ON THE COSMOLOGICAL CONSTANT, THE VACUUM ENERGY, AND DIVERGENT SERIES[1]

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Abstract

The cosmological constant was considered by Albert Einstein as the 'silliest mistake of his life' ("die grösste Eselei meines Lebens"). Quite on the contrary, nowadays it has turned out to be an absolutely necessary term in order to have the possibility to explain the observed acceleration in the expansion of the universe. Such term is unavoidably related with the energy of the quantum vacuum, from which it cannot be easily disentangled in the observational result: both contributions go together and are to be added in the final value. The concept of vacuum energy of a quantum system is most fundamental and appears in very different situations. In particular it becomes manifest in the so called Casimir force. When computing the corresponding physical quantities one has to deal, from the very beginning, with divergent series and determinants, which must be regularized through procedures that neither Einstein himself nor Paul Dirac (among other much distinguished physicists) could ever admit –but which have given the most outstanding approximation (to the fourteenth order) between any known physical theory and experiment to date. In mathematics this does not seem to be such a problem: some centuries ago Euler already maintained the idea that one should be able to assign to any given series a certain number, in some reasonable way. A method extraordinarily elegant and useful in order to do this (at least for a large family of divergent series) starts from the consideration of the ζ function associated with the Hamiltonian operator of the quantum system in question. It was Hawking who definitely introduced this method into physics, through a famous paper published in Communications on Mathematical Physics in 1975.

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