.

## Course guide

# 34961 - QQMDS - Quantitative and Qualitative Methods in Dynamical Systems

Last modified: 11/04/2024

<b>Unit in charge:</b>	School of Mathematics and Statistics	
<b>Teaching unit:</b>	749 - MAT - Department of Mathematics.	
<b>Degree:</b>	MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).	
<b>Academic year:</b> 2024	<b>ECTS Credits:</b> 7.5	<b>Languages:</b> English

### LECTURER

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<b>Coordinating lecturer:</b>	PAU MARTIN DE LA TORRE
<b>Others:</b>	Primer quadrimestre: INMACULADA CONCEPCION BALDOMA BARRACA - A PAU MARTIN DE LA TORRE - A

### PRIOR SKILLS

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Good knowledge of calculus, algebra and differential equations. It is strongly recommended a good understanding of the basic theory of ordinary differential equations as well as a basic knowledge of dynamical systems from a local point of view.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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#### Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

#### Transversal:

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.



## TEACHING METHODOLOGY

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We do not distinguish theoretical and practical classes. Some results about modern theory in Dynamical systems are presented in class. The main idea is to give basic knowledge and useful tools in the study of a dynamical system from both quantitative and qualitative points of view. We will stress the relation between different kind of systems and we will mainly focus in the use of perturbative techniques to study a dynamical system globally.

## LEARNING OBJECTIVES OF THE SUBJECT

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### STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

**Total learning time:** 187.5 h

## CONTENTS

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### Invariant objects in Dynamical Systems

**Description:**

Continuous and discrete Dynamical Systems.  
Poincaré map.  
Local behaviour of hyperbolic invariant objects. Conjugation.  
Invariant manifolds.

**Full-or-part-time:** 10h

Theory classes: 10h

### Normal forms

**Description:**

Poincaré-Dulac normal forms. Convergence: Poincaré and Siegel domains.

**Full-or-part-time:** 10h

Theory classes: 10h

### Perturbation theory in Dynamical Systems

**Description:**

Classic perturbation theory. Averaging theory. Perturbed homoclinic orbits in the plane. Melnikov method. Singular perturbation theory.

**Full-or-part-time:** 15h

Theory classes: 15h





### Bifurcations

**Description:**

Local bifurcations for planar vector fields and real maps. Saddle node and Hopf bifurcations.

**Full-or-part-time:** 10h

Theory classes: 10h

### Homoclinic points and chaotic Dynamics

**Description:**

Smale horseshoe. Homoclinic points and bifurcations. Hyperbolic sets and transversal homoclinic points. Dynamical systems with chaotic dynamics. Newhouse phenomenon.

**Full-or-part-time:** 10h

Theory classes: 10h

### Non-smooth systems

**Description:**

Introduction to non-smooth differential equations. Definition and motivating examples. Filipov's convention.

**Full-or-part-time:** 5h

Theory classes: 5h

## GRADING SYSTEM

The students have to do some problems (60%) and a research work (25%). There will be also a final exam covering on the theoretical part of the subject (15%). On the other hand they will attend the winter courses "Recent trends in non-linear science" and produce a document about them.

## EXAMINATION RULES.

There will be a final exam covering the theoretical part of the course.

## BIBLIOGRAPHY

**Basic:**

- Arrowsmith, D. K; Place, C. M. An Introduction to dynamical systems. Cambridge [England] ; New York: Cambridge University Press, 1990. ISBN 0-521-30362-1.
- Guckenheimer, John; Holmes, Philip. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. New York, NY [etc.]: Springer-Verlag, 1983. ISBN 0-387-90819-6.
- Katok, Anatole; Hasselblatt, Boris. Introduction to the modern theory of dynamical systems. Cambridge [etc.]: Cambridge University Press, 1995. ISBN 0521341876.
- Hasselblatt, Boris; Katok, A. B. A First course in dynamics : with a panorama of recent developments. Cambridge [etc.]: Cambridge University Press, 2003. ISBN 0521583047.
- Hirsch, Morris W.; Smale, Stephen; Devaney, Robert L. Differential equations, dynamical systems, and an introduction to chaos. 3rd Edition. Oxford: Elsevier, 2013. ISBN 9780123820105.



## Course guide

# 200901 - SAGDM - Seminar on Algebra, Geometry and Discrete Mathematics

Last modified: 08/05/2024

**Unit in charge:** School of Mathematics and Statistics

**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2024

**ECTS Credits:** 3.0

**Languages:** English

## LECTURER

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**Coordinating lecturer:** JUAN JOSÉ RUE PERNA

**Others:** Segon quadrimestre:  
JUAN JOSÉ RUE PERNA - A  
ORIOL SERRA ALBO - A  
ENRIC VENTURA CAPELL - A

## PRIOR SKILLS

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The student must know the basics on graph theory. Additionally, it is necessary to have some knowledge on probability theory, group theory and arithmetics.

## DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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### Specific:

MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.

MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.

MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

## TEACHING METHODOLOGY

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This seminar will be based on the presentation (from the professors of the course and the students) of material in the context of the theory of expanders. This material will be taken from specialized books, research papers and surveys.

## LEARNING OBJECTIVES OF THE SUBJECT

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The main objective of the seminar is to show an area of mathematics that intersects both algebra and geometry, discrete mathematics and other related areas, such as computer science, low-dimensional geometry and probability theory, among others.

The main objective is to get the student to gain a basic knowledge of expander theory, as well as the various applications in various branches of contemporary mathematics. In this direction, the student will also be encouraged to learn to conduct technical talks in public and in the preparation of technical scientific documents.



## STUDY LOAD

Type	Hours	Percentage
Self study	51,0	68.00
Hours large group	24,0	32.00

Total learning time: 75 h

## CONTENTS

### Spectral graph Theory and expanders

**Description:**

- Spectral graph theory.
- Spectral bounds: spectral gap, Mixing Lemma, Alon-Boppana. Ramanujan graphs.
- Existence of graph expanders: Margulis construction. Probabilistic constructions.
- Zig-zag product and construction of expanders.

**Full-or-part-time:** 20h

Theory classes: 6h 40m

Self study : 13h 20m

### Graph expanders and group theory

**Description:**

- Cayley graphs. Properties
- Random walks in Cayley graphs.

**Full-or-part-time:** 9h 20m

Theory classes: 6h

Self study : 3h 20m

### Graf expanders and Number Theory

**Description:**

- Sum-product phenomena. Balog-Szemerédi-Gowers Theorem.
- Classical algebraic groups and quasi-random groups.
- Expansion in  $SL_2(\mathbb{F}_q)$ : Helfgott Theorem and Bourgain-Gamburd.

**Full-or-part-time:** 12h

Theory classes: 6h

Self study : 6h



### Other applications of graph expanders

**Description:**

- Applications in knot theory: the Barzdin-Kolmogorov theorem.
- Applications in theoretical computer science: design of concentrators and coding theory.
- Applications in theoretical computing: design of algorithms.
- Analogues in Riemannian geometry.

**Full-or-part-time:** 12h

Theory classes: 6h

Self study : 6h

### GRADING SYSTEM

The grading of this seminar will be based on three points: (CA) Continuous evaluation, (MP) Material preparation and (PT) presentation.

(CA): will be based on the understanding of the material, as well as the meetings that will be held between the student and the responsible to prepare the student's presentation (or presentations). It will also include the fact of being active during the seminar sessions.

(MP): preparation of both the presentation, its good preparation and the summary sheet.

(PT): presentation. This will include questions from the teacher and students.

Overall grading of the seminar: 30% (CA)+20% (MP)+50% (PT)

### BIBLIOGRAPHY

**Basic:**

- Kowalski, Emmanuel. An Introduction to expander graphs. Société Mathématique de France, 2019. ISBN 9782856298985.
- Davidoff, Giuliana; Sarnak, Peter; Valette, Alain. Elementary number theory, group theory and Ramanujan graphs [on line]. London Mathematical Society, 2003 [Consultation: 10/07/2023]. Available on: <https://www.cambridge-org.recursos.biblioteca.upc.edu/core/books/elementary-number-theory-group-theory-and-ramanujan-graphs/7932F64548F1B38B95AA2593E0B986B2>. ISBN 9780521531436.
- Hoory, Shlomo; Linial, Nathan; Wigderson, Avi. "Expander graphs and their applications". Bulletin of the American Mathematical Society [on line]. [Consultation: 10/07/2023]. Available on: <https://www.ams.org/journals/bull/2006-43-04/S0273-0979-06-01126-8/S0273-0979-06-01126-8.pdf>.
- Krebs, Mike; Shaheen, Anthony. Expander families and Cayley graphs : a beginner's guide [on line]. Oxford University Press, 2011 [Consultation: 10/07/2023]. Available on: <https://web-p-ebshost-com.recursos.biblioteca.upc.edu/ehost/ebookviewer/ebook?sid=2edb9be3-0d08-4c98-8afd-062a20793602%40redis&vid=0&format=EB>.
- Ireland, K.; Rosen, M.. A Classical introduction to modern number theory. New York: Springer Verlag, 1990. ISBN 038797329X.

**Complementary:**

- Lubotzky, Alex. Discrete groups, expanding graphs and invariant measures. Birkhäuser, 1994. ISBN 376435075X.



# Course guide

## 200902 - SADEM - Seminar on Analysis, Differential Equations and Modelling

Last modified: 11/04/2024

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 748 - FIS - Department of Physics.  
749 - MAT - Department of Mathematics.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2024    **ECTS Credits:** 3.0    **Languages:** English

### LECTURER

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**Coordinating lecturer:** GEMMA HUGUET CASADES

**Others:** Segon quadrimestre:  
GISSELL ESTRADA RODRÍGUEZ - A  
GEMMA HUGUET CASADES - A  
JOSEP JOAQUIM MASDEMONT SOLER - A  
ALVARO MESEGUER SERRANO - A  
JOSE JAVIER MUÑOZ ROMERO - A

### PRIOR SKILLS

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It is important to have some background in one of the following areas: Dynamical Systems, Partial Differential Equations, Numerical methods and discretisation methods for PDEs.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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**Specific:**

MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.

MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.

MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.

MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

### TEACHING METHODOLOGY

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Students will present a topic related to Mathematical Analysis for Oscillatory Systems autonomously on a seminar format. This material will be taken from specialized books and research papers.

There will be some meetings between the instructor and the students before the oral presentation.

Students must prepare an abstract/report of the seminar to help the rest of the students attending the seminar to understand the topic.

Students must attend at least 90% of the lectures and be active in all the presentations.



## LEARNING OBJECTIVES OF THE SUBJECT

The main goal is to provide the students with a basic knowledge on mathematical methods for the analysis for oscillatory systems, as well as, several applications in different branches of applied mathematics. The topic intersects with dynamical systems, partial differential equations as well as numerical methods, amongst others.

Additionally, the students will learn how to conduct technical talks in public and prepare technical scientific documents.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	24,0	32.00
Self study	51,0	68.00

**Total learning time:** 75 h

## CONTENTS

### Mathematical tools for periodic orbits and stability analysis

**Description:**

Poincaré Section. Perturbation analysis. Characteristic equations

**Full-or-part-time:** 15h

Theory classes: 4h

Self study : 11h

### Biological oscillators

**Description:**

Reaction-Diffusion equations and Turing patterns. Biological oscillators in neural systems. Electrical + ion concentration fields. Hodgkin-Huxley equations.

**Full-or-part-time:** 30h

Theory classes: 10h

Self study : 20h

### Oscillations in mechanics

**Description:**

Chemo-mechanical oscillators in elasticity. Delay systems in mechanics and biology. Wave Propagation in solids and fluids: acoustics, surface waves, etc. Periodic orbits in celestial mechanics.

**Full-or-part-time:** 30h

Theory classes: 10h

Self study : 20h

## GRADING SYSTEM

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The grading of this seminar will be based on three aspects: (C) Continuous evaluation, (M) Material preparation and (P) Presentation.

(C): the grade will be based on the understanding of the material, as well as the meetings that will be held between the student and the instructor to prepare the student's presentation (or presentations). The goal of the seminar is to ensure that the audience understands the lectures and its technicalities.

(M): the grade will be based on the quality of both the presentation slides and the abstract/report.

(P): the grade will be based on the clarity of the presentation. This will include questions from the teacher and students.

Overall grading of the seminar: 30% (C) + 20% (M) + 50% (P)

## BIBLIOGRAPHY

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### Basic:

- Erneux, Thomas. Applied delay differential equations [on line]. New York, NY: Springer New York, 2009 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-74372-1>. ISBN 9780387743721.
- Murray, J. D. Mathematical biology, vol. 1 [on line]. 3rd ed. New York [etc.]: Springer, 2002-2003 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/b98868>. ISBN 9780387224374.
- Keener, James; Sneyd, James. Mathematical physiology [on line]. New York, NY: Springer New York, 2009 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-75847-3> (v.1) - <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-79388-7> (v.2). ISBN 9780387758473.
- Guckenheimer, John; Holmes, Philip. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. New York, NY [etc.]: Springer-Verlag, 1983. ISBN 0387908196.
- Strogatz, Steven H. Nonlinear dynamics and chaos : with applications to physics, biology, chemistry and engineering [on line]. 2nd ed. CRC Press, Taylor, 2018 [Consultation: 10/07/2023]. Available on: <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pg-origsite=primo&docID=1181622>. ISBN 9780813349107.