CARL FRIEDRICH GAUSS (1777-1855)

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Contents

1. Brunswick 1777-1795

Gauss was born in Brunswick (Braunschweig), which was then a residential town. It was the largest town in the princedom Braunschweig-Wolfenbüttel. There were about 30 thousand inhabitants, a court, a theater and high schools such as the Collegium Carolinum, which was founded in 1745.

There is only one source for Gauss as a child, that is Gauss himself together with Wolfgang Sartorius of Waltershausen (1809-1876). Sartorius had studied science under Gauss since 1830 and became a scientist in geophysics. In 1847 Sartorius was promoted professor of geology at the university of Göttingen, where he also was director of the mineralogical and palaeontological collections. During Gauss' late years, Sartorius became a close friend to him. One year after Gauss' death, in 1855, Sartorius published a memorial.[1]

[1] Wolfgang Sartorius von Waltershausen: *Gauss zum Gedächtniss.* Leipzig 1856; Wiesbaden 1965, Vaduz 1980. English translation by Helen W. Gauss: *A Memorial.* Colorado, 1966.

One finds there all those the nice stories about the wonder boy who was able to calculate before learning how to speak. Sartorius is also the source for the best known story about Gauss: the teacher in the elementary school wanted to keep a special group of pupils busy for a while; so he asked them to add up the numbers from 1 to 100. Shortly afterwards Gauss presented the result, 5050.

It is nearly unknown that this problem is a very old one. It belongs to the problem collection attributed to Alkuin (about 735 to 804), that is, to the time of Charlemagne (742-814; king of the Francs 768-814, and emperor of the West since 800). This problem collection, *Propositiones ad acuendos iuvenes*, is the oldest problem collection in the Latin language. The problem no. 42, "Propositio de scala habente gradus centum", is the same as the one asked by Gauss' teacher, and it has a similar solution:

> There was a ladder with 100 steps. On the first step one pigeon sat, on the second two pigeons; on the third three, on the fourth four, on the fifth five and so on, up to the hundredth step. Tell me, if you can, how many pigeons there were.
>
> Solution: Calculate in the following way. Take the one pigeon that sat on the first step on the 99th step, then there are 100 pigeons. Then put the two pigeons sitting on the second step together with the 98 pigeons sitting on the 98th step, you find 100 once again. All you have to do, is to put the pigeons from the highest step together with the pigeons from the lowest step and you will always have 100 pigeons. The 50th step, however, stays alone, there is no corresponding step. The same happens to the 100th step. If you add altogether you'll find $49 \cdot 100 + 50 + 100 = 5050$ pigeons.[2]

And there is a further source for Gauss as a schoolboy, especially for the period 1788-1792, when he was at the gymnasium. This source is Gauss' library. It has survived nearly complete and is kept safe in the manuscript department of the Göttingen university library. It contains about 1400 titles. Especially in younger years, Gauss wrote notes in his books, like the year when he bought or got the book. He also used the spare pages as rough paper. The oldest book I know, namely Cornelius

[2] Paul Leo Butzer; Dietrich Lohrmann (ed.): *Science in Western and Eastern Civilization in Carolingian Times.* Basel, Boston, Berlin 1993, p. 348f.

Nepos *De vita excellentium imperatorum* (Königsberg 1742),[3] dates from the year 1789. Incidentally, Gauss kept most or all of his Latin and Greek schoolbooks.

After the gymnasium Gauss changed to the Collegium Carolinum. In 1791 Gauss was presented to his patron, who had repeatedly payed in the past for his education and living expenses, as Gauss' parents were not rich enough. The patron was the duke of Brunswick, Carl Wilhelm Ferdinand (1735-1806), who reigned since 1780. On occasion of this event, Gauss got a book as a present: Johann Carl Schulze's *Sammlung logarithmischer, trigonometrischer und anderer zum Gebrauch der Mathemathik unentbehrlichen Tafeln* (2 vol., Berlin 1778).[4] This was the first logarithmic table that Gauss owned. Later he bought several others, so that he owned a whole collection.[5]

This present of the duke was much more than a logarithmic table would be today. It was so to speak the computer of the nineteenth century. Gauss used to write notes on the spare pages. Schulze's logarithmic table is full of notes from Gauss, and some of them concern the regular heptadecagon.

And mathematics? Gauss learnt mathematics primarily on his own. He was an autodidact. His library allows to follow the traces of his learning. He began with easy books, like the reckoning masters. For example, Gauss' library contains books of Johann Hemeling and Valentin Heins.[6] Mathematical textbooks came after. One of Gauss' favorite authors was Christian Wolff[7] and afterwards he turned to scientific mathematical literature, such as Jakob Bernoulli's *Ars conjectandi* (Basel 1713).[8] Gauss bought the *Ars conjectandi* in 1792, when he was only 14 or 15 years old.

Gauss learnt mostly by himself, and not so much from his teachers.

[3]Gauss library no. 557.

[4]Gauss library no. 31.

[5]Karin Reich: *Logarithmentafeln - Gauß "tägliches Arbeitsgerät"*. In E. Mittler (ed.): *Wie der Blitz einschlägt, hat sich das Räthsel gelöst—Carl Friedrich Gauß in Göttingen*. Göttingen 2005, p. 73-89.

[6]Gauss library no. 1061 and no. 429.

[7]Gauss library nos. 55, 262, 263, 264, 266.

[8]Gauss library no. 282.

2. Göttingen 1795-1798

The duke of Brunswick kept paying Gauss a scholarship, so Gauss was able to visit the university. He chose Göttingen, which belonged to another country, namely the electorate, and later Kingdom, of Hannover. The Göttingen university was founded in 1737, as a reform university, by the British king George II (he reigned from 1723-1760), who had also been the electorate of Hannover.

The Göttingen university library was very famous for its quality and for the fact, which was a novelty at that time, that students were allowed to borrow books. The first book that Gauss had borrowed was not a mathematical work, but a novel: *Clarissa*, by Samuel Richardson (1689-1761). A bestseller at Gauss' time, it is a triangular love story in which the young lady dies of a broken heart after being forced to marry the wrong man.

Of course Gauss also borrowed mathematical literature. He read books and articles from many authors. The number one among these was no doubt Leonhard Euler (1707-1783). Later Gauss owned many textbooks of Euler. In one of them, as it has been found only recently, he drew a sketch of Euler.[9]

On March 29, 1796, Gauss had an eureka experience. He woke up early in the morning and had the vision that the regular heptadecagon can be constructed with ruler and compass. The next day he began his diary,[10] and he had no further doubts that he would study mathematics. At that time, however, the main mathematician in Göttingen was Abraham Gotthelf Kästner (1719-1800), the author of many mathematical textbooks, and Gauss could not learn much from him as he did not rank among the best mathematicians.

[9]Gauss library no. 826. Karin Reich: *Gauss' geistige Väter: nicht nur* summus Newton*, sondern auch* summus Euler. In: E.Mittler: *Wie der Blitz einschlägt, hat sich das Räthsel gelöst—Carl Friedrich Gauß in Göttingen.* Göttingen 2005, p. 105-117.

[10]*Gauss's Mathematical Diary*, in: G. Waldo Dunnington: *Carl Friedrich Gauss. Titan of Science*, ed. by J. Gray and F. E. Dohse, The Association of America 2004, p. 469-484. See also *Carl Friedrich Gauss: Mathematisches Tagebuch 1796-1814.* Ostwalds Klassiker 256, Frankfurt am Main 2005.

In 1798, Gauss finished his thesis on the fundamental theorem in algebra,[11] and he was promoted at the university of Helmstedt, where Johann Friedrich Pfaff (1765-1825) was a well-respected mathematician. While Pfaff wrote an eulogium about Gauss' thesis,[12] Kästners announcement[13] was more or less a disaster. Shortly afterwards Gauss returned to Brunswick again, where the duke kept paying him a salary.

3. Brunswick 1798-1807

There followed the most fruitful years in Gauss' life. In 1801 he published at last his *Disquisitiones arithmeticae*, his arithmetical masterpiece concerning number theory.[14] His sources had mostly been French mathematicians and Euler.

During the 18th-century intellectual life, Germany had produced a Goethe and a Schiller, but no mathematicians comparable with those in France. French mathematics was on the top and French mathematicians played a dominant role.

In his *Disquisitiones arithmeticae* Gauss quoted 29 articles of Euler, 8 of Joseph-Louis Lagrange (1736-1813) and 2 of Adrien-Marie Legendre (1752-1833).

It is thus natural that French was the first language into which the *Disquisitiones arithmeticae* was translated: *Recherches arithmétiques* (Paris 1807). This translation was due to Antoine-Charles Marcel Poullet-Delisle (1778-1849), a mathematics teacher at the Lycée d'Orléans. However, the translation had been initiated by Pierre-Simon de Laplace (1749-1827). In the same year 1807, Louis Poinsot (1777-1859) published an extensive review.[15]

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[11] Carl Friedrich Gauss: *Demonstratio nova theorematis: Omnem functionem algebraicam, rationalem, integram, unius variabilis in factores reales primi vel secundi gradus resolvi posse.* Helmstedt 1799. In: *Gauss Werke* 3, p. 1-30.

[12] Paul Zimmermann: *Zum Gedächtniß Karl Friedrich Gauß. 1. Die Promotion in Helmstedt.* Braunschweigisches Magazin 5, 1899, p. 113-117.

[13] Göttingische Gelehrte Anzeigen 1800, p. 129-133 (January 25, 14.Stück).

[14] Carl Friedrich Gauss: *Disquisitiones arithemticae.* Leipzig 1801. In: *Gauss Werke* 1, p. 1-474.

[15] *Le Moniteur* 36, 1807, p. 312 (March 21).

It was mainly this French translation that made Gauss popular among French mathematicians. And Poinsot's review proves Gauss' popularity in France. In Germany there were nearly no mathematicians who were able to appreciate Gauss' difficult work.

In the same year 1801 Gauss became interested in astronomy, even though there was no observatory in Brunswick. So Gauss could only contribute to theoretical astronomy, as observations were out of questions. In February 1801 the Italian astronomer Giuseppe Piazzi (1746-1826) found a new planet, but lost it after some days. It was Gauss who made the rediscovery possible. Indeed, in November 1801 he presented a new way of calculation and soon afterwards, on December 7 and 31, 1801, his friends Xaver von Zach (1754-1832) and Wilhelm Olbers (1758-1840) were able to find the planetoid again. After the discovery of the second small planet, Pallas, in the year 1802, the name Gauss was also mentioned in a German newspaper.[16]

This second period in Brunswick was perhaps the most happiest and the most fruitful period in Gauss' life. But it was in November 1806 when this good time finished abruptly. His patron died, mortally wounded, some days after the battle of Jena and Auerstedt. Now Gauss had a money problem. But he was lucky, as the university of Göttingen offered him a position as professor of astronomy and as director of the observatory, which he accepted.

4. Göttingen 1807-1855

In autumn 1807 Gauss moved, together with his family, to Göttingen. For the first time in his life, he had his own observatory, though it was still the old observatory of Tobias Mayer (1723-1762). But there was war between the German countries and Napoleon. In 1808 Göttingen was occupied and all people with a position had to pay a war contribution. Gauss asked Laplace and Lagrange for help and both offered him the money. So also Gauss astronomical colleague, Ludwig Harding

[16] *Vossische Zeitung*, June 24, 1802.

(1765-1834), was able to benefit from the generous attitude of Laplace and Lagrange.[17]

The year 1809 was a disaster for Gauss.

The beginning was promising, despite the war, Gauss' astronomical main work *Theoria motus corporum coelestium in sectionibus conicis solem ambientium* (Theory of the motion of the heavenly bodies moving about the sun in conic sections) appeared in Hamburg in the early summer of 1809. But in October, Gauss' wife Johanna (1780-1809) gave birth to her third child and died. The only solution for a man with small children at this time was to get married again. And this happened in summer 1810, Gauss' second wife was Minna Waldeck (1788-1831).

4.1. **Colleagues at the university of Göttingen.** When Gauss went to Göttingen, Ludwig Harding was professor of astronomy at the university (he was appointed after his discovery of the third little planet, Juno, in 1805). So when Gauss arrived, there were two astronomers in Göttingen. On the other hand, there was only one mathematician, his name was Bernhard Thibaut (1775-1832). He had written several small articles and had published two textbooks, which were successful:

1) *Grundriß der reinen Mathematik* (Outline of pure mathematics). Göttingen 1801, with a fith edition in 1831.
2) *Grundriß der allgemeinen Arithmetik oder Analysis* (Outline of the general arithmetic or analysis). Göttingen 1809, second edition in 1830.

Thibaut was an extremely successful teacher. Students reported that he spoke like Goethe. At a time when the whole university had about 200 students, Thibaut's lectures were attended by one-hundred twenty students or more. Most of the students did not even study mathematics, but they just wanted to hear Thibaut. He really was an extraordinary teacher, with an outstanding teaching ability.[18]

[17]Karin Reich: *Im Umfeld der Theoria motus: Gauß' briefwechsel mit Perthes, Laplace, Delambre und Legendre.* Göttingen 2001, p. 80 and 134.

[18]Karin Reich: *Bernhard Friedrich Thibaut, der Mathematiker an Gauß' Seite.* Mitteilungen der Gauß-Gesellschaft 34, 1997, p. 45-62.

4.2. **Astronomy and mathematics.** Although Gauss was now professor of astronomy, he never forgot mathematics, on the contrary, mathematics played the most important role, even when he worked in other fields. Let us have a closer look at his astronomical investigations, mainly his determination of the orbits of planets.

As Gauss himself and his colleagues had often pointed out, the main chapters of his *Theoria motus* were paragraphs 99 and 100. Here he determined the orbit by means of a transcendental equation. That was not new at this time, this result was also achieved by other astronomers. But Gauss solved this transcendental equation by means of continuous fractions, and these converge very fast. So Gauss was able to achieve a very good result by means of a small amount of terms.

Some time later he picked up this theory once again and in 1812 he published a very important article about the so-called hypergeometric series: *Allgemeine Untersuchungen über die unendliche Reihe*

$$1+\frac{\alpha\beta}{1\cdot\gamma}x+\frac{\alpha(\alpha+1)\beta(\beta+1)}{1\cdot2\cdot\gamma(\gamma+1)}xx+\frac{\alpha(\alpha+1)(\alpha+2)\beta(\beta+1)(\beta+2)}{1\cdot2\cdot3\cdot\gamma(\gamma+1)(\gamma+2)}x^3+\text{etc.}$$

(General investigations about infinite series).[19] In this paper Gauss also advanced the investigation of the so-called Gamma-function, which had been introduced and studied by Euler.

4.3. **Surveying and mathematics.** In 1820 the King of Hannover and Great Britain, George IV (he reigned from 1820-1830) gave the order to survey the kingdom of Hannover. Gauss started immediately and presented the first results in 1823.[20] But while surveying he also published theoretical or purely mathematical papers which arose from his practical work.

In 1825 he published a first mathematical result: *Allgemeine Auflösung der Aufgabe die Theile einer gegebnen Fläche so abzubilden, dass die Abbildung dem Abgebildeten in den kleinsten Theilen ähnlich wird* (General solution of the problem of how to represent the parts of a given

[19] In: *Gauss Werke* 3, p. 123-162.

[20] Astronomische Nachrichten 1, 1823, Beylage zu no. 24.

surface on another given surface, so that the smallest parts of the representation are similar to the corresponding parts of the surface represented).[21] Here Gauss introduced for the first time the term "conformal" and developed a theory of the "conformal mapping". For this work he was awarded a medal from the Danish Academy in Copenhagen.

Three years later, in 1828, he presented his main contribution to differential geometry *Disquisitiones generales circa superficies curvas* (General investigations of curved surfaces),[22] where he introduced a new definition of the measure of curvature and a new definition of a surface which was equivalent to a two-dimensional manifold:

> When a surface is regarded, not as the boundary of a solid, but as a flexible, though not extensible solid, one dimension of which is supposed to vanish, then the properties of the surface depend in part upon the form to which we can suppose it reduced, and in part are absolute and remain invariable, whatever may be the form into which the surface is bent. To these latter properties, the study of which opens to geometry a new and fertile field, belong the measure of curvature and the integral curvature, in the sense which we have given to these expressions. To these also belong the theory of shortest lines, and a great part of what we reserve to be treated later. From this point of view, a plane surface and a surface developable on a plane, e.g., cylindrical surfaces, conical surfaces, etc., are to be regarded as essentially identical; and the generic method of defining in a general manner the nature of the surfaces thus considered is always based upon the formula
>
> $$\sqrt{Edp^2 + 2Fdpdq + Gdq^2},$$
>
> which connects the linear element with the two indeterminates p, q (§ 13).

4.4. **Physics and mathematics.** When Wilhelm Weber (1804-1891) became professor of physics at the university of Göttingen in 1831, a

[21] Astronomische Abhandlungen, Heft 3, 1825, p. 1-30. In: *Gauss Werke* 4, p. 189-216.

[22] Commentationes societatis regiae scientiarum Gottingensis recentiores 6, p. 99-146. In: *Gauss Werke* 4, p. 217-258.

new period began in Gauss' life. He had had interests for physics, especially electricity and magnetism, for a long time, but he needed help in creating experiments. The duo Gauss and Weber was exceptional, as Gauss' interest was a more theoretical one and Weber was able to add the practical know-how. In 1833 Gauss finished his *Intensitas vis magneticae terrestris ad mensuram absolutam revocata* (The intensity of the terrestrial magnetic force reduced to absolute measurement).[23] Earth magnetism became a worldwide international investigation whose heart was located in Göttingen. Gauss introduced the term "potential" and made successful use of the spherical functions within his physical investigations.

5. Gauss as a teacher

As professor of astronomy Gauss normally gave lectures only in astronomy; his most common lectures were:

75 times "Practical astronomy"
19 times "Methods of least-squares"
18 times "Instruments, measurements and calculations in surveying"
18 times "Theory of the motion of comets"
11 times "The theory of magnetical phenomena".

He gave lectures in mathematical disciplines only in rare occasions, like:

WS 1809: Selected chapters in number theory
SS 1819, WS 1823/4, WS 1826/7: Theory of probability and its applications
SS 1827, WS 1827/8, WS 1829/30, WS 1931/2: Surface theory
SS 1833 and WS 1833/4: Theory of numerical equations

However, it is rather astonishing that many of the large number of Gauss' students became well-known mathematicians. In alphabetical order:

[23]Commentationes societatis regiae scientiarum Gottingensis recentiores 8, 1841, p. 3-44. In: *Gauss Werke* 5, p. 79-118.

Moritz Cantor	Heinrich Lübsen	Moritz Abraham Stern
Richard Dedekind	Michel Reiss	Johann Tellkampf
Julius Deicke	Bernhard Riemann	Rasmus Thune
Enno Dirksen	Ernst Schering	Friedrich Toennies
Alfred Enneper	Heinrich Scherk	Georg Ulrich
Johann Gebauer	Ludwig Schnürlein	Ludwig Wachter
Carl Graeffe	Christian Schnuse	Johann Heinrich Westphal
Johann August Grunert	Friedrich Spehr	Theodor Wittstein
Heinrich Eduard Heine	Karl von Staudt	

When Gauss died, his chair at the university of Göttingen was divided: Ernst Friedrich Wilhelm Klinkerfues (1827-1884) succeeded Gauss in astronomy and Peter Lejeune-Dirichlet (1805-1859) succeeded Gauss in mathematics.

Dirichlet was followed by Bernhard Riemann (1860-1866), Alfred Clebsch (1833-1872), Lazarus Fuchs (1874-1875), Hermann Amandus Schwarz (1875-1892), Heinrich Weber (1892-1895), David Hilbert (1895-1930), Hermann Weyl (1930-1933). This is in some way a list of excellency.

One should stop here, because with the Third Reich a new time arrived, especially for the University of Göttingen.